

# Power Angle & Time Domain Plots

## 1.0 The power-angle curves

Consider a generator connected to an infinite bus through a network, as shown in Fig. 1 below (all in pu).

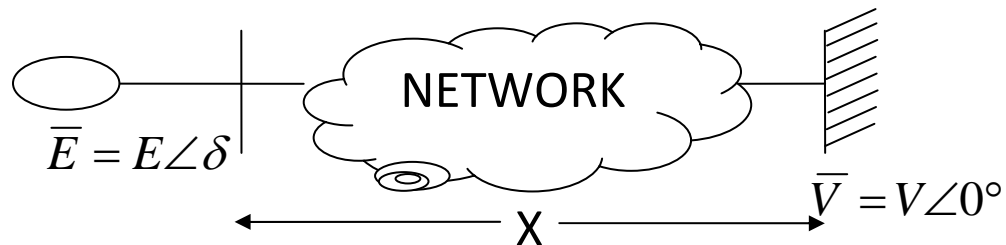


Fig. 1

We assume the network is lossless (all  $R=0$ ), the machine is modeled classically, and that  $X$  includes all network and machine reactance. We consider three situations, consistent with our previous notes.

1. Steady-state operating conditions (pre-fault):

$$P_{e1} = P_{M1} \sin \delta \quad (1)$$

2. Fault-on conditions:

$$P_{e2} = P_{M2} \sin \delta \quad (2)$$

3. Post-fault conditions:

$$P_{e2} = P_{M2} \sin \delta \quad (3)$$

Inspection of (1), (2), and (3) indicates that the electrical power out of a machine is a sinusoidal function of angle  $\delta$ . We can and will explore plots of these three power-angle curves, but before doing that, let's observe that in general, we will have:

$$P_{M2} < P_{M3} \leq P_{M1} \quad (4)$$

$$P_{Mi} = \frac{|E||V|}{X_i} \quad (5)$$

where we assume that  $|E|$  and  $|V|$  are constant throughout the sequence, and  $X_i$  is the impedance between the internal machine and the infinite bus under condition  $i=1, 2, 3$ .

Some comments about  $X_i$ :

- For a fault at the machine terminals,  $X_2 = \infty$ , and  $P_{M2} = 0$ .
- In general, faults occur away from the terminals, in the network and so  $P_{M2} > 0$ .
- It is possible for  $P_{M3} = P_{M1}$ . How?

Answer:  $P_{M3}=P_{M1}$  if the fault is only temporary, and the post-fault network is the same as the pre-fault network.

We draw the power-angle curves, Fig. 2, which show the dependence of  $P_{ei}$  on  $\delta$ . It also shows the mechanical power,  $P_m$ , the pre-fault stable equilibrium angle  $\delta_1$  ( $P_{e1}=P_m$ ), the post-fault stable equilibrium angle  $\delta_3$  ( $P_{e3}=P_m$ ), & the post-fault unstable equilibrium angle  $\delta_m$ .

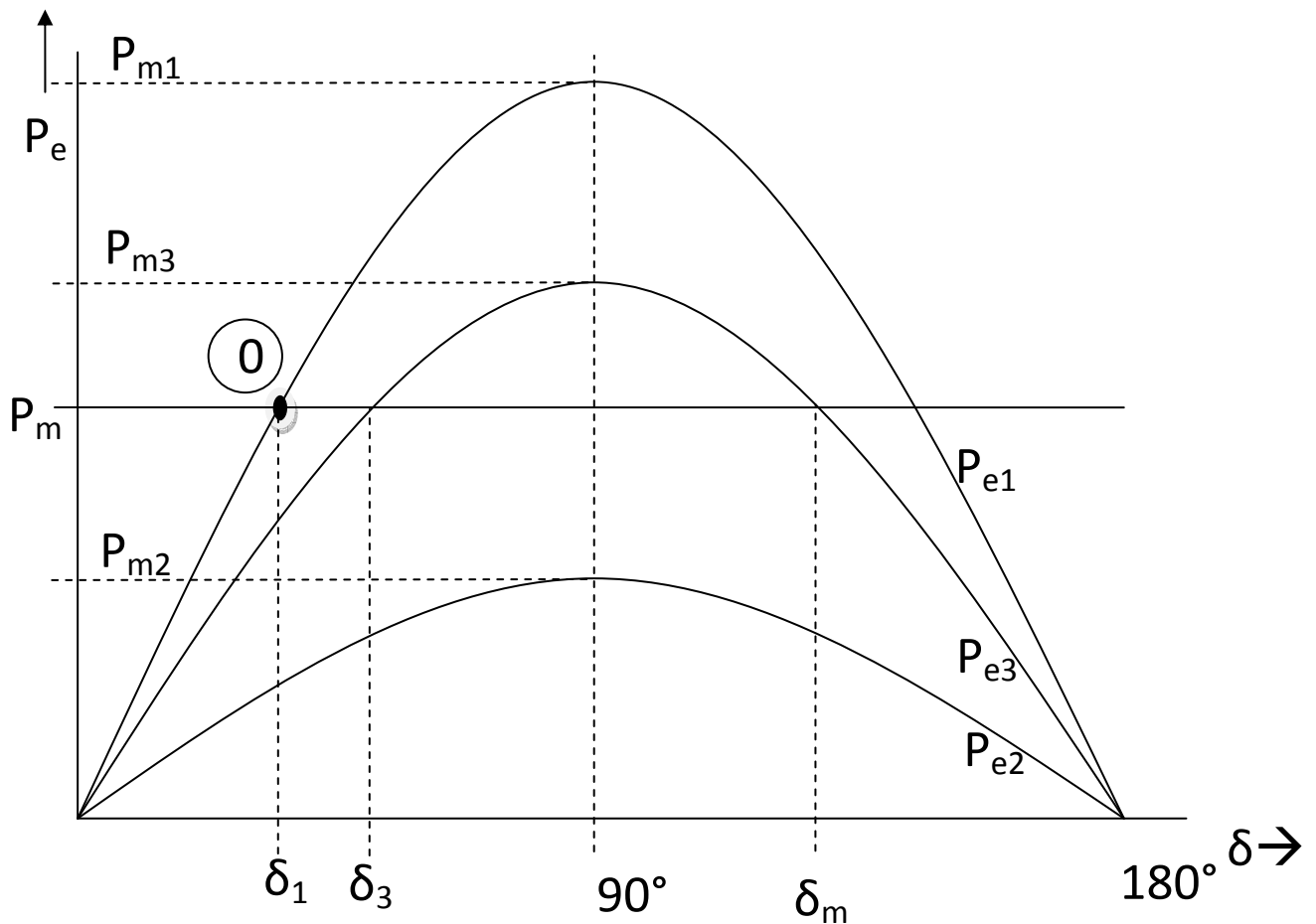


Fig. 2

Recall that  $\delta$  is relative to a moving, synchronous reference,  $\omega$  is relative speed and is zero if  $\omega_m = \omega_R$ .

Now we move through the sequence of events associated with occurrence of a fault, illustrating each change on the power-angle curves of Fig. 3.

0: At  $\delta_1$ , under the pre-fault condition,  $P_m - P_{e1} = 0 \rightarrow P_a = 0$ .

1: At  $t = 0^+$ , fault occurs, and  $P_e = P_{e2}$ . The angle remains  $\delta_1$ .

