Multi-Machine Systems with Constant Impedance Loads

The parts of the text which we have yet to cover include:

- Chapter 3: System response to small disturbances
- Chapter 6: Linear models of synchronous machines
- Chapters 7-8: Excitation systems and Effect of excitation on stability
- Chapter 9: Multimachine systems with constant impedance loads
- Chapters 10-13: Speed governing and prime movers (steam/hydro/CTs/CC units)

We will not study chapters 7 and 8 (excitation) or 10-13 (turbine-governors) at all. These chapters are covered in an advanced course on power system dynamics. We will spend some time on Chapter 9 and then move back to Chapter 3. We will very briefly look at Chapter 6 as well (note that Chapter 6 is to Chapter 3 as Chapter 4 is to Chapter 2, i.e., Chapter 4 extends the coverage of transient instability analysis done in Chapter 2 from the classical machine model to more elaborate machine models. Chapter 6 does the same thing, except instead of transient instability, it extends the coverage of small-signal instability done in Chapter 3).

So here we look at Chapter 9.

Chapter 9 consists of the following sections:

- 9.1: Introduction
- 9.2: Problem statement
- 9.3: Matrix representation of a passive network
    - Network in the transient state
    - Converting to a common reference frame
- 9.4: Converting machine coordinates to system reference
- 9.5: Relation between machine currents and voltages
- 9.6: System order
- 9.7: Machines represented by classical methods

- 9.8: Linearized model for the network
- 9.9: Hybrid formulation
- 9.10: Network equations with flux linkage model
- 9.11: Total system equations
- 9.12: Multimachine study

We will work through sections 9.1-9.5.
Note that Padiyar's book also gives good treatment of this in pp. 462-474.

<u>Section 9.1, Introduction</u>:

I will use this section to emphasize the importance of load modeling. You can refer back to a document available on the WECC web page at
www.wecc.biz/documents/library/SRWG/Transient_Stability.pdf
called "Load Representation in Transient Stability Studies." This document provides an excellent overview of load modeling issues.

[<u>Aside</u>: The URL
www.wecc.biz/modules.php?op=modload&name=Downloads&file=index&req=viewsdownload&sid=15
contains a number of useful documents on practical dynamic security assessment.]

There are two basic types of commonly used load models.
- Exponential
- Polynomial

The polynomial is probably the most common. One version of the polynomial is the so-called ZIP model:

$$P = P_0\left\{A + B\left(\frac{|V|}{|V_0|}\right) + C\left(\frac{|V|}{|V_0|}\right)^2\right\}(1 + L_P \Delta f)$$

$$P = P_0\left\{D + E\left(\frac{|V|}{|V_0|}\right) + F\left(\frac{|V|}{|V_0|}\right)^2\right\}(1 + L_Q \Delta f)$$

Typically, the frequency sensitivity coefficients obey $0<L_P<3$ and $-2<L_Q<0$ so that when frequency declines (meaning $\Delta f<0$), P decreases and Q increases, which tends to be the case for an induction motor.

The voltage sensitivity coefficients must obey A+B+C=1 and D+E+F=1. If we set A=B=D=E=0 and C=F=1, then we have a constant impedance model. This load model provides that power consumption of loads decreases as voltage drops. This characteristic typically decreases the severity of system response in terms of transient instability in that:

- We usually see voltage drop during and after a disturbance
- When voltage drops, constant Z loads consume less power according to the square of the voltage drop – which in turn improves the stability performance of the generators.

One advantage to using the constant Z-model is that it allows us to easily reduce the network to generator nodes as all loads are represented in the Y-bus. We obtain the impedance equivalents via $Z=|V_i|^2/S^*$.

One should note carefully here the difference between load modeling for transient analysis and load modeling for steady-state analysis.

Typically, for steady-state analysis (using power flow), we represent the load using constant power models. Some power flow programs do allow for using other load models, e.g., ZIP. However, if your power system contains under-load-tap-changing (ULTC) transformers connecting between the transmission system and the load (most commonly between the subtransmission and the distribution systems), and most do, then use of anything except a constant power model makes no sense unless you are also representing the ULTC transformers.

The reason for this is as follows:

Steady-state analysis of disturbances using power flow is typically done to analyze the 3-10 minute time period following the disturbance. The value of 3 minutes is chosen because this is enough time for the ULTC to operate fully, restoring the voltage levels in the distribution system, so that the loads actually see a constant voltage and therefore behave as constant power loads.

<u>Section 9.2, Problem statement</u>:
Each machine is represented by the following relation:

$$\underline{\dot{x}} = \underline{f}(\underline{\dot{x}}, \underline{v}, T_m, t)$$

where $\underline{x}$ is the state vector (could be any number of states between 2-8 depending on the choice of machine model), $\underline{v}=[v_d, v_q, v_F]^T$, T_m is the mechanical torque, and t is time.

