Subtransient & Transient Inductances
(First part of Section 4.14 in text, i.e., excluding Section 4.14.1)

We initially consider the round-rotor machine and at the end address the salient pole machine.

Direct-axis transient inductance:
Here is the scenario to consider (a sort of "thought experiment"):

1. Remove the damper windings (D and Q)
2. Short the field windings (v_F=0)
3. Rotate the rotor at synchronous speed.
4. Suddenly apply a set of balanced voltages to the stator terminals such that the phase *a* voltage is in phase with the direct axis rotation.

$$\underline{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}V \begin{bmatrix} \cos\theta \\ \cos(\theta - 120) \\ \cos(\theta + 120) \end{bmatrix} u(t) \qquad (4.164)$$

where *V* is the rms phase voltage and *u(t)* is the unit step function, as shown in Fig. 1 below.

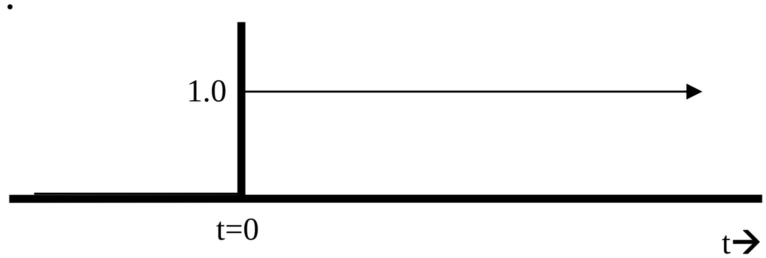

Fig. 1

Assumption: The stator winding dynamics are very fast relative to the dynamics of the field winding (T=L/R, field wdg L>>stator wdg L; field wdg R<<stator wdg R).

Implication: We know that the currents i_a, i_b, i_c appear instantaneously with application of the voltages. Because of above assumption, we can ignore the constant flux linkage theorem, CFLT (flux linkages cannot change instantaneously), and assume the stator flux linkages appear instantaneously on application of the voltages.

Then Park's transformation on $\underline{v}_{abc}$ yields

$$\underline{v}_{0dq} = \underline{P}\underline{v}_{abc} = \begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3}Vu(t) \\ 0 \end{bmatrix} \qquad (4.165)$$

Now the field winding dynamics are not fast (inductance is much larger), the CFLT applies. Thus, λ_F cannot change instantaneously.

But before application of the stator voltages, all windings were de-energized, and in particular, $\lambda_F(0)=0$. By CFLT, then, $\lambda_F(0^+)=0$.

Now recall

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_G \\ \lambda_D \\ \lambda_Q \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & kM_F & 0 & kM_D & 0 \\ 0 & 0 & L_q & 0 & kM_G & 0 & kM_Q \\ 0 & kM_F & 0 & L_F & 0 & M_R & 0 \\ 0 & 0 & kM_G & 0 & L_G & 0 & M_Y \\ 0 & kM_D & 0 & M_R & 0 & L_D & 0 \\ 0 & 0 & kM_Q & 0 & M_Y & 0 & L_Q \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_G \\ i_D \\ i_Q \end{bmatrix} \text{eq. (4.20)}$$

from which we extract the expression for λ_F, which is

$$\lambda_F = kM_F i_d + L_F i_F + M_R i_D$$

But recall we have removed the damper windings, so that $i_D=0$, then

$$\lambda_F = kM_F i_d + L_F i_F$$

Then at $t=0^+$, we have that

$$\lambda_F(0^+) = 0 = kM_F i_d + L_F i_F$$

and so we see that

$$i_F = \frac{-kM_F i_d}{L_F} \qquad (4.172)$$

Again, from eq. (4.20) above, we have that

$$\lambda_d = L_d i_d + kM_F i_F + kM_D i_D$$

But $i_D=0$,

$$\lambda_d = L_d i_d + kM_F i_F$$

and substitution of (4.172) yields

$$\lambda_d = L_d i_d - \frac{(kM_F)^2 i_d}{L_F} = \underbrace{\left(L_d - \frac{(kM_F)^2}{L_F} \right)}_{L'_d} i_d$$

The term in brackets in the last equation is defined as the d-axis transient inductance according to

$$L'_d \equiv L_d - \frac{(kM_F)^2}{L_F}$$

And with $L_{AD}=kM_F$ (from 4.108), we obtain

$$L'_d \equiv L_d - \frac{L_{AD}^2}{L_F} \qquad (4.174)$$

L'_d gives the initial d-winding flux linkage per unit of d-winding current with no damper winding, and with the field winding shorted.

In per-unit, since $\omega=1$, $X'_d = L'_d$.

<u>Direct-axis subtransient inductance</u>:

The dynamics of the damper windings are faster than those of the field circuit but not as fast as the stator circuits. It increases model accuracy to account for the effect of the damper windings.

To do so, we repeat the above procedure but

- We cannot let $i_D=0$
- We must also use the λ_D equation from eq. (4.20)
- We use $\lambda_D(0^+)=0$ and $\lambda_F(0^+)=0$, by CFLT

See equations (4.166)-(4.171), resulting in

$$L_d'' = L_d - \frac{(kM_F)^2 L_D + (kM_D)^2 L_F - 2kM_F kM_D M_R}{L_F L_D - M_R^2} \qquad (4.170)$$

With $L_{AD}=kM_F=kM_D=M_R$ (eq. 4.108), we get

$$L_d'' = L_d - \frac{L_D + L_F - 2L_{AD}}{\dfrac{L_F L_D}{(L_{AD})^2} - 1} \qquad (4.171)$$

Again, in pu, $L_d'' = X_d''$.

