Simulation of Synchronous Machines

This chapter covers:

(A) Sections 5.2-5.7: Determination of initial conditions

(B) Section 5.8: Determination of machine parameters from manufacturers' data

(C) Sections 5.9-5.10: Construction of analog and digital simulation models

We will only cover (A). This breaks down into:

- Section 5.2: Steady-state and phasor diagram
- Section 5.3: Machine connected to an infinite bus through a line
- Section 5.4: Machine connected to an infinite bus with local load at machine terminal
- Section 5.5: Determining steady-state conditions
- Section 5.6: Examples
- Section 5.7: Initial conditions for a multimachine system

Of the above, we will concentrate on Sections 5.2, 5.5, and 5.7, but I encourage you to read all of these sections 5.2-5.7.

The basic problem is motivated by the following fact:
Simulation of the transient response of any dynamical system represented by state variables requires initial conditions for those state variables.

So what are our state variables?

- In general, it depends on the machine model.
- However, there are two state variables that are common to all machine models: δ, ω

The initial condition for ω is easy: $\omega(t=0) = 1$.

But what about the initial condition for δ?

What is δ? See page 85, which says: "At t=0 the phasor V is located at the axis of phase a, i.e., at the reference axis in Fig. 4.1. The q-axis is located at an angle δ, and the d-axis is located at $\theta=\delta+\pi/2$. At t>0, the reference axis is located at an angle $\omega_R t$ with respect to the axis of phase a. The d-axis of the rotor is therefore located at $\theta=\omega_R t+\delta+\pi/2$ where ω_R is the rated (synchronous) angular frequency in rad/sec and δ is the synchronous torque angle in electrical radians." (Note $\omega_R=\omega_{Re}$ here).

The below picture, Fig. 1, illustrates the relation between θ and δ for t=0, i.e., at $\theta=\delta+\pi/2$

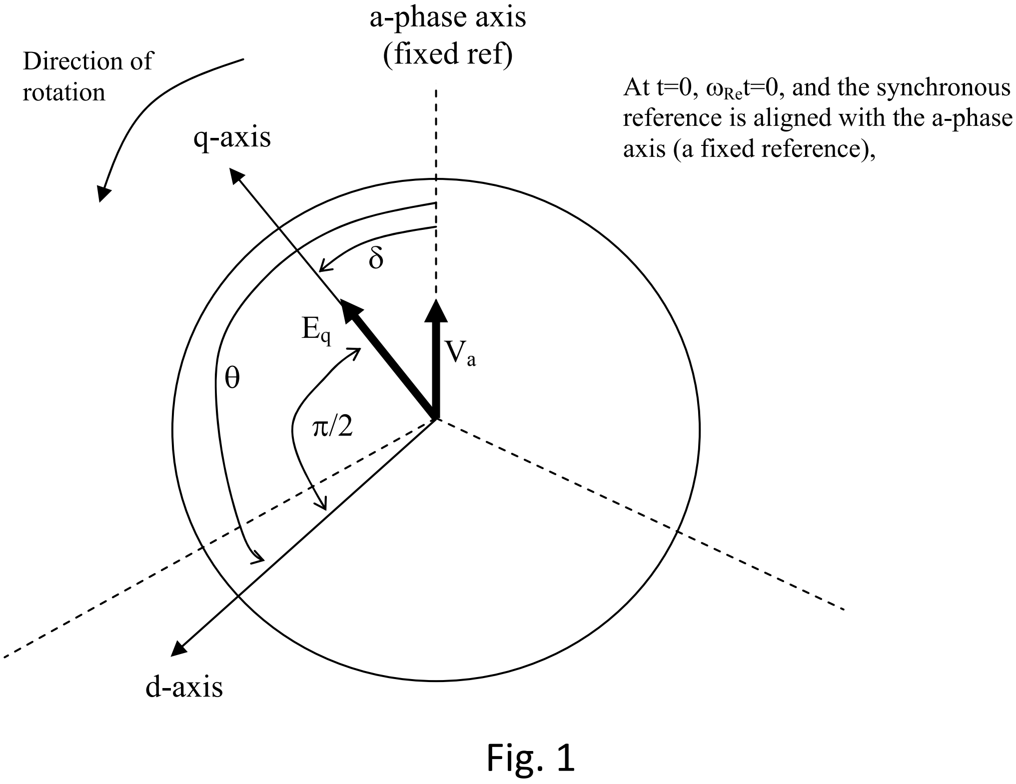


Fig. 1

So we see that δ is the angle between the *synchronous reference* and the q-axis of the machine.

So what is the reference? It is usually taken as the terminal bus voltage for one machine in the network. In the above picture, V_a identifies the reference.

So, the problem may be described by the following.

- We are about to perform a time domain simulation of a multi-machine system where each machine is represented using one of the Chapter 4 machine models. We will be simulating the electro-mechanical response of the power system to some identified disturbance.
- We have the corresponding power flow solved case to initialize the simulation. This power flow solution provides
    - V_a, the bus voltage (i.e., at the machine terminals) for all generator buses in the network, magnitude and angle, where the angle is given relative to the reference.
    - I_a, the bus current injection, magnitude and angle.

Since δ locates the q-axis for the machine, if we can find the angle of a quantity that lies along the q-axis, this angle will be δ.

What steady-state quantity lies along the q-axis?

This is the stator equivalent pu voltage corresponding to the field current i_F in pu. It is denoted by E in your text, but other books often denote it as E_q, to emphasize that it lies along the q-axis (and some books use E_I). It lies on the q-axis because it is entirely due to the field flux (see pg. 5 of “Simplified Models”).

From Section 4.7.4, we recall that $\sqrt{3}E = \omega k M_F i_F$.

So our problem is now as follows:

Given V_a and I_a, find E.