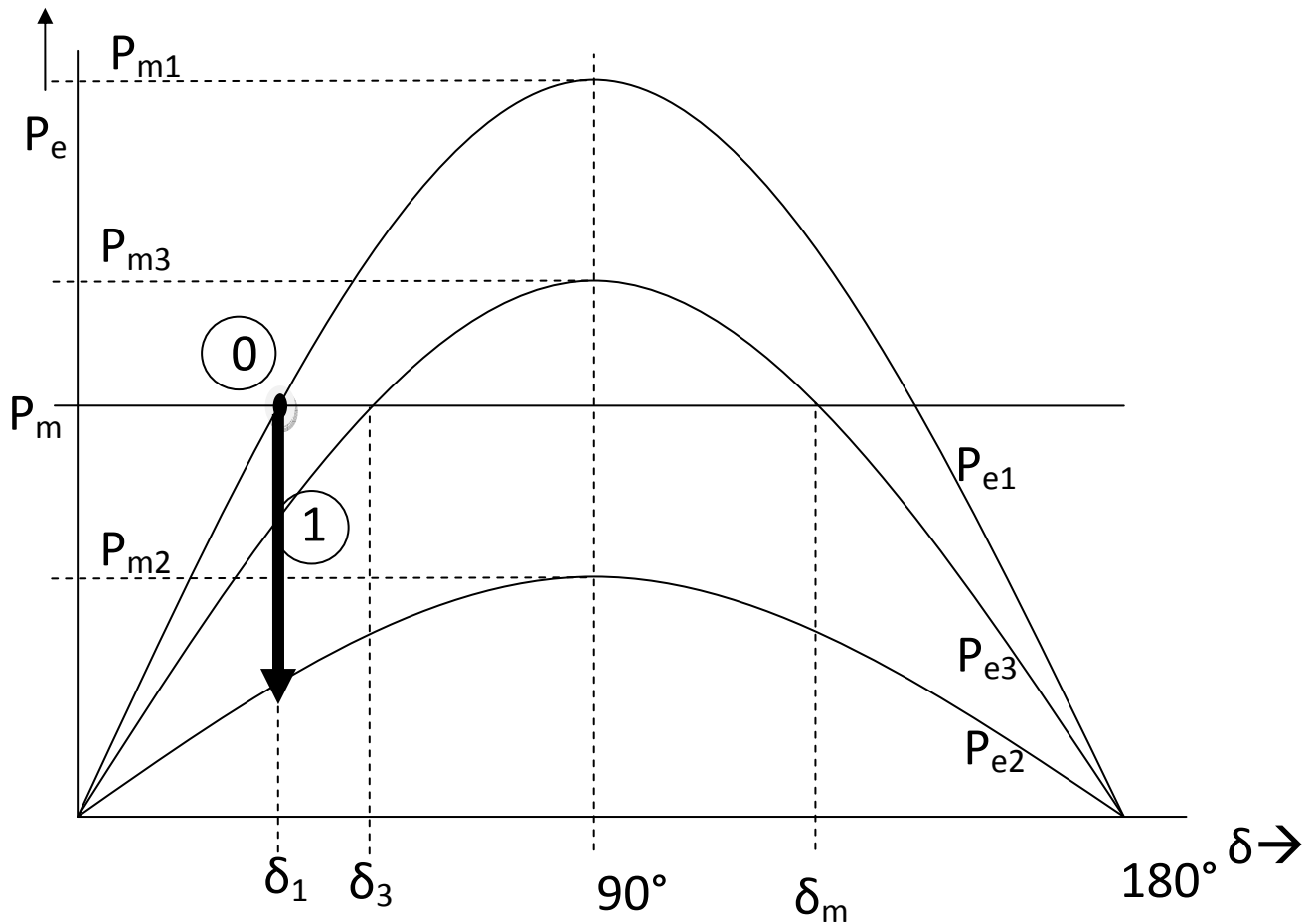


Fig. 3

2. From  $t=0^+$  to  $t_c$  (corresponding to  $\delta_c$ ),  $P_m - P_{e2} = P_{a2} > 0$ . So the machine accelerates, and  $\delta$  increases along the  $P_{e2}$  curve.

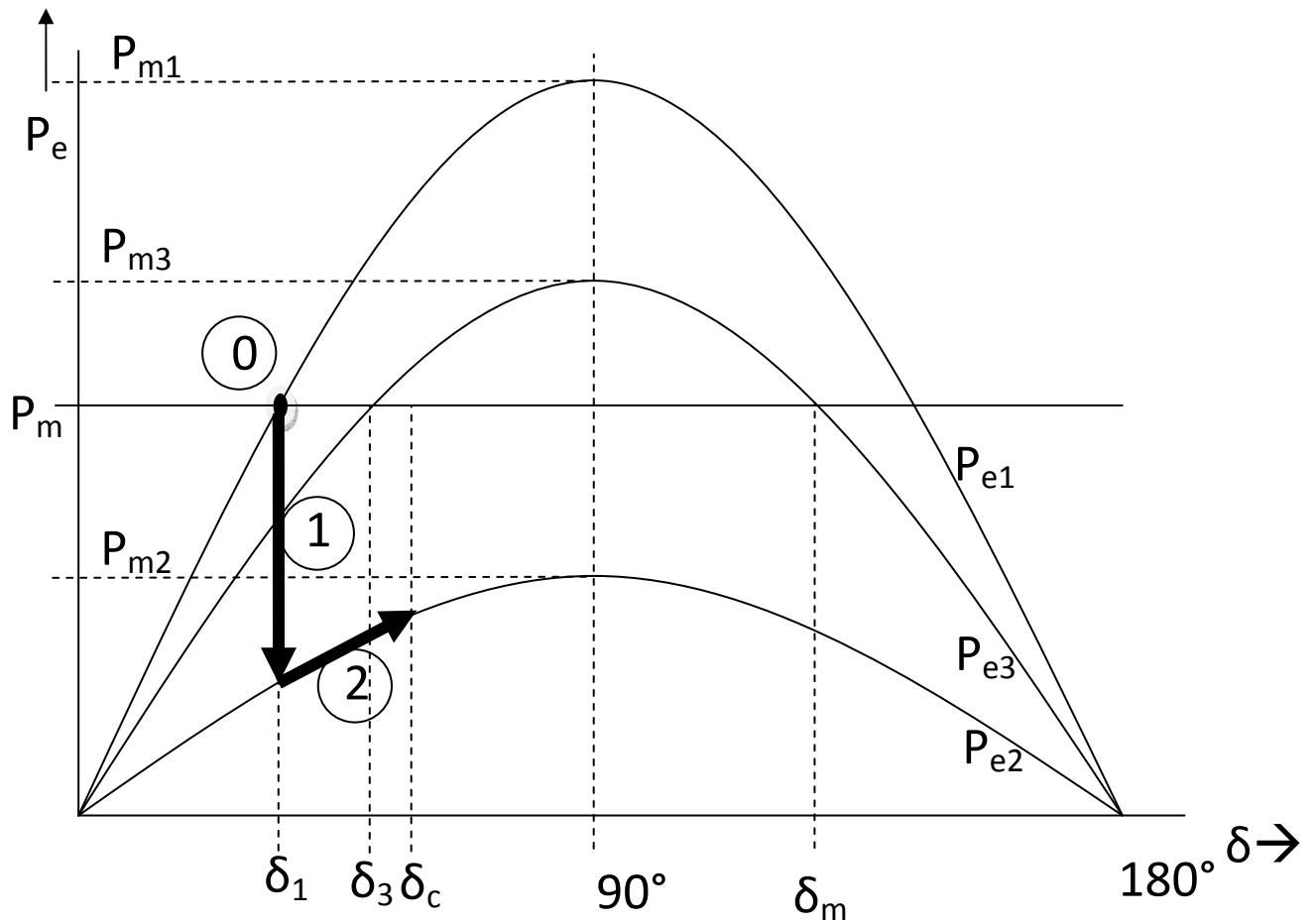


Fig. 4

3. At  $t=t_c$  (corresponding to  $\delta_c$ ), the breakers operate and remove the faulted line. Electric power  $P_e$  immediately changes (increases) to  $P_{e3}(\delta_c)$ , but  $\delta$  does not change.

