

Participation Factors

The mode shape, as indicated by the right eigenvector, gives the relative phase of each state in a particular mode.

However, it does not give the influence of each state on the mode.

We would like to be able to obtain the influence of states on modes because then we will know which states (machines) to control in order to increase damping of a certain problem mode.

Let's define a new state variable ("xi") as follows:

$$\xi_k = \underline{q}_k^T \underline{x} \Rightarrow \dot{\xi}_k = \underline{q}_k^T \dot{\underline{x}}$$

Notice that ξ_k is a scalar since the transpose of the left eigenvector is a $1 \times n$ and the state vector is an $n \times 1$.

Thus, ξ_k is a combination of all of the states, but the manner in which all of the other states is combined is through the left eigenvector elements of the k^{th} mode.

An important attribute of ξ_k , and the reason why it is of great interest here, is that it is a state which is associated with the k^{th} mode and no other mode. We can prove this as follows. Start with the system state equations:

$$\dot{\underline{x}} = \underline{A}\underline{x}$$

Pre-multiply both sides by \underline{q}_k^T .

$$\underline{q}_k^T \dot{\underline{x}} = \underline{q}_k^T \underline{A}\underline{x} \quad (\text{P-1})$$

Recall that the left eigenvector is defined as

$$\underline{q}_k^T \underline{A} = \lambda_k \underline{q}_k^T \quad (\text{P-2})$$

Notice that the left-hand-side of eq. (P-2), is on the right-hand-side of eq. (P-1). Substituting the right-hand-side of eq. (P-2) into the right-hand-side of eq. (P-1), we obtain:

$$\underline{q}_k^T \dot{\underline{x}} = \lambda_k \underline{q}_k^T \underline{x} \quad (\text{P-3})$$

Returning to the definition of our new state variable, which is $\underline{\xi}_k = \underline{q}_k^T \underline{x} \Rightarrow \dot{\underline{\xi}}_k = \underline{q}_k^T \dot{\underline{x}}$, we note the left-hand-side of eq. (P-3) is $\dot{\underline{\xi}}_k$ while the right hand side of eq. (P-3) is $\lambda_k \underline{\xi}_k$. Making these substitutions, eq. (P-3) becomes:

$$\dot{\underline{\xi}}_k = \lambda_k \underline{\xi}_k \quad (\text{P-4})$$

Now eq. (P-4) is a time-domain expression. So let's take the LaPlace transform to obtain:

$$\begin{aligned} s\underline{\Xi}_k(s) - \underline{\xi}_k(0) &= \lambda_k \underline{\Xi}_k(s) \\ \Rightarrow s\underline{\Xi}_k(s) - \lambda_k \underline{\Xi}_k(s) &= \underline{\xi}_k(0) \\ \Rightarrow (s - \lambda_k) \underline{\Xi}_k(s) &= \underline{\xi}_k(0) \\ \Rightarrow \underline{\Xi}_k(s) &= \frac{\underline{\xi}_k(0)}{(s - \lambda_k)} \end{aligned}$$

Taking the inverse LaPlace transform, we find that

$$\underline{\xi}_k(t) = \underline{\xi}_k(0)e^{\lambda_k t} \quad (\text{P-5})$$

This confirms that $\underline{\xi}_k$ is associated with only the k^{th} mode and no other mode. That is, the only dynamics associated with $\underline{\xi}_k$ are $e^{(\lambda_k)t}$.

What is the implication of this fact? The state variables that influence $\underline{\xi}_k$ are the state variables that influence the k^{th} mode. So we can study $\underline{\xi}_k$ to learn about the k^{th} mode.

Let's see if we can determine which state variables influence $\underline{\xi}_k$

Recall from the notes called "Linear system theory" eq. (L-5), which was:

$$\underline{x}(t) = \sum_{k=1}^n \left[\underline{q}_k^T \underline{x}(0) e^{\lambda_k t} \right] \underline{p}_k \quad (\text{L-5})$$

I will change the summation index so as to not confuse with the index (k) used previously in these notes:

$$\underline{x}(t) = \sum_{j=1}^n \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{p}_j \quad (\text{P-6})$$

Now substitute eq. (P-6) into the definition of ξ_k :

$$\xi_k = \underline{q}_k^T \underline{x} = \underline{q}_k^T \sum_{j=1}^n \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{p}_j \quad (\text{P-7})$$

Note that the part of the summation in brackets is a scalar. Therefore we can move the \underline{q}_k^T inside the summation, beyond the brackets, so as to pre-multiply \underline{p}_j . This results in:

$$\xi_k = \underline{q}_k^T \underline{x} = \sum_{j=1}^n \left[\underline{q}_j^T \underline{x}(0) e^{\lambda_j t} \right] \underline{q}_k^T \underline{p}_j \quad (\text{P-8})$$