Recall that the input vector for each of our machine models included v_d and v_q (or V_d and V_q where $V_d = \frac{v_d}{\sqrt{3}}$ and $V_q = \frac{v_d}{\sqrt{3}}$), which are the d- and q- axis components of the machine terminal voltage. For example, the current-state-space model for model 1 is:

$$\begin{bmatrix} \dot{i}_d \\ \dot{i}_F \\ \dot{i}_D \\ \dot{i}_q \\ \dot{i}_Q \\ \dot{i}_G \\ \dot{\omega} \\ \dot{\delta} \end{bmatrix} = \begin{bmatrix} -\underline{L}^{-1}(\underline{R}+\omega\underline{N}) & \underline{0} \\ \begin{bmatrix} \frac{-L_d i_q}{3\tau_j} & \frac{-kM_F i_q}{3\tau_j} & \frac{-kM_D i_q}{3\tau_j} & \frac{L_q i_d}{3\tau_j} & \frac{kM_Q i_d}{3\tau_j} & \frac{kM_G i_d}{3\tau_j} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \frac{-D}{\tau_j} & 0 \\ 1 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ i_G \\ \omega \\ \delta \end{bmatrix} + \begin{bmatrix} \left[-\underline{L}^{-1}\underline{v}\right] \\ \begin{bmatrix} \frac{T_m}{\tau_j} \\ -1 \end{bmatrix} \end{bmatrix}$$

where

$$\underline{R} = \begin{bmatrix} r & 0 & 0 & 0 & 0 & 0 \\ 0 & r_F & 0 & 0 & 0 & 0 \\ 0 & 0 & r_D & 0 & 0 & 0 \\ 0 & 0 & 0 & r & 0 & 0 \\ 0 & 0 & 0 & 0 & r_Q & 0 \\ 0 & 0 & 0 & 0 & 0 & r_G \end{bmatrix}; \qquad \underline{N} = \begin{bmatrix} 0 & 0 & 0 & L_q & kM_Q & kM_G \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -L_d & -kM_F & -kM_D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{L} = \begin{bmatrix} L_d & kM_F & kM_D & 0 & 0 & 0 \\ kM_F & L_F & M_R & 0 & 0 & 0 \\ kM_D & M_R & L_D & 0 & 0 & 0 \\ 0 & 0 & 0 & L_q & kM_Q & kM_G \\ 0 & 0 & 0 & kM_Q & L_Q & M_Y \\ 0 & 0 & 0 & kM_G & M_Y & L_G \end{bmatrix} \qquad \underline{v} = \begin{bmatrix} v_d \\ -v_F \\ 0 \\ v_q \\ 0 \\ 0 \end{bmatrix}; \qquad \underline{i} = \begin{bmatrix} i_d \\ i_F \\ i_D \\ i_q \\ i_Q \\ i_G \end{bmatrix}$$

These terms v_d and v_q (or V_d and V_q) are determined by the network, and we therefore need to interface the machine model with the network in order to account for them.

We assume that v_F and T_m are fixed (they are actually influenced by the excitation control and the turbine-governor control, but we have no time to study these in this course – this material is in Chapters 7-8 and 10-13).

Let's assume that we are using the current state-space model of Model 1 (which is the "full" model including the G-circuit and two damper windings, so it is called model 2.2).

Our objective is derive expressions for v_d and v_q in terms of the state variables, which in the case of the current state-space model of Model 1, would be the currents.

We will begin by recalling the stator-side equivalents to v_d, v_q, i_d, and i_q, given by:

$$V_{di} = \frac{v_{di}}{\sqrt{3}} \qquad V_{qi} = \frac{v_{qi}}{\sqrt{3}}$$

$$I_{di} = \frac{i_{di}}{\sqrt{3}} \qquad I_{qi} = \frac{i_{di}}{\sqrt{3}}$$

where the subscript "i" indicates that the relations apply to machine i.

We also have that

$$\bar{V}_i = V_{qi} + jV_{di} \qquad \bar{I}_i = I_{qi} + jI_{di}$$

for every machine i=1, …, n.

Thus we have a vector of nodal voltages and currents for every generator bus given by:

$$\underline{V} = \begin{bmatrix} V_{q1} + jV_{d1} \\ \vdots \\ V_{qn} + jV_{dn} \end{bmatrix} = \begin{bmatrix} \bar{V}_1 \\ \vdots \\ \bar{V}_n \end{bmatrix} \qquad \underline{I} = \begin{bmatrix} I_{q1} + jI_{d1} \\ \vdots \\ I_{qn} + jI_{dn} \end{bmatrix} = \begin{bmatrix} \bar{I}_1 \\ \vdots \\ \bar{I}_n \end{bmatrix} \tag{9.4}$$

(Note that we use underlines to denote vectors and matrices, and we use overbars to denote phasors).