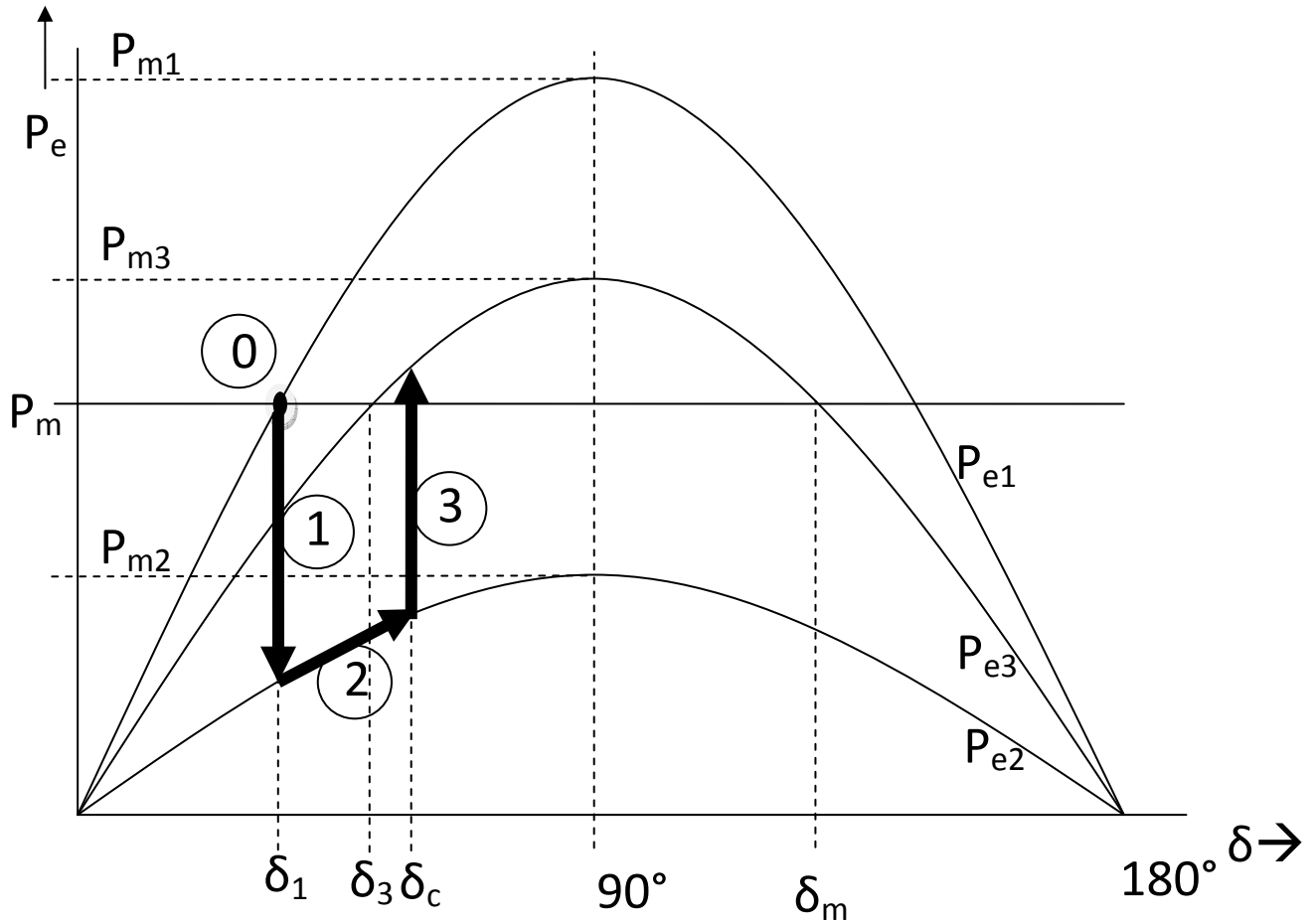


Fig. 5

4. For  $t$  just after  $t_c$ ,  $P_m - P_{e3} = P_{a3} < 0$ . The fact that accelerating power is negative means that the machine decelerates, i.e., its angular velocity decreases. However, it is important to realize that  $\omega$  is still positive, i.e., the angle is increasing and it continues to increase until the velocity reaches 0, at, say,  $\delta_r$ .

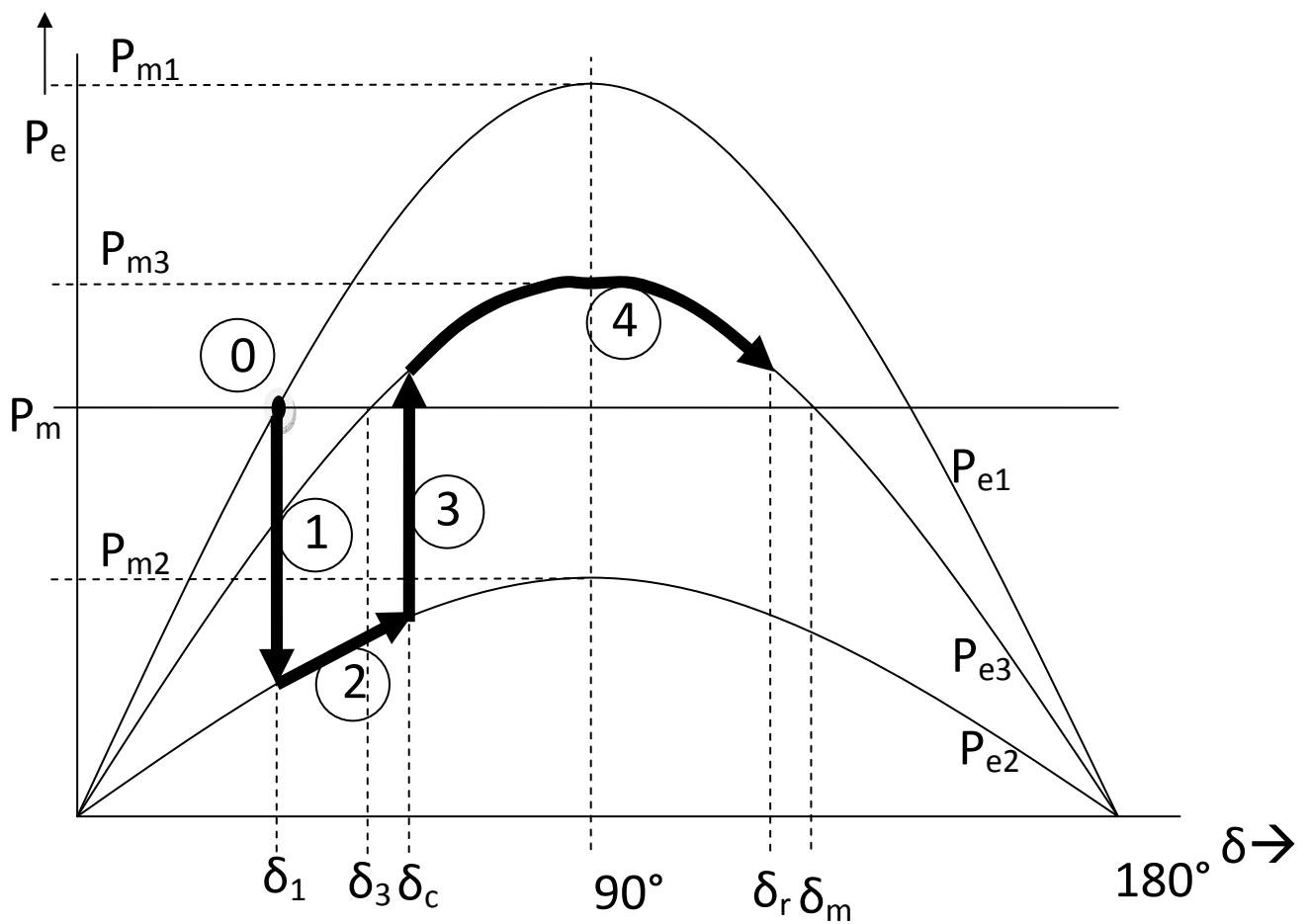


Fig. 6

And so  $\delta$  continues to increase while speed decreases, as long as  $\delta < \delta_m$ . If the system is stable, then at the angle  $\delta_r$ , the speed reaches 0 and the angle  $\delta$  begins to decrease.

5. The angle decreases from  $\delta_r$  because velocity is negative. Velocity continues to be negative because  $P_a < 0$ . This continues until  $\delta = \delta_3$ .