Recalling that the matrices \underline{P} and \underline{Q} are orthogonal, we know that

$$\underline{q}_k^T \underline{p}_j = 0 \quad \text{for } k \neq j \quad (\text{P-9})$$

Therefore, there is only one non-zero term in the summation of eq. (P-8), and that is the term for which $k=j$. As a result, eq. (P-8) is:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) e^{\lambda_k t} \right] \underline{q}_k^T \underline{p}_k \quad (\text{P-10})$$

Rearranging slightly, we have,

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) \right] \left[\underline{q}_k^T \underline{p}_k \right] e^{\lambda_k t} \quad (\text{P-11})$$

But note:

$$\underline{q}_k^T \underline{p}_k = \begin{bmatrix} q_{1k} & q_{2k} & \cdots & q_{nk} \end{bmatrix} \begin{bmatrix} p_{1k} \\ p_{2k} \\ \vdots \\ p_{nk} \end{bmatrix} \quad (\text{P-12})$$

We may express the above vector product as a summation:

$$\underline{q}_k^T \underline{p}_k = \sum_{j=1}^n q_{jk} p_{jk} \quad (\text{P-13})$$

Substitution of eq. (P-13) into eq. (P-11), we obtain:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) \right] \left[\sum_{j=1}^n q_{jk} p_{jk} \right] e^{\lambda_k t} \quad (\text{P-14})$$

Now we are in a position to make a definition:

Participation factor:

$$\rho_{jk} = q_{jk} p_{jk} \quad (\text{P-15})$$

Substitution of eq. (P-15) in eq. (P-14) results in:

$$\xi_k = \left[\underline{q}_k^T \underline{x}(0) \right] \left[\sum_{j=1}^n \rho_{jk} \right] e^{\lambda_k t} \quad (\text{P-16})$$

The participation factor ρ_{jk} indicates the participation (influence) of the j^{th} state in the k^{th} mode.

The participation factor is extremely useful. Consider that you learn through eigenvalue calculation and/or time-domain simulation that mode k is a “problem mode,” i.e., it is marginally damped or negatively damped. Then one can identify what to do about this is by inspecting the participation factors for this mode.

There will be “ n ” participation factors for this mode, ρ_{jk} , $j=1, \dots, n$ (where “ n ” is the number of states). The states having the larger participation factors are the states which should be most strongly considered to control in order to affect this problem mode k .

Note: in contrast to $\underline{q}_k^T \underline{x}(0)$, ρ_{jk} is less dependent of the initial conditions and therefore serves as more of a universal indicator of participation than does $\underline{q}_k^T \underline{x}(0)$.

So let’s look at the big picture. How does one generally proceed in a small-signal analysis study?

1. Compute eigenvalues and eigenvectors for a certain operating condition.
2. If any $\lambda_k = \sigma_k \pm j\omega_k$ has $|\sigma_k| < \varepsilon$, or $\sigma_k > 0$, then this is a problem mode at that operating condition.
3. Identify right \underline{p}_k and left \underline{q}_k eigenvectors for mode k .
 - a. Identify “groups” of generators based on mode shape using \underline{p}_k (use the angles of the elements of \underline{p}_k corresponding to the speed deviation states).
 - b. For each group, identify the speed deviation states (and thus the generators) most heavily participating (influencing) the mode, based on ρ_{jk} .
4. Install, or retune the power system stabilizer (PSS) on the generators identified in step 3-b using speed deviation as a control signal so that they increase damping of the k^{th} mode.

Last comment: This is for linearized (small-signal) analysis, not for large-signal (fault) analysis.

Going back to our example (see notes on linear system theory), we recall that

$$\begin{bmatrix} \Delta \dot{\delta}_{13} \\ \Delta \dot{\delta}_{23} \\ \Delta \dot{\omega}_{13} \\ \Delta \dot{\omega}_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -104.096 & -59.524 & 0 & 0 \\ -33.841 & -153.460 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_{13} \\ \Delta \delta_{23} \\ \Delta \omega_{13} \\ \Delta \omega_{23} \end{bmatrix}$$

Observe the eigenvalues in table 3.2.

Table 3.2. Frequencies of Oscillation of a Nine-Bus System

Quantity	Eigenvalue 1	Eigenvalue 2
λ	$\pm j8.807$	$\pm j13.416$
ω rad/s	8.807	13.416
f Hz	1.402	2.135
T s	0.713	0.468

Also observe the relative rotor angle plots of fig. 3.3-b, where we see that one mode can be clearly observed having a period of about 0.7 sec ($f=1.4$ hz). The other mode is not readily observable, although its presence is probably responsible for the distortion seen in the δ_{31} plot.

