Simplified Models of the Synchronous Machine

Below is a summary of the different models that we will discuss:

	#	Youngs ID	IEEE ID (d.q)	Description	G-cct?	Dmpr wdgs?	No. of states	States
What we derived in FluxLinkageEquations	1	Model III	2.2	Full model with G-cct.	Yes	Yes	8	$\delta,\omega,\lambda_d,\lambda_F,\lambda_D,\lambda_q,\lambda_Q,\lambda_G$
What A&F derived in Section 4.12.	2		2.1	Full model without G-cct.	No	Yes	7	$\delta,\omega,\lambda_d,\lambda_F,\lambda_D,\lambda_q,\lambda_Q$
See pg. 127 of A&F for a few remarks.	3	Model II	1.1	Machine with solid round rotor	Yes	No	6	$\delta,\omega,\lambda_d,\lambda_F,\lambda_q,\lambda_G$
	4		1.0	E'q model: Same as #3 except for a salient pole machine.	No	No	5	$\delta,\omega,\Lambda_d,\Lambda_q,E'_q$
	5		2.1	E" model: for a salient pole machine & $d\lambda_d/dt= d\lambda_q/dt=0$	No	Yes	5	$\delta,\omega,E''_d, \Lambda_d,E'_q$
	6		1.1	2-axis model: for a machine with solid round rotor & $d\lambda_d/dt= d\lambda_q/dt=0$	Yes	No	4	δ,ω,E'_d,E'_q
	7		1.0	One axis model: Same as #6 but without G-cct.	No	No	3	δ,ω,E'_q
	8	Model I	0.0	Classical	No	No	2	δ,ω

Note that the IEEE ID is useful: d.q, where d indicates how many direct-axis rotor circuits are modeled (F, D) and q indicates how many quadrature-axis rotor circuits are modeled (G, Q). Note that these numbers do not include the stator windings (d, q). You can get the number of electrical states from these numbers according to:

$$E=d+q+2-N$$

where “2” is for the d and q winding states and N is the number of these for which the transient is assumed to be too fast for consideration.
Then, the total number of states is E+M where M=2 is the number of mechanical states.

Model 1 (2.2) Full model with G-cct.

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \qquad (4.131')$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

where

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q + \frac{L_{MQ}}{l_G}\lambda_G$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q} + \frac{1}{l_G}\right]$$

Model 2 (2.1): Full model without G-cct.

Same as model 1 except omit the state equation for λ_G (4.131') & modify the auxiliary equations, resulting in:

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_D = -\frac{r_D}{l_D}\lambda_D + \frac{r_D}{l_D}\lambda_{AD} \qquad (4.129)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_Q = -\frac{r_Q}{l_Q}\lambda_Q + \frac{r_Q}{l_Q}\lambda_{AQ} \qquad (4.131)$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

where

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F + \frac{L_{MD}}{l_D}\lambda_D \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_Q}\lambda_Q$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F} + \frac{1}{l_D}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_Q}\right]$$

Model 3 (1.1): Machine with solid round rotor. (See page 127) (No D or Q-axis damper windings).

For this model, A&F text references Kimbark's Vol. III, and says,

> "The solid round rotor acts as a q axis damper winding, even with the d axis damper winding omitted. The mathematical model for this type of machine will be the same as given in Sections 4.10 and 4.12 with i_D or λ_D omitted."

This suggests that we should keep the state equation for λ_Q and drop the state equation for λ_D. On checking Kimbark Vol. III, pg. 73, one finds state equations for both λ_Q and λ_D have been dropped.

The reason for this apparent discrepancy is that A&F were assuming no G-winding, whereas Kimbark includes the G-winding. Whereas Kimbark drops both Q and D windings (but retains G), A&F drop only the D. Thus, A&F implicitly allow the G-winding to substitute for the Q-winding. Since we have developed our model with G, we will use Kimbark's approach.

