Last Topics #1: A broader look at reference frame theory

There are some additional topics that are appropriate to include in a course on power systems dynamics, and I do so here because they are highly relevant to what might be called "current events." The topic of this set of notes is "A broader look at reference frame theory."

1.0 Introduction

You recall that in developing a model for the synchronous machine that we found the differential equations included time varying coefficients that resulted from the dependence of inductances (stator-stator and stator-rotor) on rotor position.

To address this, so that we could obtain differential equations with constant coefficients, we performed a transformation on the a-b-c voltages and currents. We called that transformation "Park's transformation."

It is worthwhile to read what Paul Krause says in his very good text on electric machinery [1] (references within quotes are not included here). The below quotes are from his chapter 3, titled "Reference-frame theory." You would do well to have this book on your bookshelf.

"In the late 1920's, R. H. Park [1] introduced a new approach to electric machine analysis. He formulated a change of variables which, in effect, replaced the variables (voltages, currents, and flux linkages) associated with the stator windings of a synchronous machine with variables associated with fictitious windings rotating with the rotor. In other words, he transformer, or referred, the stator variables to a frame of reference fixed in the rotor. Park's transformation, which revolutionized electric machine analysis, has the unique property of eliminating all time-varying inductances from the voltage equations of the synchronous machine which occur due to (1) electric circuits in relative motion and (2) electric circuits with varying magnetic reluctance.

In the late 1930's, H. C. Stanley [2] employed a change of variables in the analysis of induction machines. He showed that the time-varying inductances in the voltage equations of an induction machine due to electric circuits in relative motion could be eliminated by transforming the variables associated with the rotor windings (rotor variables) to variables associated with fictitious stationary windings. In this case the rotor variables are transformer to a frame of reference fixed in the stator.

G. Kron [3] introduced a change of variables which eliminated the time-varying inductances of a symmetrical induction machine by transforming both the stator variables and the rotor variables to a reference frame rotating in synchronism with the rotating magnetic field. This reference frame is commonly referred to as the synchronously rotating reference frame.

D. S. Brereton et al. [4] employed a change of variables which also eliminated the time-varying inductances of a symmetrical induction machine by transforming the stator variables to a reference frame fixed in the rotor. This is essentially Park's transformation applied to induction machines.

Park, Stanley, Kron, and Brereton et al. developed changes of variables each of which appeared to be uniquely suited for a particular application. Consequentially, each transformation was derived and treated separately in literature until it was noted in 1965 [5] that all known real transformations used in induction machine analysis are contained in one generatl transformation which eliminates all time-varying inductances by referring the stator and rotor variables to a frame of reference which may rotate at any angular velocity or remain stationary. All known real transformations may then be obtained by simply assigning the appropriate speed of rotation to this so-called arbitrary reference frame. Later, it was noted that the stator variables of a synchronous machine could also be referred to the arbitrary reference frame [6]. However, we will find that the time-varying inductances of a synchronous machine are eliminated only if the reference frame is fixed in the rotor (Park's transformation); consequently the arbitrary reference frame does not offer the advantage in the analysis of synchronous machines that it does in the case of induction machines."

Krause also indicates later in the same chapter

"…The transformation of stationary circuits to a stationary reference frame was developed by E. Clarke [7] who used the notation f_α, f_β …"

There are three main points to be made from the above.

1. Other transformations: There have been a number of proposed transformations besides Park's.
2. Powerful tool: The ability to make transformations is apparently a powerful tool for analysis of electric machines.
3. General theory: There is a higher-level general theory for transformations under which all of them fall.

The topic of transformations and reference-frame theory has received renewed interest over the past few years as a result of the fact that

- wind turbines have primarily used induction machines;
- the doubly-fed induction generator (DFIG) has been particularly attractive;
- control of the DFIGs requires back-to-back voltage-source-inverters (VSI);
- design of VSI is most effectively done using space-vector-modulation (SVM);
- SVM requires one kind of transformation for the grid-side converter and one kind of transformation for the rotor-side converter.

Figure 1[2] illustrates a DFIG, and Fig. 2 [2] provides an expanded illustration of the inverter.