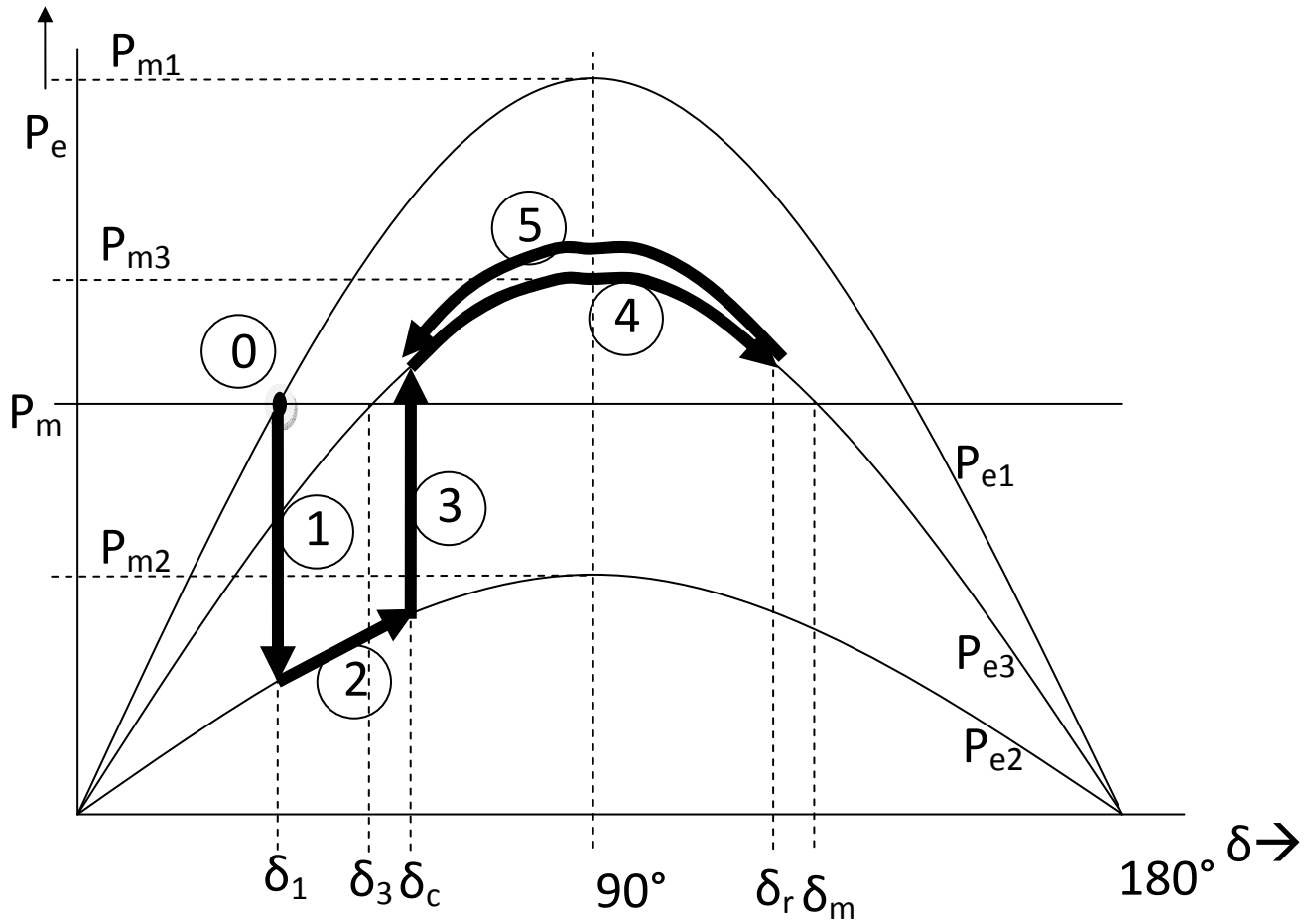


Fig. 7

6. As the rotor swings through the point  $\delta = \delta_3$ , accelerating power goes positive, i.e.,  $P_m - P_{e3} = P_{a3} > 0$ , and velocity begins to decrease (becomes less negative). The rotor will swing to some minimum angle, where its velocity will reach zero, it will turn around, and come back.

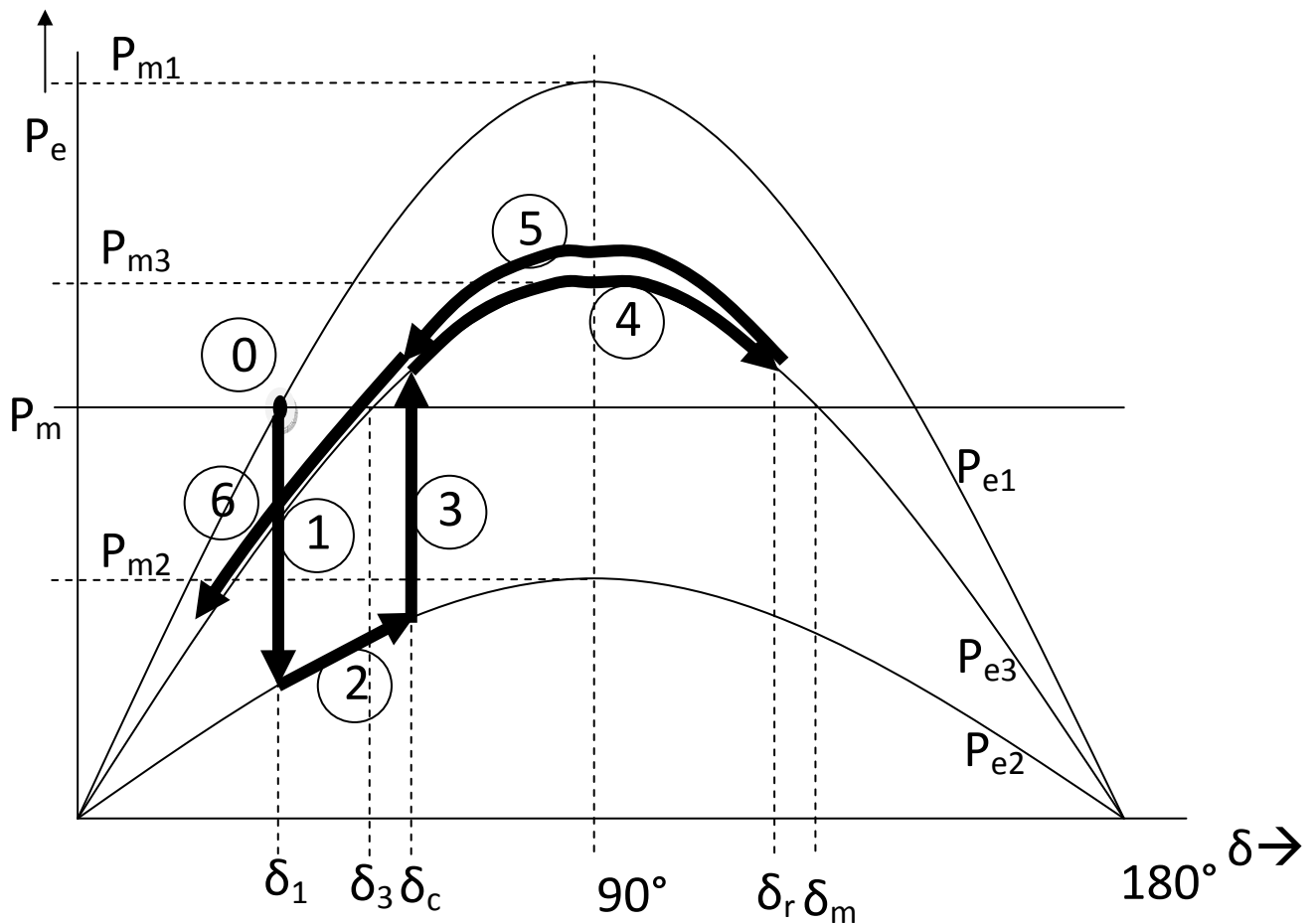


Fig. 8

7. The rotor will then oscillate back and forth about the post-fault stable equilibrium point. If there is no damping, the oscillations will continue forever. If there is damping, the oscillations will continuously decrease in amplitude and after some time, the system will stabilize at the post-fault stable equilibrium point corresponding to  $\delta = \delta_3$ .