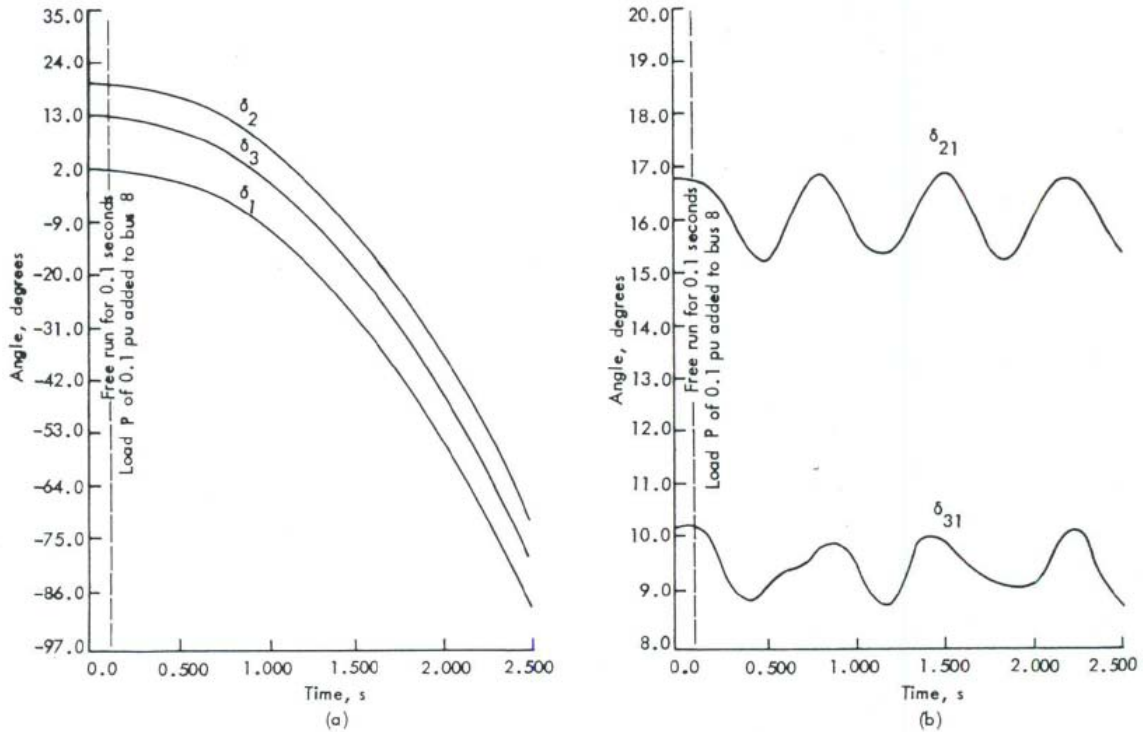


Fig. 3.3 Unregulated response of the nine-bus system to a sudden load application at bus 8: (a) absolute angles, (b) angles relative to δ_1 .

Using matlab, we use

$[P,D]=\text{eig}(A)$ where A is the matrix given above.

Then the matrix of eigenvalues D is given by

$$\begin{matrix}
 +13.4164i & 0 & 0 & 0 \\
 0 & -13.4164i & 0 & 0 \\
 0 & 0 & +8.8067i & 0 \\
 0 & 0 & 0 & -8.8067i
 \end{matrix}$$

And the matrix of right eigenvectors P is given by

$$\begin{matrix}
 -0.0459 - 0.0000i & -0.0459 + 0.0000i & -0.1030 - 0.0000i & -0.1030 + 0.0000i \\
 -0.0585 - 0.0000i & -0.0585 + 0.0000i & 0.0459 + 0.0000i & 0.0459 - 0.0000i \\
 0.0000 - 0.6154i & 0.0000 + 0.6154i & 0.0000 - 0.9075i & 0.0000 + 0.9075i \\
 0.0000 - 0.7847i & 0.0000 + 0.7847i & -0.0000 + 0.4046i & -0.0000 - 0.4046i
 \end{matrix}$$

And the matrix of left eigenvectors Q^T is given by P^{-1} , which is:

$$\begin{matrix}
 -2.8240 + 0.0000i & -6.3340 + 0.0000i & 0.0000 + 0.2105i & 0.0000 + 0.4721i \\
 -2.8240 - 0.0000i & -6.3340 - 0.0000i & 0.0000 - 0.2105i & 0.0000 - 0.4721i \\
 -3.5951 + 0.0000i & 2.8194 - 0.0000i & 0.0000 + 0.4082i & -0.0000 - 0.3201i \\
 -3.5951 - 0.0000i & 2.8194 + 0.0000i & 0.0000 - 0.4082i & -0.0000 + 0.3201i
 \end{matrix}$$