So this model is the same as model 1 except omit the state equations for λ_D (4.129) and λ_Q (4.131), and modify the auxiliary equations, resulting in:

$$\dot{\lambda}_d = -\frac{r}{l_d}\lambda_d + \frac{r}{l_d}\lambda_{AD} - \omega\lambda_q - v_d \qquad (4.126)$$

$$\dot{\lambda}_F = -\frac{r_F}{l_F}\lambda_F + \frac{r_F}{l_F}\lambda_{AD} + v_F \qquad (4.128)$$

$$\dot{\lambda}_q = -\frac{r}{l_q}\lambda_q + \frac{r}{l_q}\lambda_{AQ} + \omega\lambda_d - v_q \qquad (4.130)$$

$$\dot{\lambda}_G = -\frac{r_G}{l_G}\lambda_G + \frac{r_G}{l_G}\lambda_{AQ} \qquad (4.131')$$

$$\dot{\omega} = \frac{T_m}{\tau_j} + \left[\frac{\lambda_{AQ}}{l_q 3\tau_j}\lambda_d - \frac{\lambda_{AD}}{l_d 3\tau_j}\lambda_q\right] + \left[\frac{-D}{\tau_j}\right]\omega \qquad (4.133)$$

$$\dot{\delta} = \omega - 1 \qquad (4.102)$$

where

$$\lambda_{AD} = \frac{L_{MD}}{l_d}\lambda_d + \frac{L_{MD}}{l_F}\lambda_F \qquad \lambda_{AQ} = \frac{L_{MQ}}{l_q}\lambda_q + \frac{L_{MQ}}{l_G}\lambda_G$$

and

$$\frac{1}{L_{MD}} = \left[\frac{1}{L_{AD}} + \frac{1}{l_d} + \frac{1}{l_F}\right] \qquad \frac{1}{L_{MQ}} = \left[\frac{1}{L_{AQ}} + \frac{1}{l_q} + \frac{1}{l_G}\right]$$

Model 4 (1.0): E'q model: Same as #3 except for a salient pole machine. (See pp. 127-132)
(no D- or Q-axis dampers, and because it is salient pole, omit the G-winding)

One version of this model can be obtained from model 3 (which has no D- or Q-axis dampers) by simply omitting the state equation for λ_G (4.131') and modifying the Q-axis auxiliary equations appropriately.

But we may also describe this model in terms of some new "stator-side" states, E'_q, Λ_d, and Λ_q, which are just scaled versions of three corresponding rotor quantities, as defined below:

- E'_q is the pu value of the stator equivalent EMF corresponding to the field flux linkage λ_F, in phase with the q-axis, given by:

$$E'_q = \lambda_F \frac{L_{AD}}{\sqrt{3}L_F} \qquad (4.203)$$

  This comes about as follows. We call it E'_q because it is in phase with the q-axis. This is the case because it is a voltage due entirely to the field flux, and the field flux is generated along the d-axis. Because the corresponding induced stator winding voltage is proportional to $d\lambda/dt$, the induced voltage must be 90 degrees out of phase, meaning it must be along the q-axis.

  Its magnitude can be deduced as follows. (This expands on the discussion in A&F text, pp. 98-99). Recall that the mutual inductance between field and a-phase winding (before Park's transformation!) is given by

$$L_{aF} = M_F \cos\theta = M_F \cos\omega_{\mathrm{Re}} t \qquad (1)$$

  We know that the mutual flux linking the a-phase winding is given by

$$\lambda_{aF} = L_{aF} i_F \qquad (2)$$

  and that the time derivative of this flux linkage gives the induced voltage in the a-phase winding, i.e.,

$$\frac{d\lambda_{aF}}{dt} = \frac{d(L_{aF} i_F)}{dt} \tag{3}$$

Assuming i_F is constant, (3) becomes

$$\frac{d\lambda_{aF}}{dt} = i_F \frac{d(L_{aF})}{dt} \tag{4}$$

Substitution of (1) into (4) yields

$$\frac{d\lambda_{aF}}{dt} = i_F \frac{d(M_F \cos \omega_{\text{Re}} t)}{dt} = -i_F M_F \omega_{\text{Re}} \sin \omega_{\text{Re}} t \tag{5}$$

And we see that the peak value of the induced a-phase voltage is

$$E_{peak} = i_F M_F \omega_{\text{Re}} \tag{6}$$

The RMS value of this voltage would be V_{peak}/sqrt(2), i.e.,

$$E_{rms} = \frac{1}{\sqrt{2}} i_F M_F \omega_{\text{Re}} \tag{7}$$

If we multiple both sides by sqrt(3), we get

$$\sqrt{3} E_{rms} = \sqrt{\frac{3}{2}} i_F M_F \omega_{\text{Re}} \tag{8}$$

But recall our familiar k=sqrt(3/2), therefore

$$\sqrt{3} E_{rms} = i_F k M_F \omega_{\text{Re}} \tag{9}$$

To be consistent with the text, we will just use E for E_{rms}, so that (9) becomes

$$\sqrt{3} E = i_F k M_F \omega_{\text{Re}} \tag{10}$$

Equation (10) is given in your text at the bottom of page 98. Note that this is the voltage contribution to the a-phase from a certain value of field current.