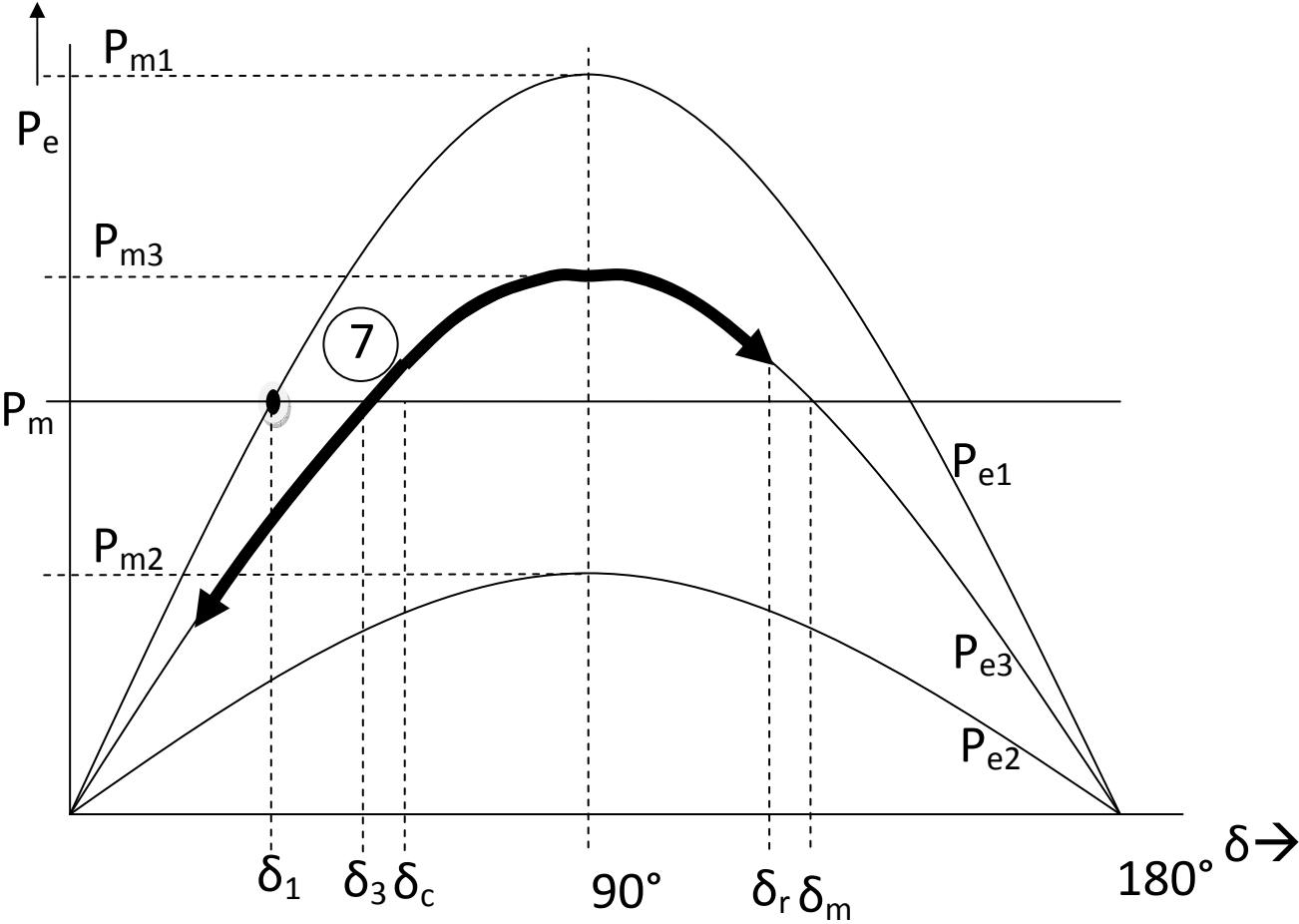


Fig. 9

## 2.0 Time-domain plots

We return to the example we worked on in the last set of notes where we had:

$$\text{Pre-fault: } \ddot{\delta}(t) = 30.16 - 83.8071 \sin \delta$$

$$\text{Fault-on: } \ddot{\delta}(t) = 30.16 - 38.0958 \sin \delta$$

$$\text{Post-fault: } \ddot{\delta}(t) = 30.16 - 59.8638 \sin \delta$$

We can observe the oscillations mentioned in the last section by plotting the two state variables against time. In order to do this, we need to convert the second-order equation to two first order equations. We will do this to the fault-on equation and to the post-fault equation. Recognizing that  $\dot{\delta}(t) = \omega(t)$  and  $\ddot{\delta}(t) = \dot{\omega}(t)$ , we may write the fault-on swing equation as:

$$\begin{aligned} \dot{\delta}(t) &= \omega(t) \\ \dot{\omega}(t) &= 30.16 - 38.0958 \sin \delta \end{aligned} \quad (29)$$

Similarly, the post-fault equation may be written as

$$\begin{aligned} \dot{\delta}(t) &= \omega(t) \\ \dot{\omega}(t) &= 30.16 - 59.8638 \sin \delta \end{aligned} \quad (30)$$

Solution to these equations must be done numerically. We have not yet discussed numerical solutions of differential equations, but we will later in the course. For now, I will simply provide you with the necessary Matlab commands.

Before doing this, however, it is necessary to realize our problem is an initial value problem, i.e., in order to solve it, we must know initial values. Since our variables are  $\delta$  and  $\omega$ , we observe that the initial values we must find are  $\delta(0)$  and  $\omega(0)$ .

Of course, the initial value of  $\omega$ , speed deviation, is very simple. Since the system is initially “at rest,” the initial speed deviation is zero.

But what about the angle  $\delta$ ? To get this, we return to the pre-fault equation, and solve it for  $P_a = P_m - P_{e1} = 0$ , i.e.,

$$\begin{aligned}\ddot{\delta}(t) &= 30.16 - 83.8071 \sin \delta = 0 \\ \Rightarrow 83.8071 \sin \delta &= 30.16 \\ \Rightarrow \delta &= 0.3681 \text{ rad}\end{aligned}$$

And so we see that  $\delta(0) = 0.3681$  radians ( $21.09^\circ$ ). This is what we have called  $\delta_1$  in all of our power-angle plots.

Now we are in a position to perform numerical solution. Note that Matlab offers a number of different solvers for ordinary differential equations. I choose ode45. The Matlab code is below (damping is zero).

```

%initial conditions have delta at 21.09degrees and omega at 0.
delta0=21.09*pi/180;
X0=[delta0;0];
% Simulation time from 0 to 4 cycles.
TCLEAR=4/60;
TSPAN = [0 TCLEAR];
%call ODE solver to integrate from 0 to TCLEAR
[t, x]=ODE45('go1',TSPAN,X0);
%
% Now prepare to perform the post-fault integration.
%
% First, get initial conditions for post-fault
% integration. This will be the final conditions
% for the fault-on integration.
last=length(t);
X0=[x(last,1); x(last,2)];
%Save solution to another matrix so it does
%not get overwritten. But do not save last point
%because last point will be first point in
%new solution.
xold=x(1:last-1,:);
told=t(1:last-1);
% Simulation time from 4 cycles to 5 seconds
TSPAN = [TCLEAR 5];
%call ODE solver to integrate from TCLEAR to 5 seconds.
[t, x]=ODE45('go2',TSPAN,X0);
% Now join xhold with x.
delta=[xold(:,1)' x(:,1)']';
delta=delta*180/pi;
omega=[xold(:,2)' x(:,2)']';
ttot=[told' t']';
plot(ttot,delta)
grid
ylabel('Angle, delta (degrees)')
xlabel('Time (seconds)')
plot(ttot,omega)
grid
xlabel('Time (seconds)')
ylabel('Speed deviation, omega (rad/sec)')

```

The two functions “go1” and “go2” are given below.

```
% FUNCTION
function xp1 = go1(t,x)
%
% In:    t (time) is a scalar.
%        x is a 2-element vector whose elements are:
%        x(1)= delta(t)
%        x(2)= omega(t)
%
% Out:   xp1 is a 2-element vector whose elements are
%        xp1(1)=d delta(t)/dt
%        xp1(2)=d omega(t)/dt
%
xp1(1)=x(2);
xp1(2)=30.16-38.095*sin(x(1));
% ODE45 expects the function to return a column vector.
xp1=xp1';
```

```
% FUNCTION
function xp2 = go2(t,x)
%
% In:    t (time) is a scalar.
%        x is a 2-element vector whose elements are:
%        x(1)= delta(t)
%        x(2)= omega(t)
%
% Out:   xp2 is a 2-element vector whose elements are
%        xp2(1)=d delta(t)/dt
%        xp2(2)=d omega(t)/dt
%
xp2(1)=x(2);
xp2(2)=30.16-59.8638*sin(x(1));
% ODE45 expects the function to return a column vector.
xp2=xp2';
```

The resulting plots are shown in Figs. 10 and 11.