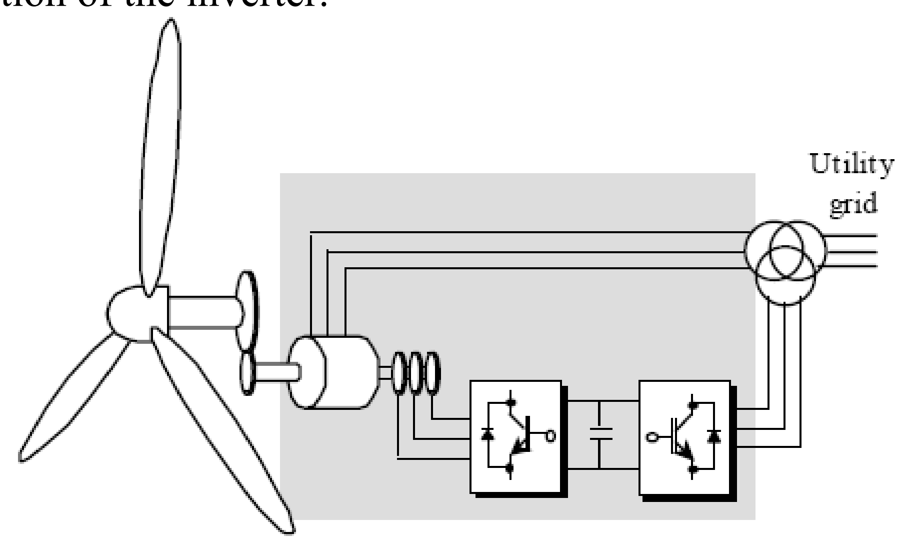

Fig. 1 [2]

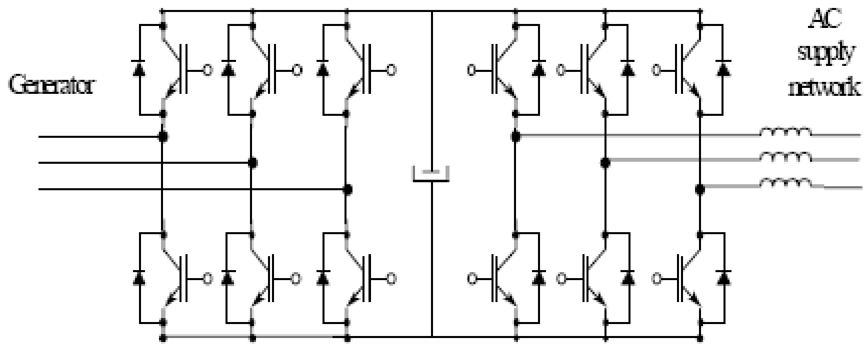

Fig. 2 [2]

Figure 2 represents a back-to-back four-quadrant VSI. Control of this converter is accomplished through space-vector modulation as illustrated in Fig. 3 [2].