A&F text now considers the case of getting the a-phase voltage from a certain amount of flux linkage seen by the field winding λ_F. Now if we consider steady-state (i_Q=i_D=0) and open circuit (i_d=i_q=0) conditions, then the only flux seen by the field winding is its own flux, and

$$\lambda_{aF} = L_F i_F \Rightarrow i_F = \frac{\lambda_F}{L_F} \tag{11}$$

Substitution of (11) into (10) results in

$$\sqrt{3} E = \frac{\lambda_F}{L_F} k M_F \omega_{\text{Re}} \tag{12}$$

The corresponding voltage is what the book calls E'_q, to remind us that it is a voltage in phase with the q-axis, i.e.,

$$\sqrt{3}E'_q = \frac{\lambda_F}{L_F} kM_F \omega_{Re} \tag{4.59}$$

We would like to per-unitize the above relation. To do so, recall the per-unit relations:

$$E'_q = E'_{qu}V_B, \qquad kM_F = kM_{Fu}M_{FB}, \qquad L_F = L_{Fu}L_{FB}, \qquad \lambda_F = \lambda_{Fu}\lambda_{FB} = \lambda_{Fu}L_{FB}I_{FB}$$

Substituting into 4.59, we have that

$$\sqrt{3}E'_{qu}V_B = \frac{\lambda_{Fu}L_{FB}I_{FB}}{L_{Fu}L_{FB}} kM_{Fu}M_{FB}\omega_{Re}$$

Bring over V_B to the right-hand-side and simplify, to obtain

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu}I_{FB}}{V_B L_{Fu}} kM_{Fu}M_{FB}\omega_{Re}$$

Rearranging the right-hand-side

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu}M_{FB} \frac{I_{FB}\omega_{Re}}{V_B}$$

Noting that

$$\frac{I_{FB}}{V_B t_B} = \frac{1}{M_{FB}}$$

we may substitute to obtain

$$\sqrt{3}E'_{qu} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu}M_{FB} \frac{1}{M_{FB}} = \frac{\lambda_{Fu}}{L_{Fu}} kM_{Fu}$$

But in per-unit, $kM_{Fu}=L_{AD}$, and dropping the per-unit notation results in

$$\sqrt{3}E'_q = \frac{\lambda_F}{L_F} L_{AD} \tag{4.203}$$

- We also need to define a "stator-side" quantity corresponding to the field voltage v_F, as v_F is our forcing function (and so we need to obtain the forcing function on the stator-side). This can be obtained by recognizing that in steady-state, $i_F=v_F/r_F$, Using (10), repeated here for convenience,

$$\sqrt{3}E = i_F kM_F \omega_{Re}$$

and denoting the stator-side emf as E_{FD}, we have

$$\sqrt{3}E_{FD} = \frac{v_F}{r_F} kM_F \omega_{Re}$$

Going through a similar process as in previous bullet to convert to per-unit, we get

$$E_{FD} = \frac{L_{AD} v_F}{\sqrt{3} r_F} \tag{4.209}$$

- Finally, we need to obtain stator-side quantities of
  - o Λ_d, the pu value of the stator equivalent flux linkage corresponding to the d-winding flux linkage λ_d,
  - o Λ_q, the pu value of the stator equivalent flux linkage corresponding to the q-winding flux linkage λ_q
  - o The d- and q- winding voltages v_d and v_q.