<u>Quadrature-axis transient and subtransient inductances</u>:

Similar analysis as before can be done to obtain L'_q and L''_q.
However, for the q-axis inductance, we must impress voltages on the stator terminals such that v_q becomes non-zero and v_d becomes zero. This is done by impressing voltages on the stator terminals such that the phase a voltage is 90° behind the voltages impressed for the d-axis inductances. In other words, instead of using (4.164),

$$\underline{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}V \begin{bmatrix} \cos\theta \\ \cos(\theta-120) \\ \cos(\theta+120) \end{bmatrix} u(t) \qquad (4.164)$$

we will use

$$\underline{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}V \begin{bmatrix} \sin\theta \\ \sin(\theta-120) \\ \sin(\theta+120) \end{bmatrix} u(t)$$

Then Park's transformation on $\underline{v}_{abc}$ yields

$$\underline{v}_{0dq} = \underline{P}\underline{v}_{abc} = \begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \sqrt{2}V\begin{bmatrix} 0 \\ 0 \\ \sqrt{3}Vu(t) \end{bmatrix}$$

I derive the transient inductance, as follows.

$$\lambda_G = kM_G i_q + M_Y i_Q + L_G i_G$$

Let's assume there is no q-axis damper winding, so i_Q=0. Then, at t=0$^+$,

$$\lambda_G(0^+) = 0 = kM_G i_q + L_G i_G \qquad \Rightarrow \qquad i_G = \frac{-kM_G}{L_G} i_q$$

Again, from 4.20, we have that

$$\lambda_q = L_q i_q + kM_Q i_Q + kM_G i_G$$

and with no damper, $i_Q=0$, and

$$\lambda_q = L_q i_q + kM_G i_G$$

Substitution for i_G from above expression yields:

$$\lambda_q = L_q i_q - kM_G \frac{kM_G}{L_G} i_q = \underbrace{\left(L_q - \frac{(kM_G)^2}{L_G} \right)}_{L_q'} i_q$$

From this last expression we observe that $L_q' = L_q - \frac{(kM_G)^2}{L_G}$

Recalling that $L_{AQ}=kM_G$, we have that

$$L_q' = L_q - \frac{{L_{AQ}}^2}{L_G} \tag{4.180}$$

Repeating the above procedure but

- We cannot let $i_Q=0$
- We must also use the λ_Q equation from eq. (4.20)
- We use $\lambda_Q(0^+)=0$ and $\lambda_G(0^+)=0$, by CFLT

See equations (4.175)-(4.179a), resulting in

$$L_q'' = L_q - \frac{L_G(\mathrm{k}M_G)^2 + (\mathrm{k}M_G)^2 L_Q - 2\left(\mathrm{k}\,M_Q\right)\left(\mathrm{k}\,M_G\right)\left(M_Y\right)}{L_G L_Q - M_Y^2} \tag{4.179a}$$

With $L_{AQ}=kM_G=kM_Q=M_Y$ (eq. 4.109), we get

$$L_q'' = L_q - \frac{L_Q + L_G - 2L_{AQ}}{\frac{L_G L_Q}{L_{AQ}^{\;2}} - 1} \tag{4.179b}$$

SUMMARY:

$$L_d' \equiv L_d - \frac{L_{AD}^2}{L_F} \tag{4.174}$$

$$L_d'' = L_d - \frac{L_D + L_F - 2L_{AD}}{\frac{L_F L_D}{(L_{AD})^2} - 1} \tag{4.171}$$

$$L_q' = L_q - \frac{L_{AQ}^{\;2}}{L_G} \tag{4.180}$$

$$L_q'' = L_q - \frac{L_Q + L_G - 2L_{AQ}}{\frac{L_G L_Q}{L_{AQ}^{\;2}} - 1} \tag{4.179b}$$

Final comments:

1. Remember that in these notes we developed the inductances used within the so-called "standard" parameter set from inductances used within the so-called "fundamental" (sometimes also called "basic") parameter set. A good way to remember the difference between these two sets of parameters is that the fundamental parameters are represented by the equivalent circuits of Figures 4.5 and 4.6 in VMAF (and also shown on pp. 33-34 of the "perunitization" notes). The standard parameter set, which can be expressed as a function of the fundamental set (as we have done in these notes), can be obtained via short-circuit, decrement, and standstill frequency response tests. In contrast, as Kundur writes on p. 139 of his text, "While the fundamental

parameters completely specify the machine electrical characteristics, they cannot be directly determined from measured responses of the machine." You should review the "Last comments" on p. 35 of the "perunitization" notes and the attached section 4.16 (from VMAF) in those notes.

2. In the above, we obtained the transient quantities by ignoring the damper windings. Because the damper windings have "fast" dynamics, we may say that transient quantities characterize the machine following decay of those "fast" dynamics, i.e., it characterizes the machine a few cycles after the initialization of the disturbance.
3. On the other hand, in the above, we obtained the subtransient quantities by including the damper windings. These quantities therefore characterize the machine during the first 2-3 cycles following the disturbance.

The figure below illustrates the difference between the subtransient and transient time frames.