Recall eq. 4.74' which was derived in the notes on per-unitization.

$$\begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r & 0 & 0 & \omega L_q & \omega kM_Q & \omega kM_G \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ -\omega L_d & -\omega kM_F & -\omega kM_D & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_Q & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ i_G \end{bmatrix}$$

$$-\begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q & kM_G \\ 0 & 0 & 0 & kM_Q & L_Q & M_Y \\ 0 & 0 & 0 & kM_G & M_Y & L_G \end{bmatrix} \begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_q \\ \dot{i}_Q \\ \dot{i}_G \end{bmatrix}$$

Recall also that 4.74' is correct independent of whether units are MKS or per-unit. We will assume that we are in MKS.

We can obtain from 4.74' the steady-state relations between the d-q voltages and currents, by setting

- All derivatives to zero.
- $i_D=i_Q=i_G=0$

because we are analyzing steady-state conditions.

The resulting equations are:

$$v_d = -ri_d - \omega L_q i_q = -ri_d - x_q i_q \qquad (*)$$

$$v_q = -ri_q + \omega L_d i_d + \omega kM_F i_F = -ri_q + x_d i_d + \sqrt{3}E \qquad (**)$$

From Park's relation $\underline{v}_{abc}=\underline{P}^{-1}\underline{v}_{0dq}$, with $v_0=0$, which is

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \cos\theta & \sin\theta \\ \frac{1}{\sqrt{2}} & \cos(\theta-120) & \sin(\theta-120) \\ \frac{1}{\sqrt{2}} & \cos(\theta+120) & \sin(\theta+120) \end{bmatrix} \begin{bmatrix} 0 \\ v_d \\ v_q \end{bmatrix}$$

This provides that

$$v_a = \sqrt{\frac{2}{3}}\left[v_d \cos\theta + v_q \sin\theta\right]$$

where θ is the angle of the D-axis given by $\theta = \omega_{\mathrm{Re}} + \delta + \pi/2$.

Substituting v_d, v_q given as in (*) and (**), we obtain:

$$v_a = \sqrt{\frac{2}{3}}\left[\left(-ri_d - x_q i_q\right)\cos\left(\omega_{\mathrm{Re}}t + \delta + \pi/2\right) + \left(-ri_q + x_d i_d + \sqrt{3}E\right)\sin\left(\omega_{\mathrm{Re}}t + \delta + \pi/2\right)\right]$$

Noting that the sin term in the above equation can be written as: $\sin\left(\omega_{\mathrm{Re}}t + \delta + \pi/2\right) = \cos\left(\omega_{\mathrm{Re}}t + \delta\right)$, we have that:

$$v_a = \sqrt{\frac{2}{3}}\left[\left(-ri_d - x_q i_q\right)\cos\left(\omega_{\mathrm{Re}}t + \delta + \pi/2\right) + \left(-ri_q + x_d i_d + \sqrt{3}E\right)\cos\left(\omega_{\mathrm{Re}}t + \delta\right)\right]$$

Now the above expression is the instantaneous expression, so that its magnitude is a peak quantity. To obtain RMS quantities, we need to divide by √2, resulting in:

$$V_a = \frac{1}{\sqrt{3}}\left[\left(-ri_d - x_q i_q\right)\cos\left(\omega_{\mathrm{Re}}t + \delta + \pi/2\right) + \left(-ri_q + x_d i_d + \sqrt{3}E\right)\cos\left(\omega_{\mathrm{Re}}t + \delta\right)\right]$$

Converting to phasor notation, we have:

$$\overline{V_a} = \frac{\left(-ri_d - x_q i_q\right)}{\sqrt{3}}\angle(\delta+\pi/2) + \frac{-ri_q + x_d i_d}{\sqrt{3}}\angle(\delta) + E\angle(\delta)$$

Combining terms in r yields:

$$\overline{V_a} = -r\left[\frac{i_d}{\sqrt{3}}\angle(\delta+\pi/2) + \frac{i_q}{\sqrt{3}}\angle(\delta)\right] - x_q\frac{i_q}{\sqrt{3}}\angle(\delta+\pi/2) + x_d\frac{i_d}{\sqrt{3}}\angle(\delta) + E\angle(\delta)$$

Recognizing that $\angle(\delta+\pi/2) = j\angle\delta$, and defining the RMS equivalent d- and q-axis currents reflected to the stator as:

$$I_d = \frac{i_d}{\sqrt{3}} \qquad I_q = \frac{i_q}{\sqrt{3}}$$

we have that

$$\overline{V_a} = -r\left[jI_d\angle\delta + I_q\angle\delta\right] - jx_q I_q\angle\delta + x_d I_d\angle\delta + E\angle(\delta)$$

The quantity $jI_d\angle\delta + I_q\angle\delta$ is the stator current phasor decomposed into the d- and q-axes, i.e.,

$$\bar{I}_a = jI_d\angle\delta + I_q\angle\delta = \bar{I}_d + \bar{I}_q$$

where the j in front of the I_d term provides the necessary 90 degree rotation ahead of the q-axis for the d-axis component of the current.

Thus we can write the a-phase voltage phasor as:

$$\overline{V_a} = -r\bar{I}_a - jx_q I_q\angle\delta + x_d I_d\angle\delta + E\angle\delta$$

Solving for $E\angle\delta$, we have:

$$\overline{E} = E\angle\delta = \overline{V_a} + r\bar{I}_a + jx_q I_q\angle\delta - x_d I_d\angle\delta$$

Now let's focus on the last two terms of the above equation.

Clearly, $I_q \angle \delta = \bar{I}_q$. But what about $\bar{I}_d$?

Recall that $\bar{I}_d = jI_d \angle \delta \Rightarrow \frac{1}{j}\bar{I}_d = I_d \angle \delta \Rightarrow -j\bar{I}_d = I_d \angle \delta$

Therefore we can write:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\bar{I}_q + jx_d\bar{I}_d \tag{5.14}$$

Now what has all of this work bought us?

If we have, from the power flow solution, $\overline{V}_a$ and $\bar{I}_a$, we can compute the first part of (5.14).