  To obtain these, we recall that when we applied Park's transformation to a set of balanced a-b-c (stator-side) voltage, we got:

$$\begin{bmatrix} v_0 \\ v_d \\ v_q \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{3} V \sin\alpha \\ \sqrt{3} V \cos\alpha \end{bmatrix} \tag{4.43}$$

  From (4.203), (4.209), and the above, we conclude that the pu value of any d or q axis quantity is numerically equal to √3 times the pu quantity on the stator side. Therefore, the stator-side per-unit equivalents of rotor side quantities are the rotor side quantity divided by √3. And so we have:

$$\Lambda_d = \frac{\lambda_d}{\sqrt{3}} \quad \Lambda_q = \frac{\lambda_q}{\sqrt{3}} \qquad V_d = \frac{v_d}{\sqrt{3}} \quad V_q = \frac{v_d}{\sqrt{3}} \tag{4.212}$$

One confusing issue in the book is whether E'_q, Λ_d, Λ_q, and E_{FD} are given as RMS quantities or pu. All of the above equations (4.203, 4.212, and 4.209) are in per-unit.

With the above relations, we may perform substitution into the model 3 state equations and then perform a considerable amount of algebra to obtain the state equations for the E'_q model, given as follows:

$$
\begin{bmatrix} \dot{\Lambda}_d \\ \dot{\Lambda}_q \\ \dot{E}'_q \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} =
\begin{bmatrix}
\frac{-r}{L'_d} & -\omega & \frac{r}{L'_d} & 0 & 0 \\
\omega & \frac{-r}{L_q} & 0 & 0 & 0 \\
\frac{L_d - L'_d}{\tau'_{d0} L'_d} & 0 & \frac{-L_d}{\tau'_{d0} L'_d} & 0 & 0 \\
\frac{1}{\tau_j}\Lambda_q\left(\frac{1}{L'_d} - \frac{1}{L_q}\right) & \frac{-E'_q}{\tau_j L'_d} & 0 & \frac{-D}{\tau_j} & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix} \Lambda_d \\ \Lambda_q \\ E'_q \\ \omega \\ \delta \end{bmatrix} +
\begin{bmatrix} -V_d \\ -V_q \\ \frac{1}{\tau'_{d0}} E_{FD} \\ \frac{1}{\tau_j} T_m \\ -1 \end{bmatrix}
$$

We may generate a block diagram for this model by taking the LaPlace transform of all equations. The resulting block diagram is shown in Fig. 1.

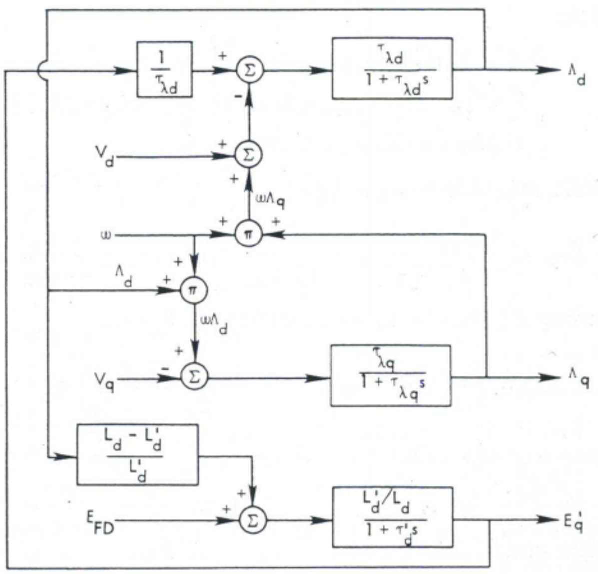

Fig. 4.9 Block diagram representation of the E'_q model.

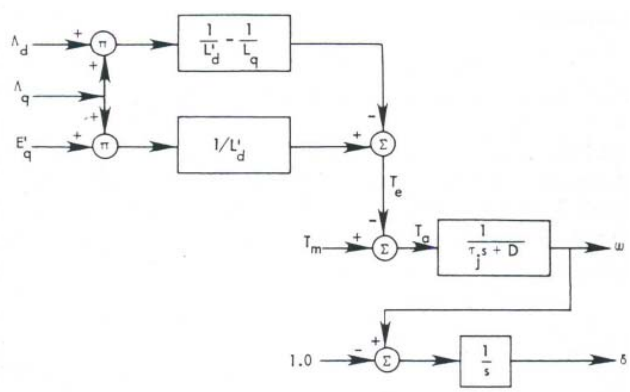

Fig. 4.10 Block diagram representation of (4.218)–(4.220).

