

Subtransient & Transient Inductances & Time Constants (Section 4.14 in text)

We consider only a salient pole machine for now. We will come back to the round-rotor machine.

Direct-axis transient inductance:

Here is the scenario to consider (a sort of “thought experiment”):

1. Remove the damper windings (D and Q)
2. Short the field windings ($v_F=0$)
3. Rotate the rotor at synchronous speed.
4. Suddenly apply a set of balanced voltages to the stator terminals such that the phase a voltage is in phase with the direct axis rotation.

$$\underline{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}V \begin{bmatrix} \cos \theta \\ \cos(\theta - 120) \\ \cos(\theta + 120) \end{bmatrix} u(t) \quad (4.164)$$

where V is the rms phase voltage and $u(t)$ is the unit step function, as shown in Fig. 1 below.

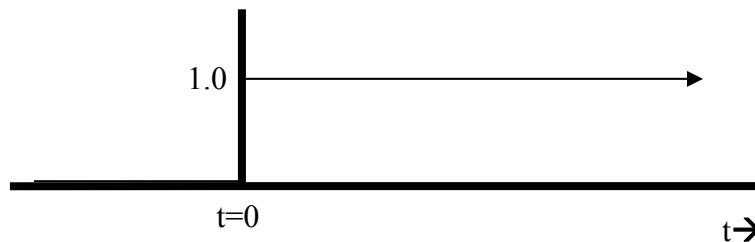


Fig. 1

Assumption: The stator winding dynamics are very fast relative to the dynamics of the field winding.

Implication: We know that the currents i_a , i_b , i_c appear instantaneously with application of the voltages. Because of above assumption, we can ignore the constant flux linkage theorem, CFLT (flux linkages cannot change instantaneously), and assume the stator flux linkages appear instantaneously on application of the voltages.

Then Park's transformation on \underline{v}_{abc} yields

$$\underline{v}_{0dq} = \underline{P}\underline{v}_{abc} = \begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \sqrt{2}V \begin{bmatrix} 0 \\ \sqrt{3}Vu(t) \\ 0 \end{bmatrix} \quad (4.165)$$

Now the field winding dynamics are not fast (inductance is much larger), the CFLT applies. Thus, λ_F cannot change instantaneously.

But before application of the stator voltages, all windings were de-energized, and in particular, $\lambda_F(0)=0$. By CFLT, then, $\lambda_F(0^+)=0$.

Now recall

$$\begin{bmatrix} \lambda_0 \\ \lambda_d \\ \lambda_q \\ \lambda_F \\ \lambda_D \\ \lambda_Q \\ \lambda_G \end{bmatrix} = \begin{bmatrix} L_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_d & 0 & \sqrt{\frac{3}{2}}M_F & \sqrt{\frac{3}{2}}M_D & 0 & 0 \\ 0 & 0 & L_q & 0 & 0 & \sqrt{\frac{3}{2}}M_Q & \sqrt{\frac{3}{2}}M_G \\ 0 & \sqrt{\frac{3}{2}}M_F & 0 & L_F & M_R & 0 & 0 \\ 0 & \sqrt{\frac{3}{2}}M_D & 0 & M_R & L_D & 0 & 0 \\ 0 & 0 & \sqrt{\frac{3}{2}}M_Q & 0 & 0 & L_Q & M_Y \\ 0 & 0 & \sqrt{\frac{3}{2}}M_G & 0 & 0 & M_Y & L_G \end{bmatrix} \begin{bmatrix} i_0 \\ i_d \\ i_q \\ i_F \\ i_D \\ i_Q \\ i_G \end{bmatrix} \text{ eq.} \quad (4.20')$$

from which we extract the expression for λ_F , which is

$$\lambda_F = kM_F i_d + L_F i_F + M_R i_D$$

But recall we have removed the damper windings, so that $i_D=0$, then

$$\lambda_F = kM_F i_d + L_F i_F$$

Then at $t=0^+$, we have that

$$\lambda_F(0^+) = 0 = kM_F i_d + L_F i_F$$

and so we see that

$$i_F = \frac{-kM_F i_d}{L_F} \quad (4.172)$$

Again, from eq. (4.20') above, we have that

$$\lambda_d = L_d i_d + kM_F i_F + kM_D i_D$$

But $i_D=0$,

$$\lambda_d = L_d i_d + kM_F i_F$$

and substitution of (4.172) yields

$$\lambda_d = L_d i_d - \frac{(kM_F)^2 i_d}{L_F} = \left(L_d - \frac{(kM_F)^2}{L_F} \right) i_d$$

The term in brackets in the last equation is defined as the d-axis transient inductance according to

$$L'_d \equiv L_d - \frac{(kM_F)^2}{L_F}$$

And with $L_{AD}=kM_F$ (from 4.108), we obtain

$$L'_d \equiv L_d - \frac{L_{AD}^2}{L_F}$$

And it gives the initial d-winding flux linkage per unit of d-winding current with no damper winding, and with the field winding shorted.

In per-unit, since $\omega=1$, $X'_d = L'_d$.

Direct-axis subtransient inductance:

The dynamics of the damper windings are faster than those of the field circuit but not as fast as the stator circuits. It increases model accuracy to account for the effect of the damper windings.

To do so, we repeat the above procedure but

- We cannot let $i_D=0$
- We must also use the λ_D equation from eq. (4.20')
- We use $\lambda_D(0^+)=0$ and $\lambda_F(0^+)=0$, by CFLT

The text lays this out very well, resulting in

$$L_d'' = L_d - \frac{(kM_F)^2 L_D + (kM_D)^2 L_F - 2kM_F kM_D M_R}{L_F L_D - M_R^2}$$

With $L_{AD}=kM_F=kM_D=M_R$ (eq. 4.108), we get

$$L_d'' = L_d - \frac{L_D + L_F - 2L_{AD}}{\frac{L_F L_D}{(L_{AD})^2} - 1}$$

Again, in pu, $L_d'' = X_d''$.

Quadrature-axis inductance:

Similar analysis as before can be done to obtain L_q' and L_q'' .

However, for the q-axis inductance, we must impress voltages on the stator terminals such that v_q becomes non-zero and v_d becomes zero. This is easily done by impressing voltages on the stator terminals such that the phase a voltage is 90° behind the voltages impressed for the d-axis inductances. In other words, we will use

$$\underline{v}_{abc} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{2}V \begin{bmatrix} \sin \theta \\ \sin(\theta - 120) \\ \sin(\theta + 120) \end{bmatrix} u(t)$$

Then Park's transformation on \underline{v}_{abc} yields

$$\underline{v}_{0dq} = \underline{P}\underline{v}_{abc} = \begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \sqrt{2}V \begin{bmatrix} 0 \\ 0 \\ \sqrt{3}Vu(t) \end{bmatrix}$$

I will not go through the details here but give the following results.

$$L'_q = L_q - \frac{L_{AQ}^2}{L_G}$$

$$L''_q = L_q - \frac{L_Q + L_G - 2L_{AQ}}{\frac{L_G L_Q}{L_{AQ}^2} - 1}$$

Suggestion: YOU derive the above two equations.

Note: They differ from the text because we have included the G-circuit.