Our problem is to express $\underline{V}$ in terms of $\underline{I}$. One might think that this is an easy problem, based on recollection of the Y-bus relation which has that $\underline{I}=\underline{YV}$.

However, there is a big problem in doing this….

The elements of these two vectors, e.g., $V_{q1}+jV_{d1}$ and $I_{q1}+jI_{d1}$, are, by definition, expressed on the d-q reference frame of the corresponding machine. We have done nothing at this point to relate the d-q frame of one machine to that of another.

So the elements of $\underline{V}$ (and the elements of $\underline{I}$) are expressed on different reference frames. Any analysis using these numbers "as is" would have relative angles between nodes in the network that mean absolutely nothing. Since relative angles determine power flow, this in unacceptable.

Section 9.3, Matrix representation of a passive network:

In consideration of a multimachine system in Chapter 2, using the classical machine representation, because the machine internal EMF is constant, we could reduce the network to its *internal machine nodes*, thus eliminating the nodes corresponding to each machines terminal voltage V_a.

Now, however, we need to retain the node corresponding to each machine's terminal voltage V_a because all of our higher-order models require it through the presence in the models of v_d and v_q.

Then we represent all loads using constant impedance shunts.

Then we use network reduction (Gaussian elimination) to eliminate all network nodes except the machine terminal node.

We have already recognized that we cannot express $\underline{I}=\underline{YV}$ using eq. (9.4) because the various vector elements are all on different reference frames.

So let's consider a new set of nodal voltages and currents that are expressed to a common reference frame where one of the quantities, often one of the voltages, has an angle designated as 0°.

We will refer to this set of nodal voltages and currents as $\underline{\hat{V}}$ and $\underline{\hat{I}}$, articulated as V-hat and I-hat. So the underline indicates "vector," and the hat indicates that all elements are referred to the *network reference frame*.

So on the network reference frame, it is acceptable to write that

$$\underline{\hat{I}} = \underline{Y}\underline{\hat{V}} \tag{9.5}$$

where $\underline{Y}$ is the network admittance matrix. Of course, at this point, we are simply conjecturing that we can express all voltages and currents to a common reference frame, but we have not yet done it.

But Dr. Anderson is so careful….. he recognizes that eq. (9.5) is a steady-state relation, and he takes a little aside to check: under what conditions can we use eq. (9.5) for transient analysis?

To answer this question, section 9.3.1, he writes the time-domain voltage drop equation for a network branch, and then transforms this equation using Park's transformation. This transformation is based on an assumed synchronously rotating reference frame which, at t=0, is aligned with the a-phase of a chosen machine. This action, then, locates the machine's rotor, and thus the machine's d-axis, at

$$\theta_i = \omega_{\text{Re}} t + \pi / 2 + \delta_i$$

Fig. 2 illustrates.

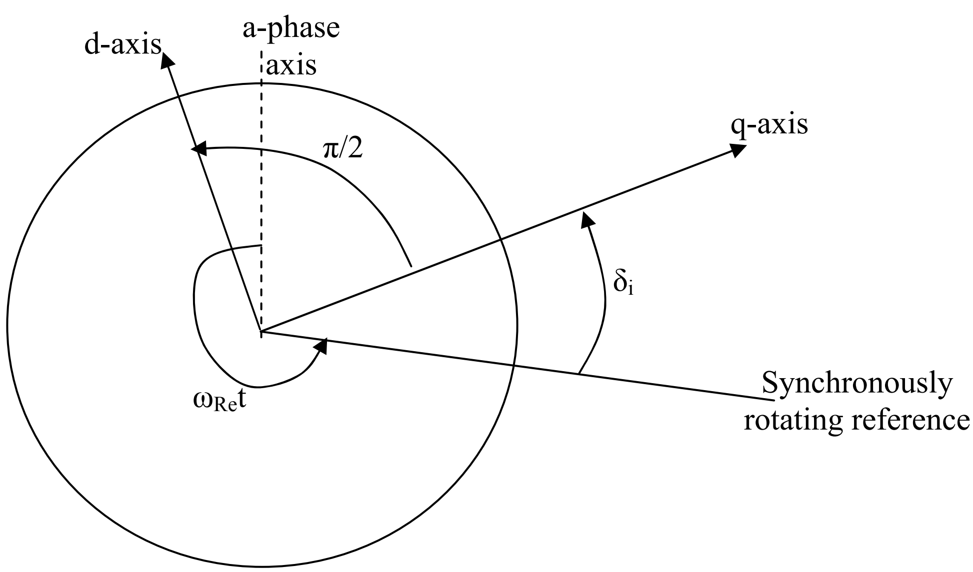


Fig. 2

I will not go through this analysis but rather will simply state the conclusions.