However, we do not yet know $\bar{I}_d$ and $\bar{I}_q$, because we do not know the location of the q-axis!

What to do?

Here is a trick. Add and subtract $jx_q\bar{I}_d$ to obtain:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\bar{I}_q \underbrace{+ jx_q\bar{I}_d}_{Added} \underbrace{- jx_q\bar{I}_d}_{subtracted} + jx_d\bar{I}_d$$

Collect terms in (jx_q) and in (jI_d) to yield:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right) \tag{*}$$

To see the significance of eqt. (*), let’s do two exercises in drawing phasor diagrams.

These exercises will use eqs. (5.14) and (*) as “instruction manuals” for drawing the phasor diagrams.

In both exercises, we will use the fact that we know the angle of V_a so that it can be our reference angle, and, without loss of generality, we can assume that this reference is 0 degrees.

Exercise 1: Use eq. (5.14). Let’s assume that we know the phasors for I_d and I_q.

$$\overline{E} = E\angle\delta = \overline{V}_a + r\overline{I}_a + jx_q\overline{I}_q + jx_d\overline{I}_d \tag{5.14}$$

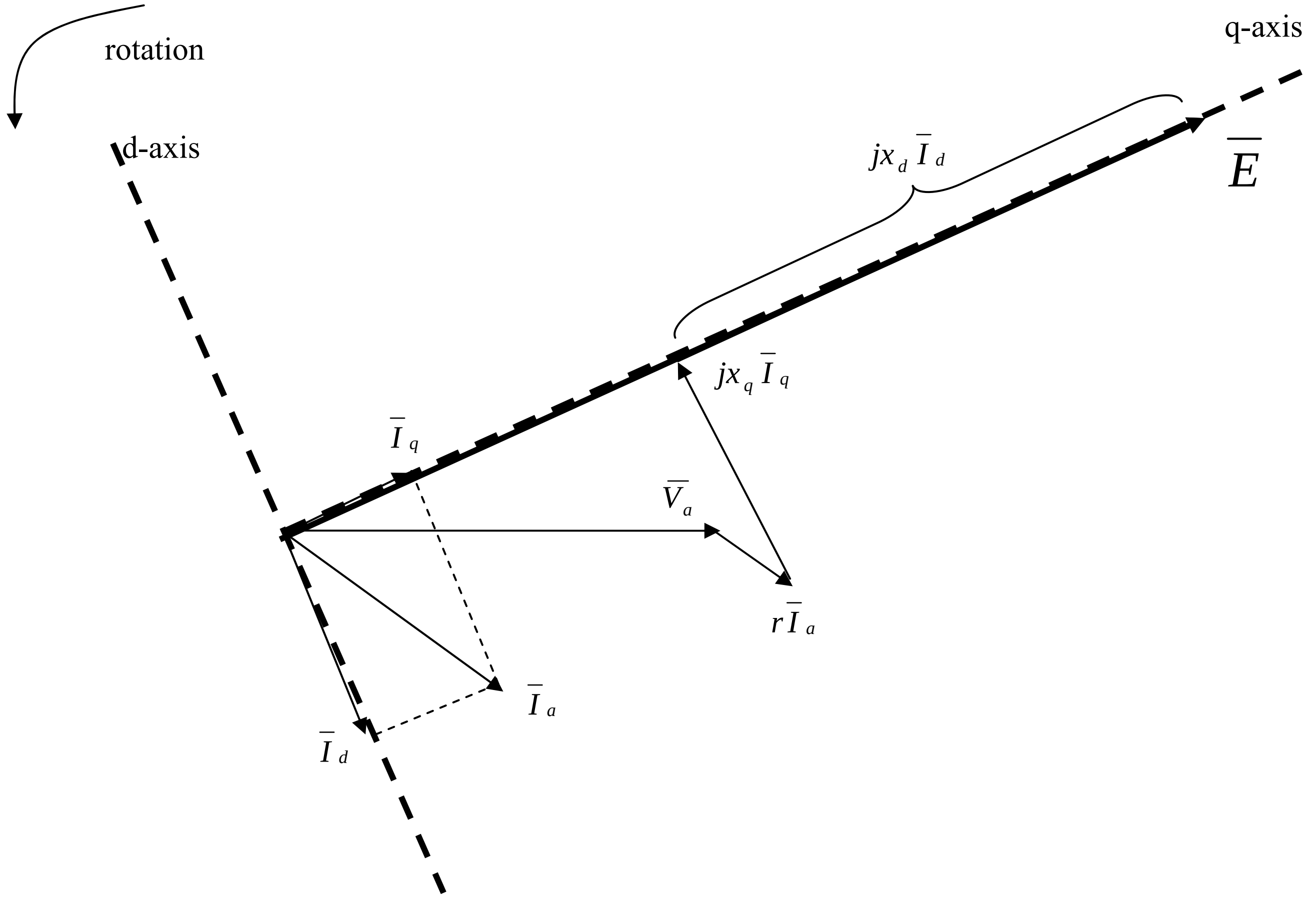


Fig. 2

Exercise 2: Use eq. (*). Again, assume that we know the phasors for I_d and I_q.

$$\overline{E} = E\angle\delta = \overline{V_a} + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right) \qquad (*)$$