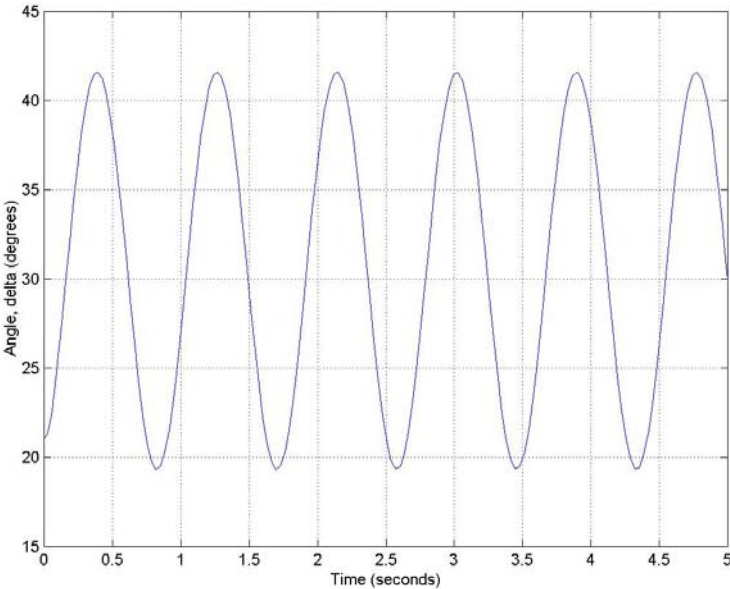


Fig.10: Angle

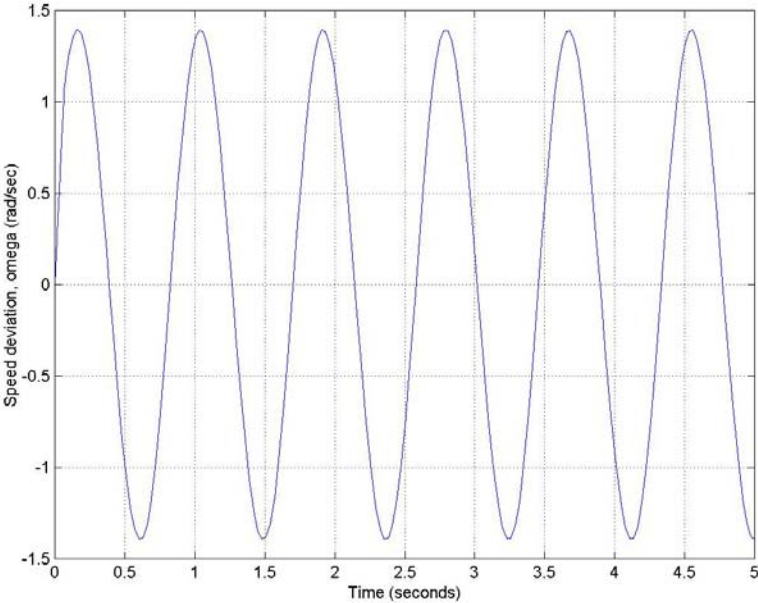


Fig. 11: Speed deviation

Observe in the plots the following:

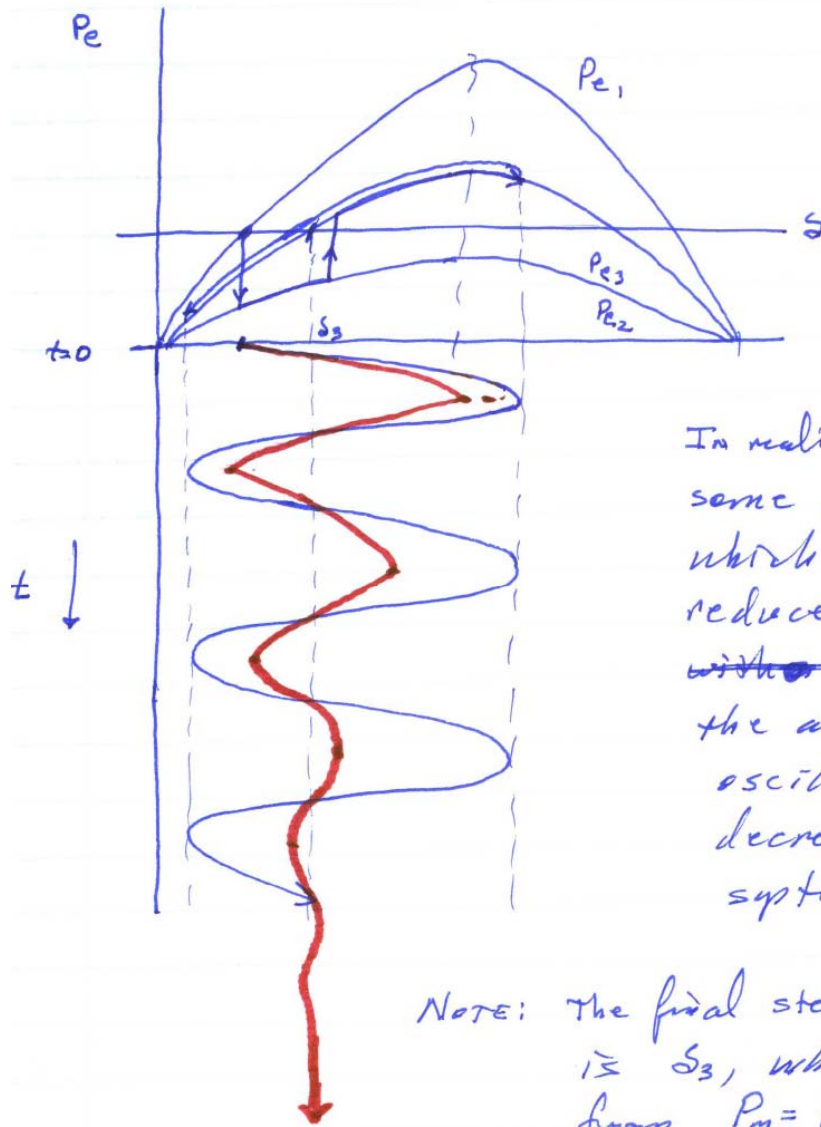
- Initial angle is  $21.09^\circ$ ; initial speed deviation is 0.
- Oscillations are undamped.
- Angle oscillates about a value just above  $30^\circ$ . We can verify this by solving the post-fault equation for the equilibrium as follows:

$$\begin{aligned}\ddot{\delta}(t) &= 30.16 - 59.8638 \sin \delta = 0 \\ \Rightarrow 59.8638 \sin \delta &= 30.16 \\ \Rightarrow \delta &= 0.528 \text{ rad}\end{aligned}$$

This is  $30.2524^\circ$ .

- Speed deviation oscillates about a value of 0.

We have developed two different ways of understanding synchronous machine response to faults: the power angle curve and the time-domain simulation. You should understand how these two ways of viewing the situation are related. Fig. 12 on the next page illustrates.



In reality, we will have some positive damping which serves to always reduce the acceleration ~~with each~~ so that the amplitude of the oscillation always decreases ~~to~~ for a stable system. See red curve.

NOTE: The final steady state angle is  $S_3$ , which is found from  $P_m = P_{e3} = \frac{|E||V_0|}{X_3} \sin S_3$

Fig. 12

### 3.0 Additional observations & clarifications

We discuss two significant concepts in this section.

#### 3.1 Stability vs. instability

It is possible for the machine to accelerate too much so that the angle does not “come back” after the first swing. What causes this to happen?

➔ It is too much acceleration, or, equivalently, not enough deceleration.

Notice that

- $P_m - P_{e2}$  causes the acceleration, and this occurs between  $t=0^+$  and  $t=t_c$ . During this time interval, our system will be “safer” if we decrease  $P_m$  and increase  $P_{m2}$  (recall  $P_{m2}$  is the maximum value of the fault-on-power-angle curve ).
- $P_{e3} - P_m$  causes the deceleration, and this just after  $t=t_c$ . During this time interval, our system will be “safer” if we increase  $P_{e3}$  and decrease  $P_m$ .

(Aside: Indeed, there are “special protection systems” which perform transient excitation boosting and fast valving to do just these things!)

Overall conclusion:

The system is “less stable” when

- $P_m$  is high; then acceleration is larger and deceleration is smaller.

→ Be a little careful when you are operating a machine at maximum output.

- $P_{e2}$  is low.

→ The worst case is when  $P_{e2}=0$ . This occurs when the fault is at the machine terminals. Fig. 13 illustrates.