Dr. Anderson's conclusion is that:

$$\bar{V}_{k(i)} = z_k \bar{I}_{k(i)}, \text{ k=1, …, b} \tag{9.16}$$

where

- $\bar{V}_{k(i)}$ and $\bar{I}_{k(i)}$ are the *branch* voltage drops and *branch* currents, respectively,
- expressed on the d-q axis reference frame of machine i, that is, the reference is the q-axis of the i^{th} machine located at angle δ_i with respect to a synchronously rotating system reference,
- z_k is the impedance of branch k, and
- b is the total number of branches in the network.

Equation (9.16), which is our standard Ohm's Law relation, is applicable for transient analysis if the following two conditions are satisfied:

1. The frequency, and therefore the reactances of the branches, are constant.
2. Current derivatives are much less than speed-current products.

$$\left|\dot{i}_d\right| << \left|\omega i_q\right|$$

$$\left|\dot{i}_q\right| << \left|\omega i_d\right|$$

This is analogous to where we assumed that transformer voltages are much less than speed voltage drops (svd), i.e., the d-q voltage components due to transformer action (i.e., variation in d-q currents or in d-q flux linkages) is much less than the d-q voltage components due to the speed. We used this in deriving the E'' model, expressed as:

$$\left|\dot{\lambda}_d\right| << \left|\omega \lambda_q\right|$$

$$\left|\dot{\lambda}_q\right| << \left|\omega \lambda_d\right|$$

In addition to identifying the conditions under which we can use our familiar steady-state form of Ohm's Law (and thus the Y-bus relation), eq. (9.16) also provides that we may express the network to a particular machine's d-q reference frame.

But this does not do us too much good since we have all the machine models expressed to their own frame.

So a better approach is to express all of the machine d-q reference frames to a ***network*** reference frame. Let's try that (Section 9.3.2).

We have already defined the d-q reference frame of the machine.

Now we define the network reference frame, and we will denote the network reference frame as D-Q (do NOT confuse this notation with the upper-case D,Q notation used for the damper windings!!!!).

So our question is: how to convert a voltage (or current) on the d-q reference frame to a voltage (or current) on the D-Q (network) reference frame?

Fig. 3 (Fig 9.4 in text) illustrates.

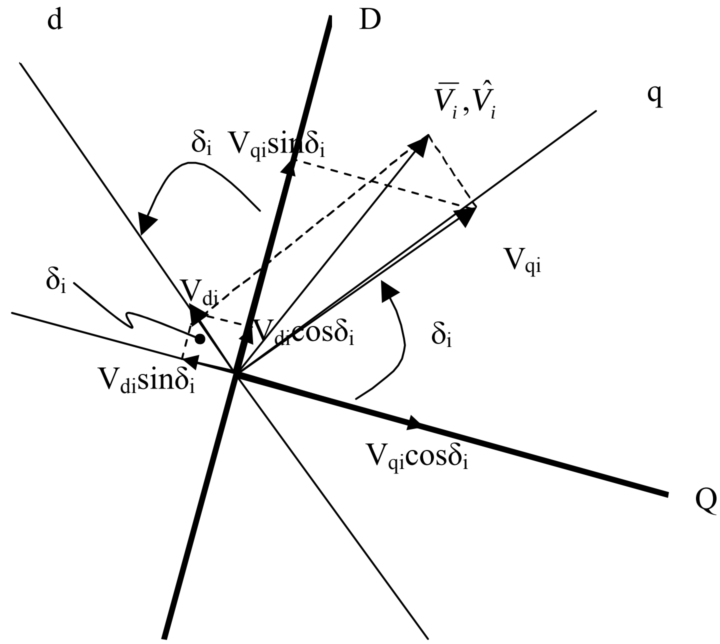


Fig. 3

From this picture, it is easy to see how to compute V_{Qi} and V_{Di} from V_{qi} and V_{di}.

It is important to recognize that we are NOT getting V_{Qi} and V_{Di} from $\bar{V}_i$ (or $\hat{V}_i$) directly but rather getting it from V_{di} from V_{qi}, which are the d-q axis components of $\bar{V}_i$ (or $\hat{V}_i$).

For example, consider getting V_{Qi}. By inspection, we see that

$$V_{Qi} = V_{qi} \cos\delta_i - V_{di} \sin\delta_i$$

where the angle δ_i is the angle between the machine i q-axis and the network frame.