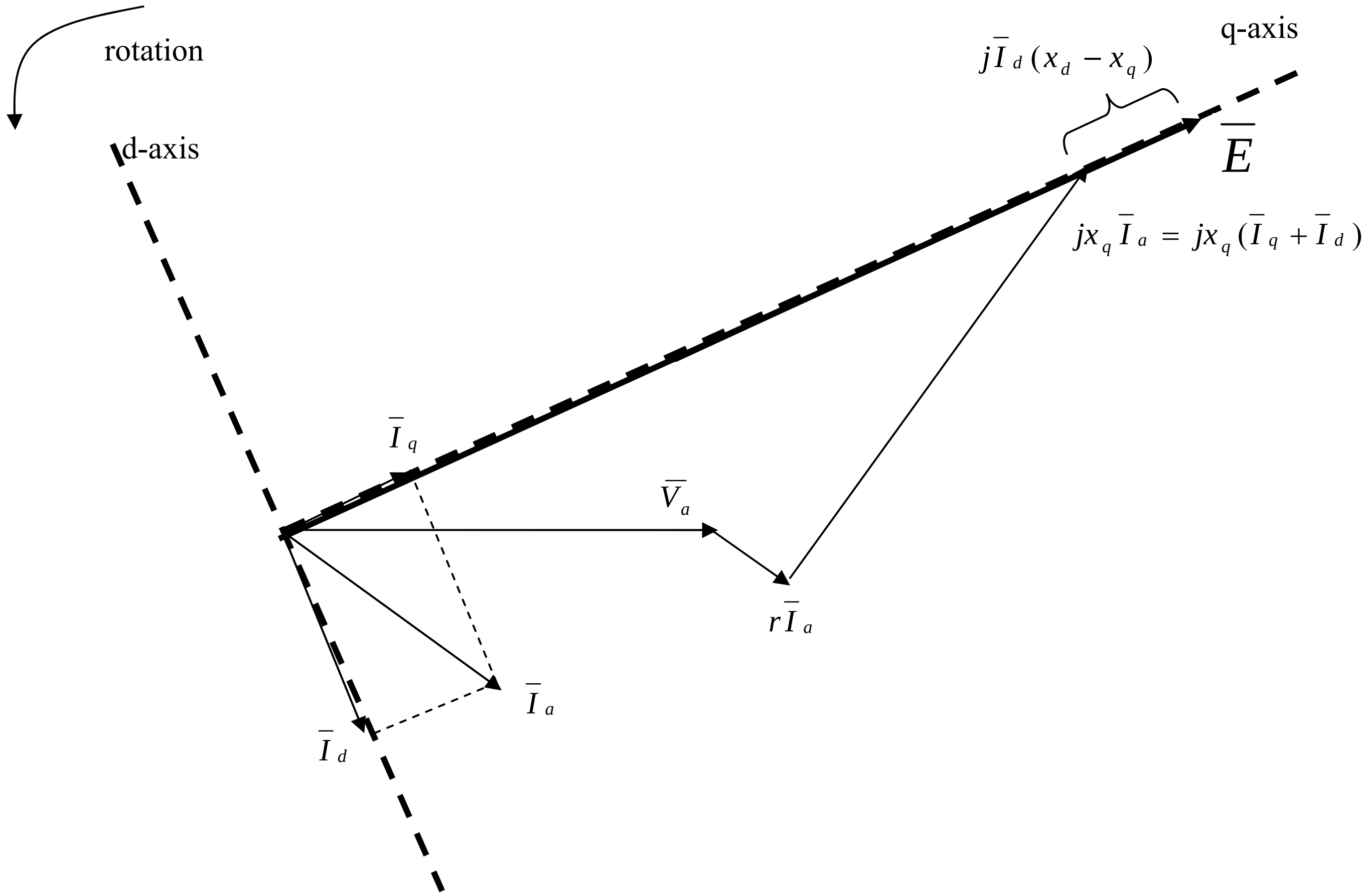

rotation
d-axis
q-axis
$j\bar{I}_d(x_d - x_q)$
$\bar{E}$
$jx_q\bar{I}_a = jx_q(\bar{I}_q + \bar{I}_d)$
$\bar{I}_q$
$\bar{V}_a$
$r\bar{I}_a$
$\bar{I}_a$
$\bar{I}_d$

Fig. 3

Note that in exercise 2, we can express eq. (*) as

$$\bar{E} = E\angle\delta = \underbrace{\bar{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right)}_{\bar{E}_a} + j\bar{I}_d\left(x_d - x_q\right) = \bar{E}_a + j\bar{I}_d\left(x_d - x_q\right)$$

where the first part of eq. (*) is given by:

$$\bar{E}_a = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_a$$

where $\bar{I}_a = \bar{I}_d + \bar{I}_q$.

If $\bar{E}$ is on the q-axis (and we have already proven that it is), then $\bar{E}_a$ must also be on the q-axis because the only difference between them is $j\bar{I}_d\left(x_d - x_q\right)$ which is a component along the q-axis (if a vector on the q-axis is added to another vector on the q-axis, the resultant vector must also be on the q-axis).

The important point here is that $\bar{E}_a$ requires only $\bar{V}_a$ and $\bar{I}_a$ to compute it, which are known from the power flow solution! So we may locate the q-axis.

Once we do that, we may compute $\bar{I}_d$ as follows….

Define the familiar power factor angle as ϕ, the angle by which I_a lags V_a (see page 157 in text), or the angle by which V_a leads I_a. The power factor angle is greater than zero for lagging current.

Let's also define β as the angle of V_a, relative to the reference. Then it is the case that

$$\angle\bar{I}_a = \beta - \phi$$

The phasor diagram below illustrates the situation (see Fig. 5.1):