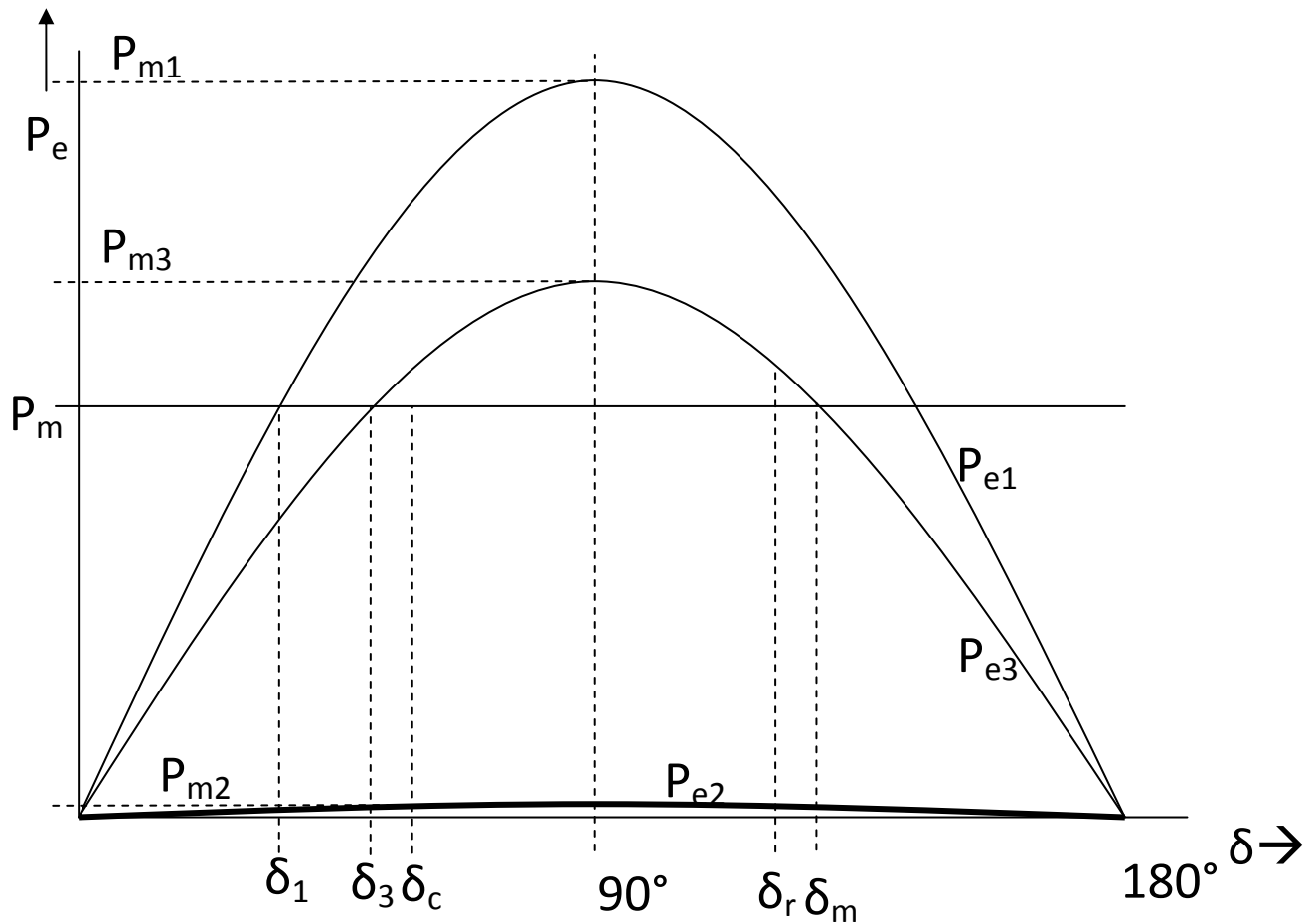


Fig. 13

- $t_c$  is large; then we will see  $P_m - P_{e2} = P_{a2} > 0$  for a longer time, & the machine has more time to accelerate.

→ So-called “stuck breaker operation” requires backup protection to clear the fault, and backup protection always takes longer time than primary protection.

- $P_{e3}$  is low.

→ Losing “strong” lines is worse than losing “weak” lines.

### 3.2 Unstable equilibrium point

What about the point  $\delta_m$ ?

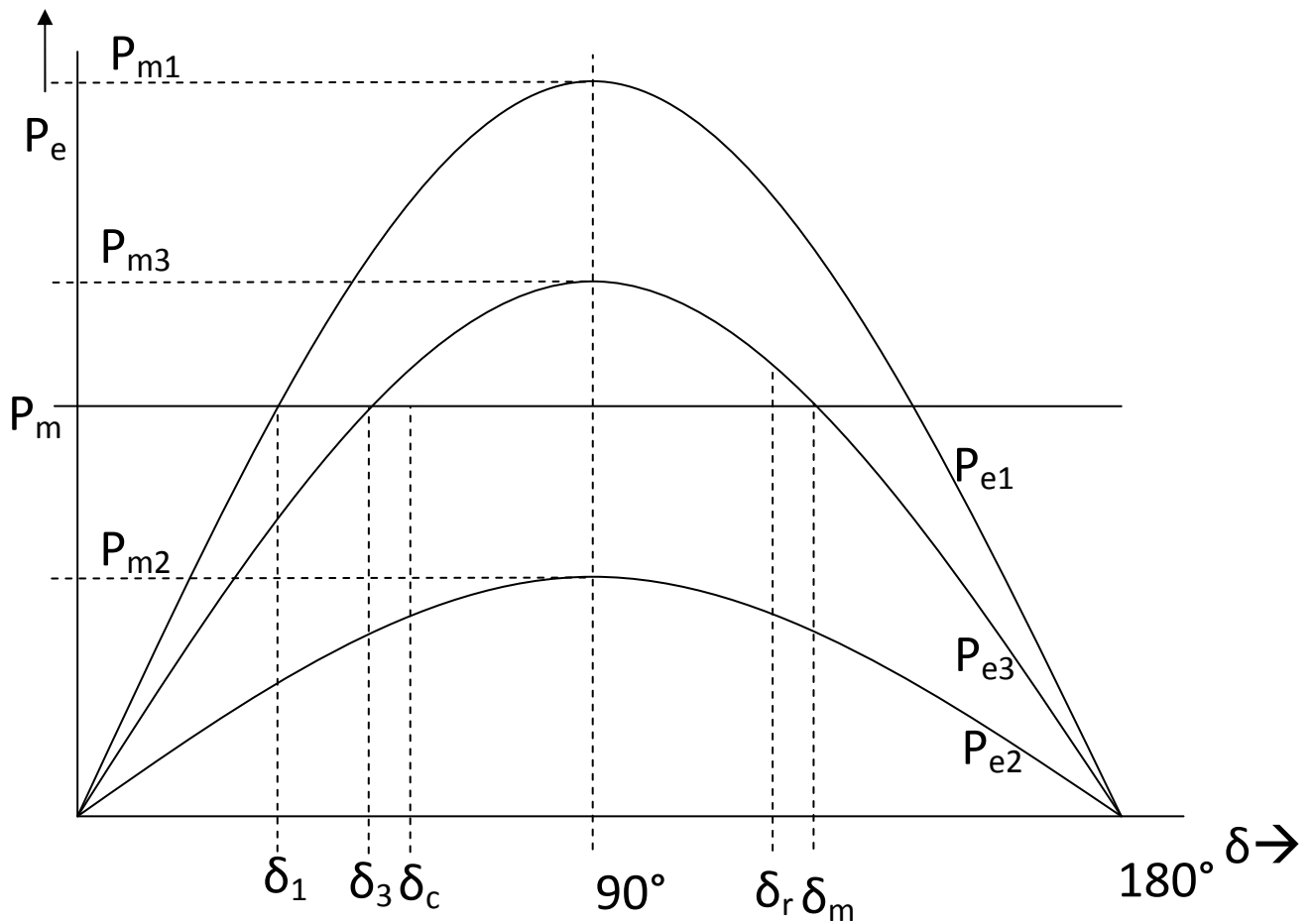


Fig. 14

- Notice that it can be computed as  $\delta_m = 180 - \delta_3$ . In the case of our example, this is  $180 - 30.25 = 149.75^\circ$ .
- This is an unstable equilibrium point. Why?

➔ If you were ever able to maneuver a generator to this point, with  $\omega=0$  (and thus in equilibrium), then

- Any disturbance causing an angle decrease would immediately result in deceleration because  $P_m < P_{e3}$ . The deceleration would cause further angle decrease, until the machine reached  $\delta_3$ . At that point, if further angle decrease occurred, then the machine would begin accelerating, tending to increase the angle once again. Eventually, the machine would settle on the stable equilibrium point at  $\delta_3$ .
- Any disturbance causing an angle increase would immediately result in acceleration because  $P_m > P_{e3}$ . The acceleration would cause further angle increase, which would cause more acceleration, and so on, with the angle increasing without bound. This is an unstable situation.

Note that this is equivalent to the case of the inverted pendulum, movement in either direction causes the system to find a new equilibrium point, as shown in Fig. 15.

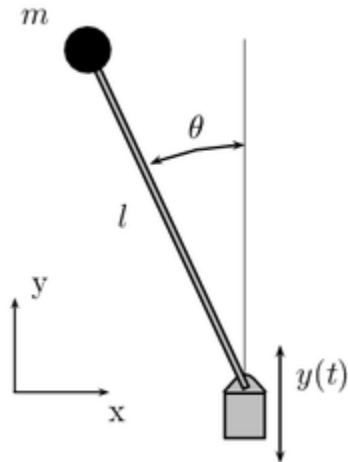


Fig. 15

- Is it possible to get the machine to operate at  $\delta_m$  by using gradual changes to  $P_m$ ? (Assume we are in post-fault state and so we the electrical power out is characterized by  $P_{e3}$ ).

➔ Answer: No! Because at  $\delta=90^\circ$ , any further increase in  $P_m$  causes a permanent state of acceleration, and the machine loses synchronism, as illustrated in Fig. 16.