Similarly, consider getting V_{Di}. Again, by inspection, we see that:

$$V_{Di} = V_{qi} \sin\delta_i + V_{di} \cos\delta_i$$

Therefore, the voltage $\overline{V_i}$ when expressed to the network reference frame, becomes $\hat{V}_i$, expressed as:

$$\hat{V}_i = V_{Qi} + jV_{Di} = (V_{qi}\cos\delta_i - V_{di}\sin\delta_i) + j(V_{qi}\sin\delta_i + V_{di}\cos\delta_i)$$

which can be factored to provide:

$$\hat{V}_i = V_{Qi} + jV_{Di} = (V_{qi} + jV_{di})(\cos\delta_i + j\sin\delta_i) = \overline{V_i}e^{j\delta_i}$$

In summary, the transformation that we are making is from one set of coordinate axes

$$\overline{V_i} = V_{qi} + jV_{di}$$

(where the positive q-axis is assigned 0 degrees),

to another set of coordinate axes

$$\hat{V}_i = V_{Qi} + jV_{Di}$$

where the positive Q-axis is assigned 0 degrees.

Here, the positive q-axis leads the +Q axis by δ_i degrees.

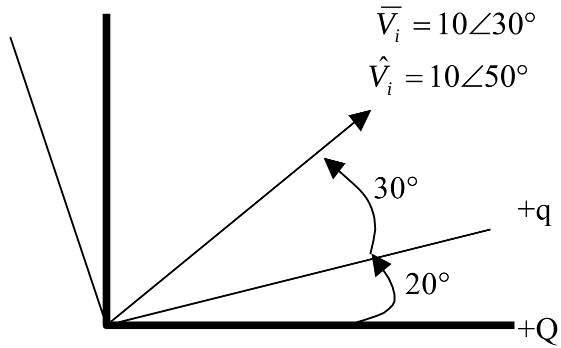
And we have found that

$$\hat{V}_i = \overline{V_i}e^{j\delta_i}$$

As an example, let $\overline{V_i} = 10\angle 30°$ (expressed on the d-q frame), and let q lead Q by δ_i=20°. Then

$$\hat{V}_i = \overline{V}_i e^{j20°} = 10\angle 30° e^{j20°} = 10\angle 50°$$

which is illustrated below.



Now recall the equation relating branch voltage drops to branch currents:

$$\overline{V}_{k(i)} = z_k \bar{I}_{k(i)}, \quad \text{k=1, …, b} \tag{9.16}$$

Remember what the (i) notation indicates – that the quantity is expressed to the d-q coordinate axes of machine i.

But we want all quantities on the D-Q (network) coordinate axes, and now we know how to achieve this….

$$\hat{V}_k = \overline{V}_{k(i)} e^{j\delta_i} \Rightarrow\Rightarrow \overline{V}_{k(i)} = \hat{V}_k e^{-j\delta_i}$$

$$\hat{I}_k = \bar{I}_{k(i)} e^{j\delta_i} \Rightarrow\Rightarrow \bar{I}_{k(i)} = \hat{I}_k e^{-j\delta_i}$$

Substitution into (9.16) yields:

$$\hat{V}_k e^{-j\delta_i} = z_k \hat{I}_k e^{-j\delta_i}$$

And we see that the exponentials cancel so that:

$$\hat{V}_k = z_k \hat{I}_k \qquad \text{k=1, …, b} \tag{9.18}$$

Combining (9.18) with (9.16) we see that

$$z_k = \frac{\hat{V}_k}{\hat{I}_k} = \frac{\overline{V}_k}{\overline{I}_k} \qquad \text{k=1, …, b}$$

This is expected – it says that the ratio of a voltage drop across an element to the current through the element will remain the same if we rotate all voltages and all currents by a particular angle.

Writing the above equation for every branch in the network results in the following matrix relation:

$$\begin{bmatrix} \hat{V}_1 \\ \hat{V}_2 \\ \vdots \\ \hat{V}_b \end{bmatrix} = \begin{bmatrix} z_{11} & 0 & 0 & 0 \\ 0 & z_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & z_{bb} \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \\ \vdots \\ \hat{I}_b \end{bmatrix}$$

We may write the above relation in more compact form:

$$\underline{\hat{V}}_b = \underline{z}_b \underline{\hat{I}}_b \tag{9.19}$$

Some comments about the above:

- Since all off-diagonal elements are zero, we have assumed that there is no mutual coupling in the network. Mutual coupling can exist, however, between lines that are physically parallel and located in close proximity, a condition that is found when several circuits share a common right-of-way.
- The matrix $\underline{z}_b$ is square with non-zero values along the diagonal and is therefore invertible. We denote its inverse as $\underline{y}_b$, such that:

$$\underline{\hat{I}}_b = \underline{y}_b \underline{\hat{V}}_b \tag{9.20}$$

- The matrix of impedances $\underline{x}_b$ is called the *primitive impedance matrix*, the matrix of admittances $\underline{y}_b$ the *primitive admittance matrix*, and the equations the z- and y- forms of the *primitive network equation*, named by Gabriel Kron.