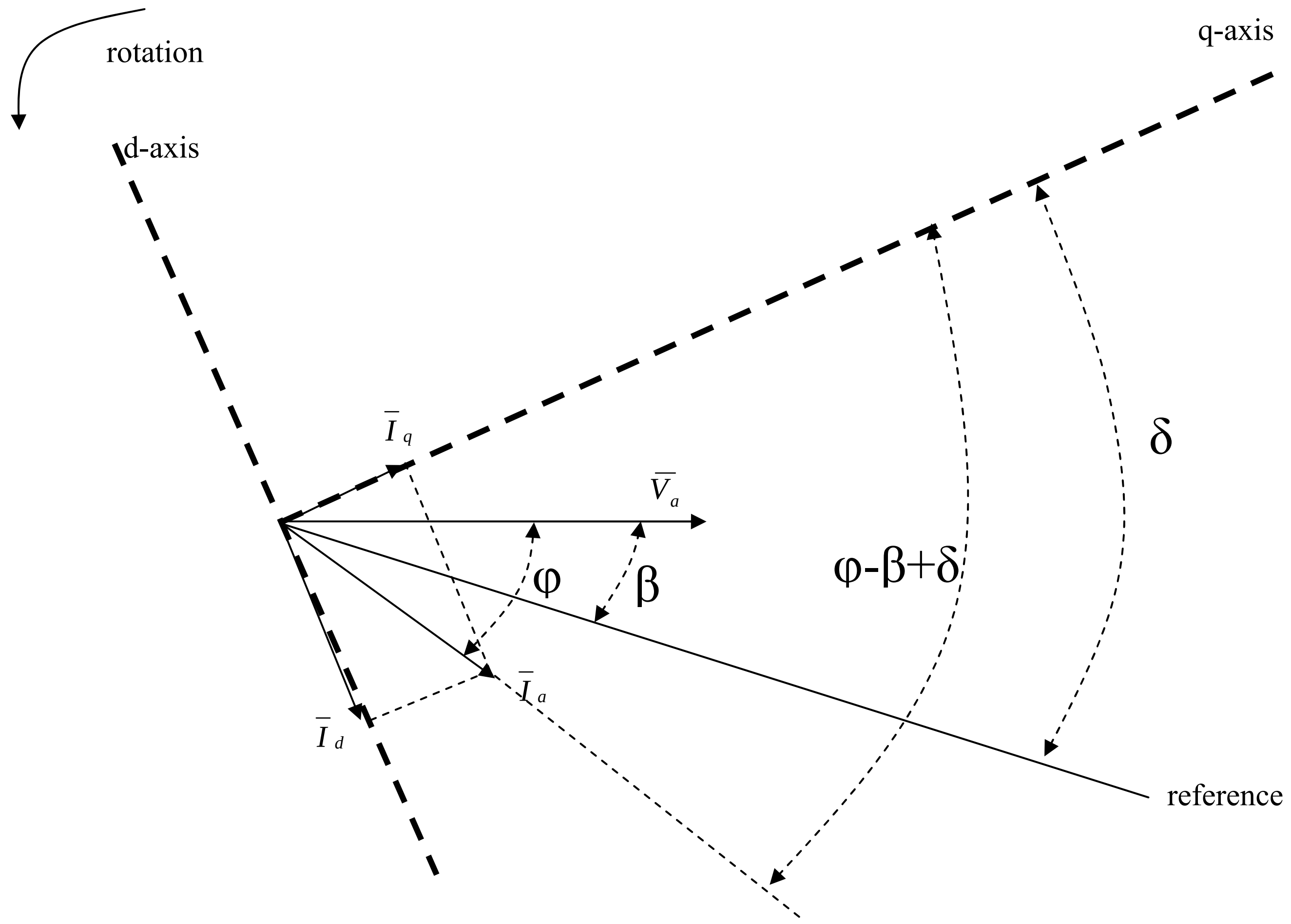


Fig. 4

From the phasor diagram, we can observe that

$$\angle \bar{I}_d = \delta - 90^\circ$$

$$\left|\bar{I}_d\right| = \left|\bar{I}_a\right| \sin(\phi - \beta + \delta)$$

The above relations provide us with I_d, from which we may compute E from

$$\overline{E} = E\angle\delta = \overline{E}_a + j\bar{I}_d\left(x_d - x_q\right)$$

Some remarks on this….

Remark 1: $I_d = -\left|\bar{I}_d\right|$

Note that eq. 5.44 in your text indicates that

$$I_d = -I_a \sin(\phi - \beta + \delta)$$

which is different than the expression given above for $\left|\bar{I}_d\right|$, as the text is assigning a sign to the magnitude of $\bar{I}_d$. Why is this?

We have said that $\bar{I}_a = \bar{I}_d + \bar{I}_q$ where:

$$\bar{I}_d = \left|I_d\right|\angle\delta - 90$$

$$\bar{I}_q = \left|I_q\right|\angle\delta$$

$$I_d = \frac{i_d}{\sqrt{3}} \qquad\qquad I_q = \frac{i_q}{\sqrt{3}}$$

Note that the text indicates, in eq. 5.12, that:

$$I_a = \left(I_q + jI_d\right)e^{j\delta} = I_q\angle\delta + jI_d\angle\delta = I_q\angle\delta \boxed{+ I_d\angle\delta + 90} = \bar{I}_q + \bar{I}_d$$

But we have said that $\bar{I}_d = \left|I_d\right|\angle\delta - 90$. The implication is that $I_d = -\left|\bar{I}_d\right|$, as below:

$$\bar{I}_d = I_d\angle\delta + 90 = -\left|\bar{I}_d\right|\angle\delta + 90$$

$$= \left|\bar{I}_d\right|\angle -180\angle\delta + 90 = \left|\bar{I}_d\right|\angle\delta + 90 - 180 = \left|\bar{I}_d\right|\angle\delta - 90$$

Remark 2: Phasors

$\bar{I}_a$ is a *phasor* getting its rotation from the sinusoidal variation of the alternating currents.

On the other hand, I_d and I_q are equivalent RMS values of i_d and i_q, respectively, and i_d and i_q are direct currents. So what are $\bar{I}_d$ and $\bar{I}_q$?

They are *phasors*, but their rotation comes from the rotor motion, not from the current variation.

Remark 3: Saliency

Recall eq. (*), where we found that

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_q + \bar{I}_d\right) + j\bar{I}_d\left(x_d - x_q\right)$$

and with $\bar{I}_a = \bar{I}_d + \bar{I}_q$, we have that:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\left(\bar{I}_a\right) + j\bar{I}_d\left(x_d - x_q\right)$$

An equivalent circuit for this appears in Fig. 5.