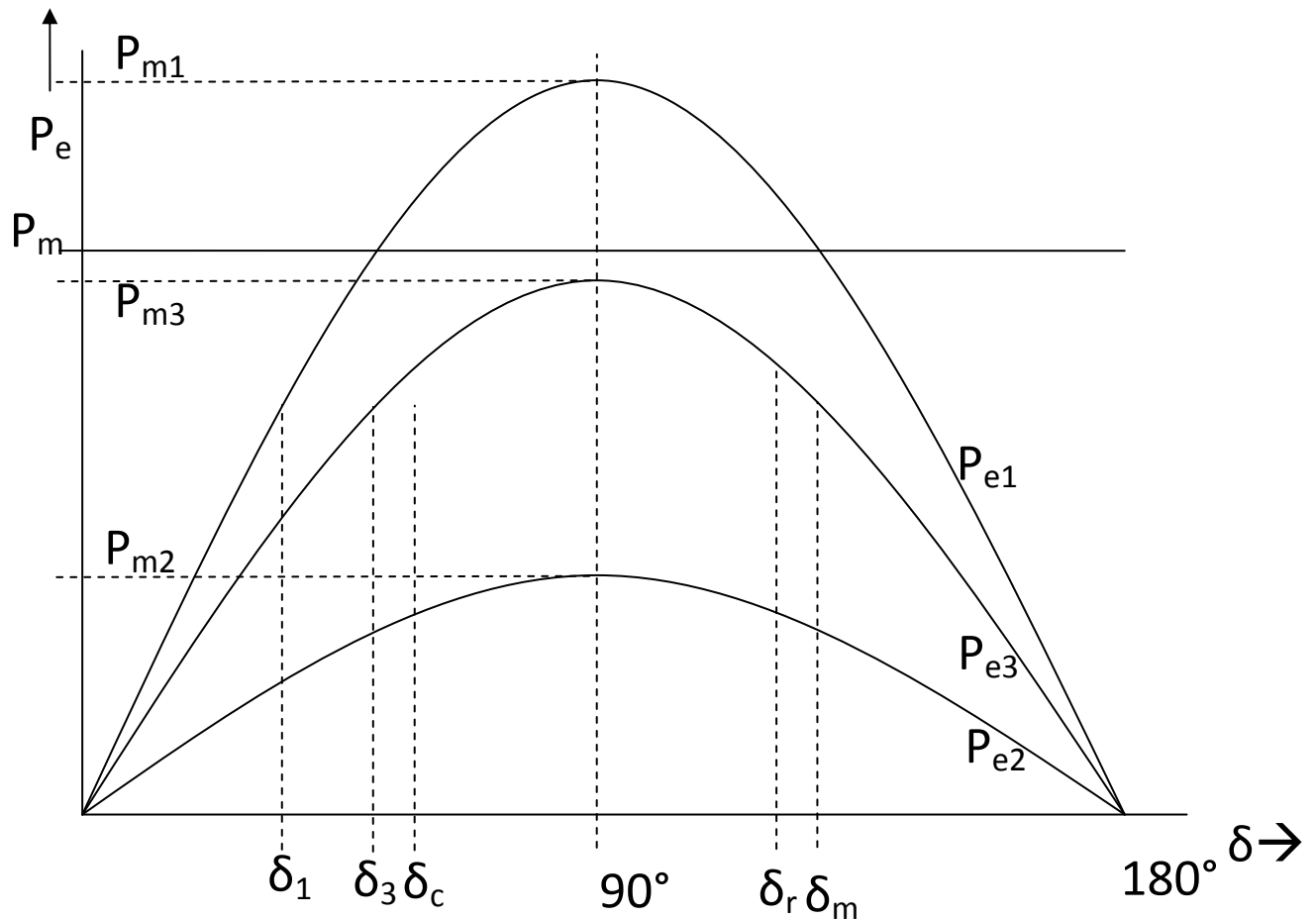


Fig. 16

- Is it possible to that the machine reaches  $\delta_m$  during a post-fault transient?

➔ Yes! If the fault-on acceleration is enough, the angle could swing to  $\delta_m$ , as shown in Fig. 17.

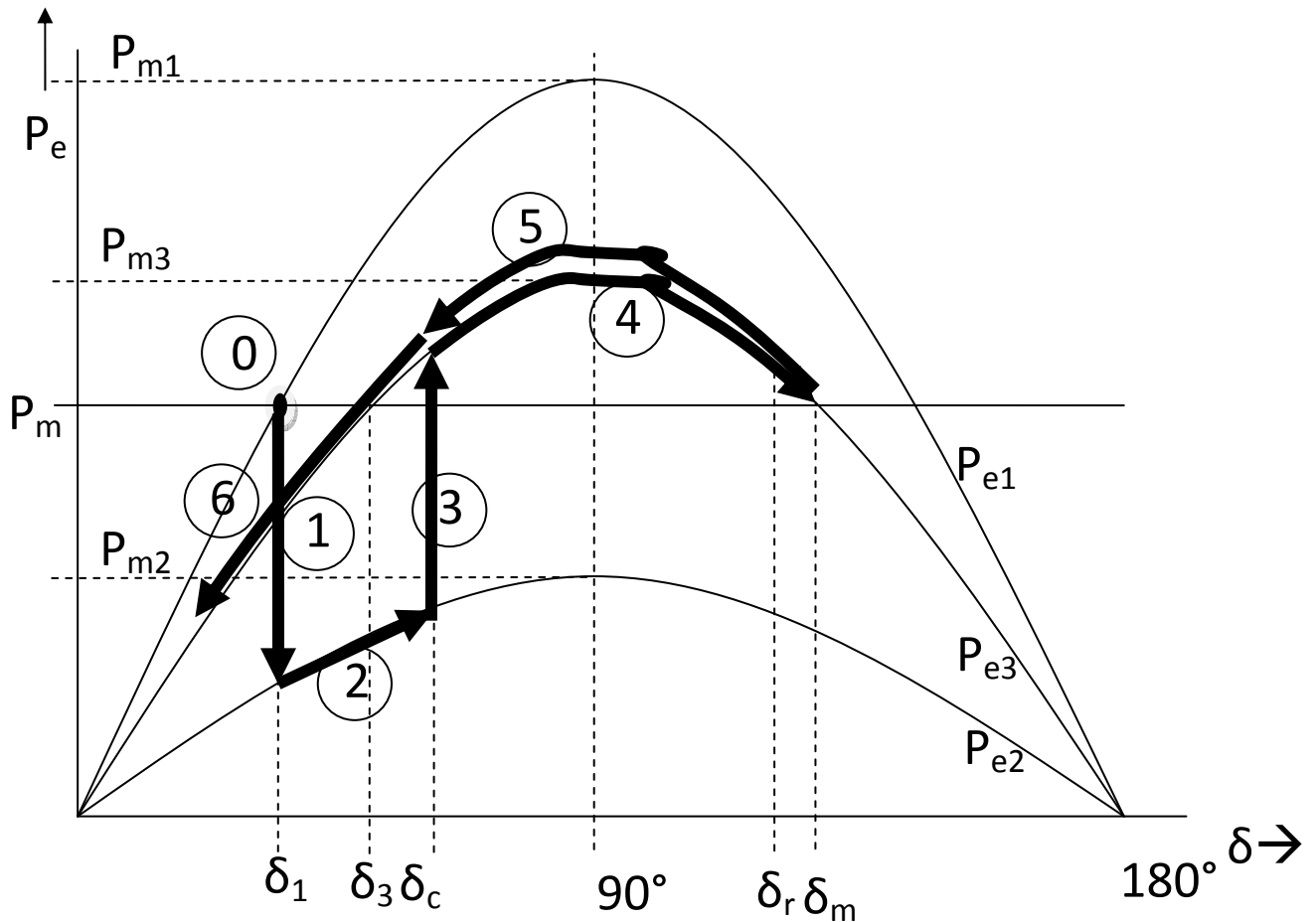


Fig. 17

- What is the significance of this point,  $\delta_m$ , during a transient?

➔ It is the maximum angular swing for stability.

If when  $\delta = \delta_m$ ,  $\omega > 0$  (which it will be on a “first swing”), the angle increases beyond  $\delta_m$ . At this point, the rotor begins to accelerate again, and there is no way to obtain deceleration because  $P_m > P_{e3}$  permanently.

And so the condition for stability is that

- $\delta_{\max} < \delta_m \rightarrow$  STABLE!
- $\delta_{\max} = \delta_m \rightarrow$  MARGINALLY STABLE
- $\delta_{\max} > \delta_m \rightarrow$  UNSTABLE