Two final notes with respect to this model:

1. The inputs are E_{FD} (stator-side field voltage which is regulated by the excitation system) and T_m (mechanical power which is regulated by the turbine-governor system).
2. The voltages V_d and V_q are functions of the external network.

Model 5 (2.1): E'' model, for a salient pole machine & $d\lambda_d/dt= d\lambda_q/dt=0$.

The intent of this model is to develop a simple model but one that does account for the effects of both damper windings. So this model includes both transient and subtransient effects. However, we do not represent the G-winding, therefore it should only be used for a salient pole machine.

Recall from our fundamental voltage equations (4.36) that:

$$v_d = -ri_d - \dot{\lambda}_d - \omega\lambda_q$$

$$v_q = -ri_q - \dot{\lambda}_q + \omega\lambda_d$$

(Note the above are in MKS, not per-unit)
An important simplifying assumption for this model is that

$$\left|\dot{\lambda}_d\right| << \left|\omega\lambda_q\right|$$

$$\left|\dot{\lambda}_q\right| << \left|\omega\lambda_d\right|$$

We will use E'_q as a state, as defined for model 4. But we will also define one new state:

- The d-axis component of the EMF produced by subtransient flux voltages:

$$e''_d = \omega\lambda_q$$

(There is a corresponding q-axis component, defined by $e''_q=-\omega\lambda_q$, but it will not be used as a state, since we have E'_q.)

There are three basic steps to the development of this model given in the book. I refer to these steps as Step A, Step B, and Step C. Each one has several sub-steps, as summarized in what follows:

Step A: Derive the auxiliary equations.

Step A-1: Derive auxiliary equations for e''_q and e''_d.

1. Substitute expressions for currents i_d and i_q (4.134) into an the equations for λ''_d and λ''_q (4.230).
2. Use $\sqrt{3}E'_q=L_{AD}\lambda_F/L_F$ to write in terms of E'_q.
3. Express $e''_q=\omega\lambda_q$ and $e''_d=-\omega\lambda_d$ to get (4.243, 4.245).

Step A-2: Derive the auxiliary equation for E (4.248).
Step A-3: Derive the auxiliary equation for i_D.

Step B: Derive the differential equations.

For each of these, we begin from the voltage equation from the corresponding winding.

Step B-1: D-axis damper: Derive differential equation for λ_D.
Step B-2: Q-axis damper: Derive differential equation for e''_d (which is produced by $d\lambda_q/dt$).
Step B-3: Field winding: Derive differential equation for E'_q (which is produced by $d\lambda_F/dt$).

State equations are given by (4.263, 4.264, 4.265, 4.267, 4.268).

Step C: Convert to a state-space form:

Step C-1: Convert the state variables to stator-side equivalents by dividing by $\sqrt{3}$, and define the constants K_1-K_4.
Step C-2: Bring in the inertial equations. This results in the equations of (4.270) in the text, which are written in the LaPlace domain (with "s" indicating differentiation).
Step C-3: Write equations (4.270) in the time domain and express them in matrix form to get a state-space model (your book does not do this part, so you do it).

$$\begin{bmatrix} \dot{E}_d'' \\ \dot{\Lambda}_d \\ \dot{E}_q' \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{bmatrix} E_d'' \\ \Lambda_d \\ E_q' \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} \\ \\ \\ \\ \\ \end{bmatrix}$$

Note: We have 5 states in this model:

$$\delta, \omega, E''_d, \Lambda_d, E'_q$$

but we are modeling the following windings:

d, q, F, D, Q

With the E'_q model (model 4), we only had 3 windings:

d, q, F

but we also had 5 states

$$\delta, \omega, \Lambda_d, \Lambda_q, E'_q$$

Why is it that we are modeling more windings in the E'' model than in the E'_q model, but we have the same number of states????

Because in the E'' model, we set $d\lambda_d/dt = d\lambda_q/dt = 0$, thus eliminating two stator states.

Model 6 (1.1): "Two-axis model" for a machine with solid round rotor & $d\lambda_d/dt = d\lambda_q/dt = 0$ (pg. 138-139)

This model accounts for the transient effects but not the subtransient effects. It includes only two rotor circuits, F and G. So we are neglecting the D- and Q- damper windings.

Also, we let $d\lambda_d/dt = d\lambda_q/dt = 0$, as in the E'' model.

Model 7 (1.0): One-axis model.