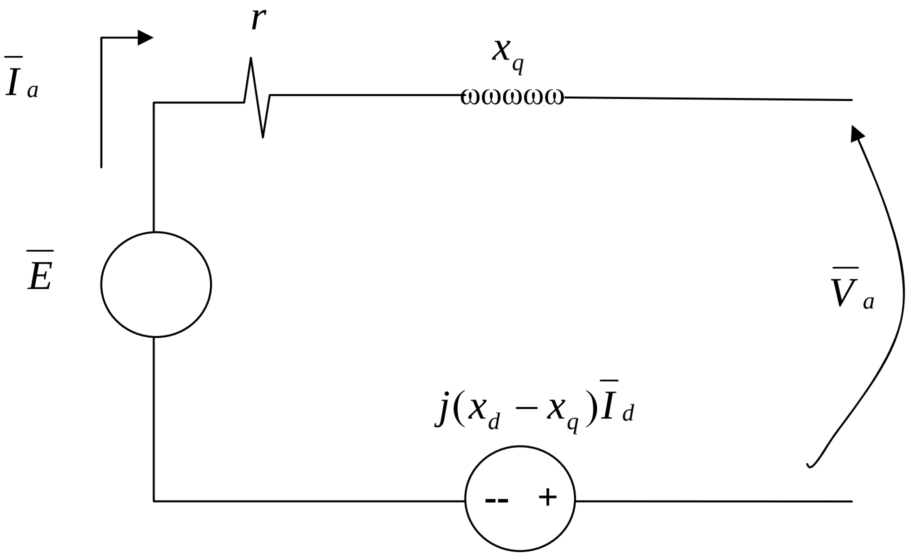


Fig. 5

Here, x_d and x_q are the synchronous machine reactances in the d- and q- axes. For a salient-pole machine, $x_d>>x_q$, and the lower voltage source is significant. For a round-rotor machine, $x_d \approx x_q$, and the lower voltage source is insignificant. We sometimes call the lower voltage the "**voltage due to saliency**."

Recall that for round rotor machines, the equivalent circuit for steady-state analysis is as in Fig. 6.

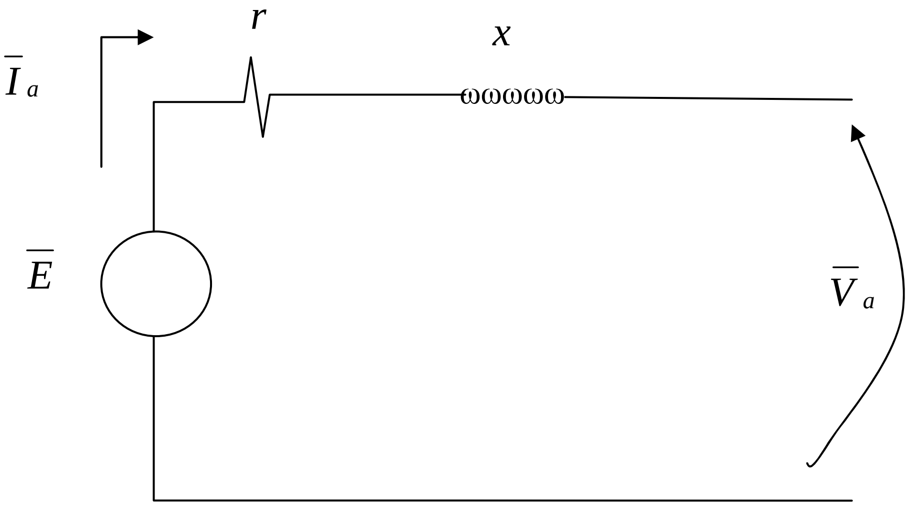


Fig. 6

The above circuit is likely quite familiar based on an undergraduate course in electromechanical energy conversion.

When r=0, we may derive for the round-rotor machine the two familiar per-unit expressions:

$$P_{out} = \frac{|\overline{E}||\overline{V}_a|}{x_q}\sin(\delta - \beta)$$

$$Q_{out} = \frac{|\overline{E}||\overline{V}_a|}{x_q}\cos(\delta - \beta) - \frac{|\overline{V}_a|^2}{x_q}$$

If $\overline{V}_a$ is the reference, then $\beta=0$ in the above relations.

But what about the case of the salient-pole machine? The voltage due to saliency should change these expressions. Let’s find out….

Letting r=0 as in the round-rotor case, and returning to eq. (5.14), which was eq. (*) before we performed the “add and subtract” trick. This equation was:

$$\overline{E} = E\angle\delta = \overline{V}_a + r\bar{I}_a + jx_q\bar{I}_q + jx_d\bar{I}_d \qquad (5.14)$$

To simplify the developement, let

$$E = |E|\angle 0^\circ \qquad \overline{V}_a = |V_a|\angle -\delta$$

Thus we can write that:

$$\overline{V}_a = |V_a|e^{-j\delta} = |V_a|\cos\delta - j|V_a|\sin\delta$$

We want $S_{out} = \overline{V}_a\bar{I}_a^{\,*} = \overline{V}_a\left(\bar{I}_d + \bar{I}_q\right)^*$

We can obtain $\bar{I}_d$ and $\bar{I}_q$ from inspecting the phasor diagram resulting from eq. (5.14) (with r=0):