- The primitive network equation does not describe the network at all, i.e., it gives absolutely no information as to how the individual branches are interconnected in the network.

In order to provide network connection information, we need the node-incidence matrix A, given by:

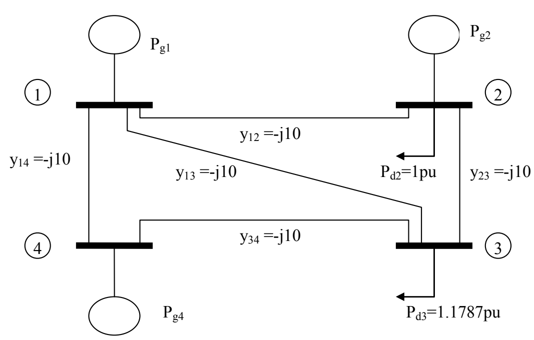
$$\underline{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{b1} & a_{b2} & \cdots & a_{bn} \end{bmatrix}$$

where
- b is the number of branches in the network.
- n is the number of nodes in the network.
- a_{ki} is given by:

$$a_{ki} = \begin{cases} +1 \text{ if current in branch k is leaving node i} \\ -1 \text{ if current in branch k is entering node i} \\ 0 \text{ if branch k is not connected to node i} \end{cases}$$

Note that A is a b × n matrix:
- Number of branches = number of rows
- Number of nodes = number of columns

$$\underline{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$



Let's denote the nodal voltages and currents, expressed to the network frame, as $\underline{\hat{V}}$ and $\underline{\hat{I}}$.

The nodal currents may be related to the branch currents by summing over all currents leaving node i. Since a node corresponds to a *column* of the node-incidence matrix, we can relate the nodal currents to the branch currents through a multiplication of $\underline{A}^T$ with the branch current vector, i.e.,

$$\hat{I} = \underline{A}^T \hat{I}_b \qquad (*)$$

The matrix $\underline{A}^T$ has each row corresponding to a node, and therefore the elements of each row will pick out of $\hat{I}_b$ the appropriate branch flows emanating from that node to provide the total injected current into that node.

Note dimensions of terms in this relation, we obtain an $n \times 1$ matrix from the product of an $n \times b$ matrix with a $b \times 1$ matrix.

So the above relation illustrates that the node-incidence matrix can be used to sum quantities. In this particular case, we summed branch currents to get the nodal currents according to KCL.

What about relating nodal voltages to branch voltage drops? In this case, we consider KVL and recall that we need to "sum" the nodal voltages to obtain the voltage drops. So we need to express $\underline{\hat{V}}_b$ as a product of $\underline{\hat{V}}$ and $\underline{A}$ in some fashion.

If you toy with these matrices from purely a dimensional point of view, you will see that

$$\underline{A}\underline{\hat{V}} = \underline{\hat{V}}_b \qquad (**)$$

where the dimensions indicate that we obtain a b × 1 from the product of an b × n with an n × 1. We may also derive this from power relations (ref: P. Anderson, "Analysis of Faulted Power Systems," pp. 371-372).

But we observe in (**) that each row of A corresponds to a particular branch, and the non-zero elements of that row correspond to a bus that is connected to that branch. There will only be two such buses, and the product AV will pick off the two voltages at either end of the branch to find their difference, which is contained in $\underline{V}_b$.

Substitution of eq. (9.20), $\underline{\hat{I}}_b = \underline{y}_b \underline{\hat{V}}_b$, into eq. (*), $\hat{I} = \underline{A}^T \hat{I}_b$, yields:

$$\hat{I} = \underline{A}^T \hat{I}_b = \underline{A}^T \underline{y}_b \underline{\hat{V}}_b \qquad (***)$$

and substitution of eq. (**) into (***) yields:

$$\hat{I} = \underline{A}^T \underline{y}_b \underline{\hat{V}}_b = \underline{A}^T \underline{y}_b \underline{A}\underline{\hat{V}}$$

Here, we clearly see that the familiar Y-bus (admittance matrix) is obtained from the primitive admittance matrix from:

$$\underline{Y} = \underline{A}^T \underline{y}_b \underline{A}$$

so that we have, finally, that

$$\underline{\hat{I}} = \underline{Y}\underline{\hat{V}} \qquad (9.21)$$

which relates nodal voltages and current injections given on the D-Q (network) coordinate axes.