Note that here, the eigenvectors are along the rows. Taking transpose, we get Q, which is

```
-2.8240 + 0.0000i -2.8240 - 0.0000i -3.5951 + 0.0000i -3.5951 - 0.0000i
-6.3340 + 0.0000i -6.3340 - 0.0000i 2.8194 - 0.0000i 2.8194 + 0.0000i
0.0000 + 0.2105i 0.0000 - 0.2105i 0.0000 + 0.4082i 0.0000 - 0.4082i
0.0000 + 0.4721i 0.0000 - 0.4721i -0.0000 - 0.3201i -0.0000 + 0.3201i
```

Now I compute the *participation matrix* below.

```
a=[0 0 1 0; 0 0 0 1; -104.096 -59.524 0 0; -33.841 -153.46 0 0];
[P, D]=eig(a);
QT=inv(P);
Q=QT';
j=1;
% j is index on columns (modes)
% i is index on rows (states)
while j<5,
    i=1;
    while i<5,
        pf(i,j)=Q(i,j)*P(i,j);
        i=i+1;
    end
    j=j+1;
end
pf
```

This gives

```
DELTA13  0.1295 + 0.0000i  0.1295 - 0.0000i  0.3705 + 0.0000i  0.3705 - 0.0000i
DELTA23  0.3705 + 0.0000i  0.3705 - 0.0000i  0.1295 + 0.0000i  0.1295 - 0.0000i
OMEGA13  -0.1295 - 0.0000i -0.1295 + 0.0000i -0.3705 - 0.0000i -0.3705 + 0.0000i
OMEGA23  -0.3705 - 0.0000i -0.3705 + 0.0000i -0.1295 - 0.0000i -0.1295 + 0.0000i
                MODE 1          MODE 1          MODE 2          MODE2
```

From this, we see that ω_{23} participates most heavily in mode 1.

ω_{13} participates most heavily in mode 2.

This is the information that we would use to decide where to place a PSS to enhance damping of a particular mode, although there is some ambiguity regarding whether ω_{1k} is a state associated with unit 1 or unit k.

Returning to Fig. 3.3 in the book (and given above), we observe that although mode 1 is clearly visible in both plots, mode 2 is only visible in the ω_{13} plot. This is consistent with the indication from the participation factors.

Recall that with uniform damping, we were able to eliminate one speed deviation state. In general, this is not possible, and so you end up with one speed generation state for each generator, a development which solves the ambiguity problem mentioned above.

As an example, the paper by Mansour provides participation factors for several cases, as indicated below. Note that for all participation “vectors” the participation factor is a normalized magnitude.

Table 4
1988 HEAVY WINTER CASE
PALO VERDE - DEVERS LINE OUT OF SERVICE
0.72 Hz MODE (-0.057 + J 4.43)

EIGEN VECTOR				PARTICIPATION VECTOR		
GEN	AREA	MAG	PH	GEN	AREA	MAG
GEYSER34	PG&E	1.0	-176	HELMS G	PG&E	1.0
HELMS G	PG&E	0.9	169	BIGCREEK	S.CAL	0.6
GEYSER78	PG&E	0.9	-176	KEMANO	S.C.	0.5
BIGCREEK	S.CAL	0.8	-165	GEYSER D	PG&E	0.4
MELONES	PG&E	0.7	-156	FCNGN4CC	ARIZ	0.3
GEYSER B	PG&E	0.7	-175	RNCHECO	PG&E	0.3
GEYSER D	PG&E	0.7	-175	GEYSER C	PG&E	0.3
GEYSER A	PG&E	0.6	-175	MOSS 7	PG&E	0.3
GEYSER C	PG&E	0.6	-175	GEYSER B	PG&E	0.2
FOLSOM1	PG&E	0.6	-178	MELONES	PG&E	0.2
FOLSOM23	PG&E	0.6	-178	GEYSER34	PG&E	0.2
KERCKHOF	PG&E	0.6	-179	DIABLO1	PG&E	0.2
-----	"			GEYSER78	PG&E	0.2
	"			DIABLO2	PG&E	0.2
	"			PTSEWNG	PG&E	0.2
	"			COMANCHE	COLED	0.2
INTERM20	UTAH	0.3	19.5			
EHUNTR 1	UTAH	0.2	5.0			
INTERM10	UTAH	0.3	19.5			
EHUNTR 2	UTAH	0.2	4.8			
NEWMANG8	N.MEX	0.3	-25.4			
MOHAV2CC	S.CAL	0.3	-22.3			
MOHAV1CC	S.CAL	0.3	-22.3			
APACHCT3	ARIZ	0.3	-21.5			
FCNGN4CC	ARIZ	0.3	-26.8			
NOVRA3A4	WAPA	0.4	-17.4			
NEWMAN45	N.MEX	0.5	-32.3			
BLNDL G1	UTAH	0.5	4.5			