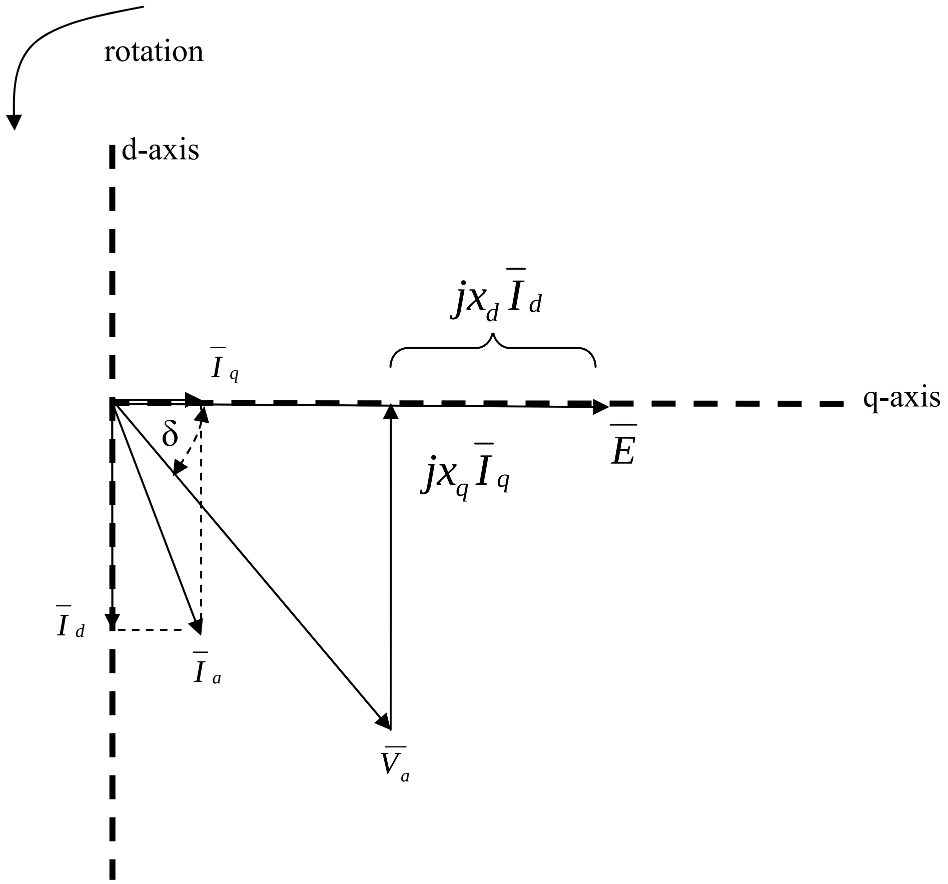


Fig. 7

From the above, we can see that

$$\bar{E} = \left|\bar{E}\right|\angle 0^\circ = \left|V_a\right|\cos\delta + jx_d\bar{I}_d \Rightarrow\Rightarrow \bar{I}_d = \frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{jx_d}$$

$$0 = -j\left|V_a\right|\sin\delta + jx_q\bar{I}_q \Rightarrow\Rightarrow \bar{I}_q = \frac{\left|V_a\right|\sin\delta}{x_q}$$

Substitution into the expression for S_{out} yields:

$$S_{out} = \left(\left|\bar{V}_a\right|\angle -\delta\right)\left(\frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{jx_d} + \frac{\left|V_a\right|\sin\delta}{x_q}\right)^* = \left(\left|\bar{V}_a\right|\left(\cos\delta - j\sin\delta\right)\right)\left(\frac{\left|V_a\right|\sin\delta}{x_q} - j\frac{\left|\bar{E}\right| - \left|V_a\right|\cos\delta}{x_d}\right)^*$$

Now taking care of the conjugation yields:

$$\Rightarrow\Rightarrow S_{out} = \left(\left|\overline{V}_a\right|(\cos\delta - j\sin\delta)\right)\left(\frac{|V_a|\sin\delta}{x_q} + j\frac{\left|\overline{E}\right| - |V_a|\cos\delta}{x_d}\right)$$

Taking the real part to find P_{out}:

$$P_{out} = \left|\overline{V}_a\right|\left[\frac{\left|\overline{V}_a\right|\cos\delta\sin\delta}{x_q} + \frac{\left|\overline{E}\right|\sin\delta - \left|\overline{V}_a\right|\cos\delta\sin\delta}{x_d}\right]$$

Multiplying through by $\left|\overline{V}_a\right|$ and rearranging the order of the terms yields:

$$P_{out} = \frac{\left|\overline{E}\right|\left|\overline{V}_a\right|\sin\delta}{x_d} + \left|\overline{V}_a\right|^2\cos\delta\sin\delta\left[\frac{1}{x_q} - \frac{1}{x_d}\right]$$

Recalling the trigonometric identity sin2x=2 cosx sinx, we have:

$$P_{out} = \frac{\left|\overline{E}\right|\left|\overline{V}_a\right|\sin\delta}{x_d} + \frac{\left|\overline{V}_a\right|^2}{2}\left[\frac{1}{x_q} - \frac{1}{x_d}\right]\sin 2\delta$$

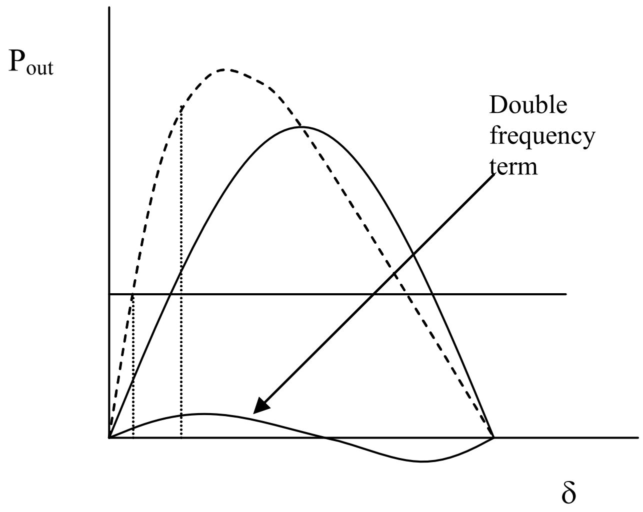
Similarly, we may derive from S_{out} the expression for reactive power out of a salient-pole machine, as:

$$Q_{out} = \frac{\left|\overline{E}\right|\left|\overline{V}_a\right|\cos\delta}{x_d} - \left|\overline{V}_a\right|^2\left[\frac{\cos^2\delta}{x_d} - \frac{\sin^2\delta}{x_q}\right]$$

Note that both P_{out} and Q_{out} collapse to round-rotor equations if $x_d = x_q$.