Now define a square n × n transformation matrix T according to:

$$\underline{T} = \begin{bmatrix} e^{j\delta_1} & 0 & 0 & 0 \\ 0 & e^{j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{j\delta_n} \end{bmatrix} \Rightarrow \underline{T}^{-1} = \begin{bmatrix} e^{-j\delta_1} & 0 & 0 & 0 \\ 0 & e^{-j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-j\delta_n} \end{bmatrix}$$

Then we can obtain the nodal currents and voltages expressed on D-Q (network) coordinate axes from the nodal currents and voltages expressed on d-q (individual machine i) coordinate axes from:

$$\underline{\hat{I}} = \underline{T}\,\underline{I} \text{ and } \underline{\hat{V}} = \underline{T}\,\underline{V}$$

Substitution into eq. (9.21), $\underline{\hat{I}} = \underline{Y}\underline{\hat{V}}$, yields:

$$\underline{T}\,\underline{I} = \underline{Y}\,\underline{T}\,\underline{V} \Rightarrow \underline{I} = \underline{T}^{-1}\underline{Y}\,\underline{T}\,\underline{V} = \underline{M}\,\underline{V} \Rightarrow \underline{I} = \underline{M}\,\underline{V}$$

where clearly,

$$\underline{M} = \underline{T}^{-1}\underline{Y}\,\underline{T}$$

What does the transformation do?

It allows us to relate currents in the d-q coordinate frame of one machine, $\overline{I}_1, \overline{I}_2, ..., \overline{I}_n$ to voltages in the d-q coordinate frame of all other machines.

You see, $\underline{I} = \underline{Y}\,\underline{V}$ does not work!

$\underline{I} = \underline{M}\,\underline{V}$ is the replacement we need.

Example 9.1: The matrix M can be evaluated by performing the appropriate matrix multiplications:

$$M = \underline{T}^{-1}\underline{Y}\underline{T} =$$

$$\begin{bmatrix} e^{-j\delta_1} & 0 & 0 & 0 \\ 0 & e^{-j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-j\delta_n} \end{bmatrix} \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j\theta_{12}} & \cdots & Y_{1n}e^{j\theta_{1n}} \\ Y_{21}e^{j\theta_{21}} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2n}e^{j\theta_{2n}} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1}e^{j\theta_{n1}} & Y_{n1}e^{j\theta_{n2}} & \cdots & Y_{nn}e^{j\theta_{nn}} \end{bmatrix} \begin{bmatrix} e^{j\delta_1} & 0 & 0 & 0 \\ 0 & e^{j\delta_2} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{j\delta_n} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}e^{j(\theta_{12}-\delta_{12})} & \cdots & Y_{1n}e^{j(\theta_{1n}-\delta_{1n})} \\ Y_{21}e^{j(\theta_{21}-\delta_{21})} & Y_{22}e^{j\theta_{22}} & \cdots & Y_{2n}e^{j(\theta_{2n}-\delta_{2n})} \\ \vdots & \vdots & \cdots & \vdots \\ Y_{n1}e^{j(\theta_{n1}-\delta_{n1})} & Y_{n1}e^{j(\theta_{n2}-\delta_{n2})} & \cdots & Y_{nn}e^{j\theta_{nn}} \end{bmatrix}$$

where $\delta_{ik}=\delta_i - \delta_k$.

The general form of the term in row i, col k, in the matrix M is:

$$M_{ij} = Y_{ik}e^{j(\theta_{ik}-\delta_{ik})} = Y_{ik}e^{j\theta_{ik}}e^{-j\delta_{ik}} = \left(G_{ik} + jB_{ik}\right)\left(\cos\delta_{ik} - j\sin\delta_{ik}\right)$$

➔

$$M_{ij} = \underbrace{\left(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}\right)}_{F_{G+B}(\delta_{ik})} + j\underbrace{\left(B_{ik}\cos\delta_{ik} - G_{ik}\sin\delta_{ik}\right)}_{F_{B-G}(\delta_{ik})}$$

So the i-k^th^ term in matrix M is given by $F_{G+B}(\delta_{ik})+j\,F_{B-G}(\delta_{ik})$.

This simplifies for the diagonal elements, since $\delta_{ii}=0$, to $G_{ii}+jB_{ii}$.