Question: What does saliency do to stability?

Refer back to the expression for P_{out} and call the first term “term 1” and the second term “term 2.”



From the above figure, we observe that:

- P_{max} occurs for $\delta<90$ degrees.
- For a given power P_{out}, the angle is smaller for a salient-pole machine relative to a round-rotor machine.
- P_{max} is greater for a salient-pole machine relative to a round-rotor machine. This fact means that, for a given level, a salient-pole machine will typically have more decelerating energy available than a corresponding round-rotor machine, with all other things being equal. ➔ Saliency tends to improve stability.

See pp. 80-89 of Kimbark Vol. III – it provides additional info regarding the above conclusion that you should check.

<u>Initial conditions for a multi-machine system (Section 5.7)</u>:

Assume that the power flow solution give us $\overline{V}_a$ and $\bar{I}_a$ for every generator such that

$\overline{V}_a = |\overline{V}_a| \angle \beta \qquad \bar{I}_a = |\bar{I}_a| \angle \beta - \phi$

Then, for each generator, we need to perform the following procedure in order to obtain the initial conditions:

1. Compute $\bar{E}_a = \bar{V}_a + r\bar{I}_a + jx_q\bar{I}_a$
2. Compute $\bar{I}_d$ and $\bar{I}_q$ from:

   $$\left|\bar{I}_d\right| = \left|\bar{I}_a\right| \sin(\phi - \beta + \delta) \qquad \left|\bar{I}_q\right| = \left|\bar{I}_a\right| \cos(\phi - \beta + \delta)$$

   where $\delta = \angle \bar{E}_a$, $\phi = \beta - \angle \bar{I}_a$, and $\bar{I}_d = \left|I_d\right| \angle \delta - 90$
3. Compute $\bar{E} = E\angle\delta = \bar{E}_a + j\bar{I}_d\left(x_d - x_q\right)$
4. Compute: $I_d = -\left|\bar{I}_d\right|$, and $I_q = \left|\bar{I}_q\right|$
5. Compute $i_d = \sqrt{3}I_d$, $i_q = \sqrt{3}I_q$, and $i_F = \dfrac{\sqrt{3}\left|E\right|}{L_{AD}}$
6. Now compute v_d and v_q. From the phasor diagram (fig. 5.1), we can decompose $\bar{V}_a$ into its component in phase with the d-axis and its component in phase with the q-axis. This results in:

   $$\left|\bar{V}_q\right| = \left|\bar{V}_a\right| \cos(\delta - \beta) \qquad \left|\bar{V}_d\right| = \left|\bar{V}_a\right| \sin(\delta - \beta)$$

   $$V_q = \left|\bar{V}_q\right| \qquad V_d = -\left|\bar{V}_d\right|$$

   $$v_q = \sqrt{3}V_q \qquad v_d = \sqrt{3}V_d$$

7. Compute $v_F = i_F r_F$

All of the above steps are "generic;" they apply to all of the machines. The remaining steps, however, depend on the particular model being used for the generator at this bus.

Let's assume that we are using the E'_q model (mode 1.0). In this model, we neglect damper windings in both the D- and Q-axes, so that the only rotor winding accounted for is the main field winding.

8. From 4.104, we obtain λ_d, λ_q, and λ_F from:

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_F \end{bmatrix} = \begin{bmatrix} L_d & 0 & kM_F \\ 0 & L_q & 0 \\ kM_F & 0 & L_F \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_F \end{bmatrix}$$

9. We also need E_{FD} as an input. We can obtain it from

$$E_{FD} = \frac{1}{\sqrt{3}} \frac{L_{AD}}{r_F} v_F$$

10. Get the initial conditions on the other states:

$$E'_q = \frac{1}{\sqrt{3}} \frac{kM_F}{L_F} \lambda_F \qquad \Lambda_d = \frac{\lambda_d}{\sqrt{3}} \qquad \Lambda_q = \frac{\lambda_q}{\sqrt{3}}$$

and these, along with δ (see step 2) and ω=1 comprise the initial conditions.

Additional comment regarding step 2 above…

If the angle β (angle of V_a) is not explicitly given, then the calculation can still be made except it is necessary to think a bit more about how to make it (see pg.157).

Consider decomposing the current I_a into components I_r in phase and I_x in quadrature with the terminal voltage V_a so that

$$\overline{I}_a = I_r + jI_x$$

With ϕ as the power factor angle (the angle by which I_a lags V_a, positive for lagging power factor), then

$$I_r = | I_a | \cos\phi \qquad I_x = - | I_a | \sin\phi$$

The minus sign on the expression for I_x is to account for the fact that when ϕ is positive, current is lagging the voltage so that the x-component should be negative in this case.

Now recall our E_a vector is

$$\overline{E}_a = \overline{V}_a + r\overline{I}_a + jx_q\overline{I}_a \text{.}$$

Substituting for I_a, we have:

$$\overline{E}_a = \overline{V}_a + r(I_r + jI_x) + jx_q(I_r + jI_x)$$

Collecting real and imaginary parts, and moving V_a to the we have:

$$\overline{E}_a = \overline{V}_a + (rI_r - x_qI_x) + j(rI_x + x_qI_r)$$

With V_a having an angle of β, the above calculation results in an E_a with an angle of δ.

But let's rotate V_a by $-\beta$ so that it has an angle of $\beta-\beta=0$. In this case, the computed quantity on the left-hand-side, E_a, will have an angle of $\delta-\beta$, and we may rewrite the above relation with V_a as an entirely real part (since it has angle of 0)

$$\overline{E}_a = (V_a + rI_r - x_qI_x) + j(rI_x + x_qI_r)$$

Thus, we have that

$$\delta - \beta = \tan^{-1}\left(\frac{rI_x + x_qI_r}{V_a + rI_r - x_qI_x}\right)$$