So

$$M_{ij}= F_{G+B}(\delta_{ik})+j\, F_{B-G}(\delta_{ik})$$
$$M_{ii}= G_{ii}+jB_{ii}$$

Separating real and imaginary parts, we obtain $\underline{M}=\underline{H}+j\underline{S}$ where

$H_{ik}= F_{G+B}(\delta_{ik})$
$H_{ii}=G_{ii}$

$S_{ik}=F_{B-G}(\delta_{ik})$
$S_{ii}=B_{ii}$

You should review examples 9.2 and 9.3 in the text.

Additional comments:

The overall problem is given by

$$\underline{\dot{x}} = \underline{f}(\underline{\dot{x}}, \underline{v}, T_m, t)$$
$$\underline{I} = \underline{M}\underline{V}$$

Where $\underline{M}$ is formulated as follows:

$$\underline{M} = \underline{T}^{-1}\underline{Y}\underline{T}$$

And because

$$\underline{Y} = \underline{A}^T \underline{y}_b \underline{A}$$

we have that

$$\underline{M} = \underline{T}^{-1}\underline{Y}\underline{T} = \underline{T}^{-1} \underline{A}^T \underline{y}_b \underline{A}\underline{T}$$

Now here is an issue. If we have entirely constant impedance loads, then all loads can be included into the matrix $\underline{Y}$, and the above formulation is OK.

If we have constant current loads, then those loads may be included in the vector I.

But if we have constant power loads, then those loads, when converted to a constant current representation through I=(S/V)*, are a function of voltage. In that case, the problem we are solving is

$$\underline{\dot{x}} = \underline{f}(\underline{\dot{x}}, \underline{v}, T_m, t)$$
$$\underline{I}(\underline{V}) = \underline{M}\,\underline{V}$$

where the algebraic equations must be solved iteratively.

Either way, we have the interface problem, illustrated in a figure from Brian Stott's paper below.

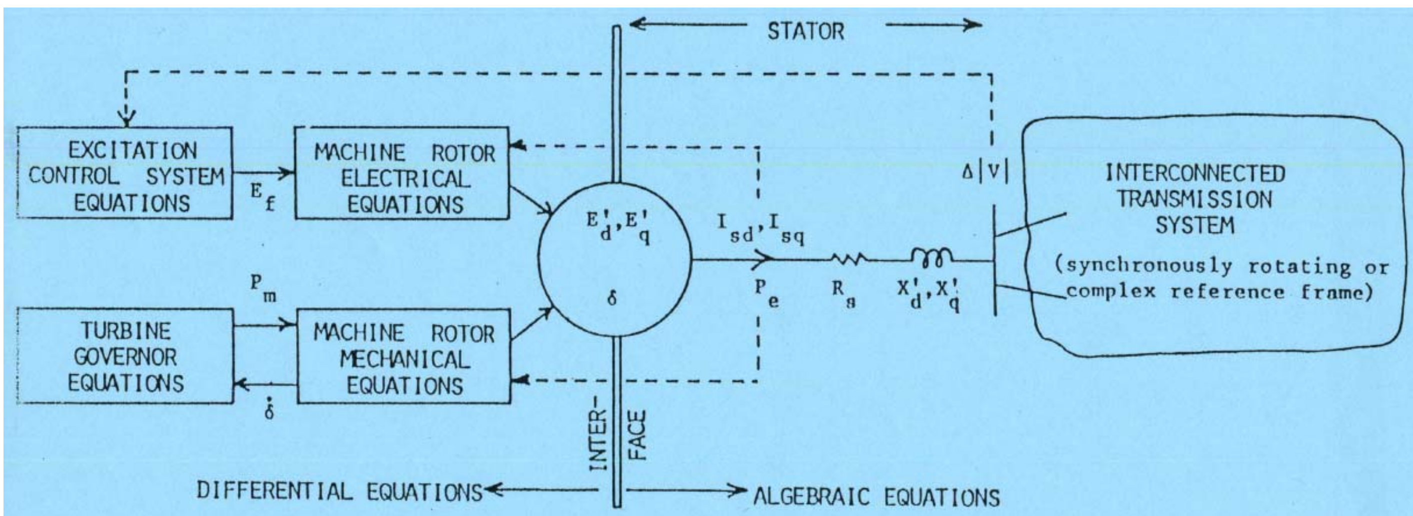


Fig. 1. Schematic of transient model of synchronous generator connected to transmission network.

Stott, Section IV of his paper, introduces a classification system for solving a differential-algebraic equation (DAE), which is what we have.

He says that solution approaches are characterized by three attributes:

1. The way in which machine an network equations are interfaced with each other:
   a. Partitioned: alternating
   b. Simultaneous (combined or algebraically)
2. The integration method used:
   a. Explicit
   b. Implicit
3. The technique for solving the algebraic equations (an issue if you have constant power loads and you solve using the alternating method.