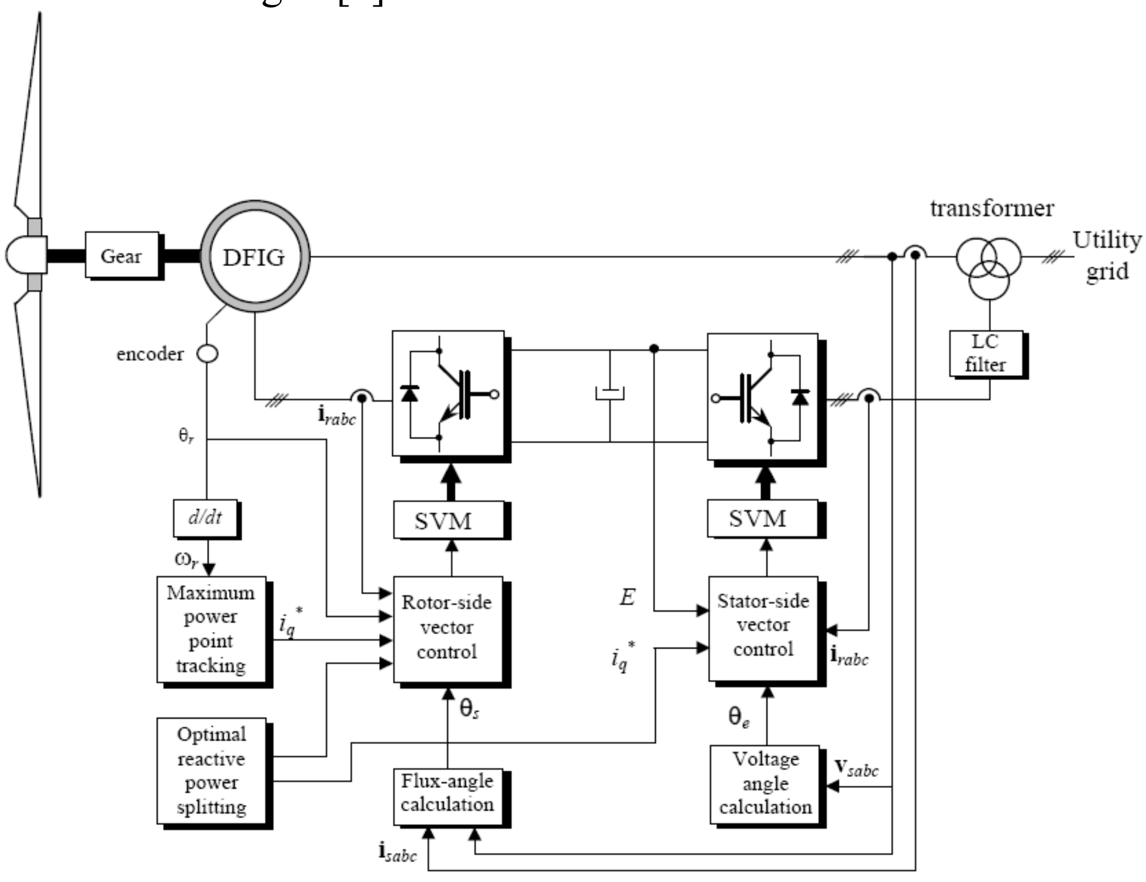

Fig. 3 [2]

In these notes, I want to provide you with enough material to appreciate the main points associated with Dr. Krause's text (other transformations, powerful tool, and general theory) as listed above, together with an appreciation of the relevance of reference frame theory to the analysis of DFIG.

2.0 A&F vs. Krause

We recall from our work earlier in the semester that we used Park's transformation as

$$\underline{P} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\theta & \cos(\theta - 120) & \cos(\theta + 120) \\ \sin\theta & \sin(\theta - 120) & \sin(\theta + 120) \end{bmatrix} \tag{1}$$

This enabled us to transform any a-b-c quantities into a rotating reference frame fixed on the rotor, according to

$$\underline{f}_{0dq} = \underline{P}\,\underline{f}_{abc} \tag{2}$$

Krause uses a slightly different transformation, calling it the "transformation to the arbitrary reference frame," given by

$$\underline{K}_S = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120) & \cos(\theta + 120) \\ \sin\theta & \sin(\theta - 120) & \sin(\theta + 120) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \tag{3}$$

where the angle θ is given by

$$\theta = \int_0^t \omega(\tau)d\tau + \theta(0) \tag{4}$$

It enables us to transform any a-b-c- quantities into a reference frame according to

$$\underline{f}_{qd0} = \underline{K}_S\,\underline{f}_{abc} \tag{5}$$

Krause explains that

> "The connotation of arbitrary stems from the fact that the angular velocity of the transformation is unspecified and can be selected arbitrarily to expedite the solution of the system equations or to satisfy the system constraints."

In this sense, A&F's transformation P is also a "transformation to the arbitrary reference frame." Other significant differences between Krause's transformations and A&F's include:

- The q-axis quantities of Krause are obtained through the row that starts with cosθ, whereas the d-axis quantities of A&F are obtained through the row that starts with cosθ. This is because Krause has the q-axis 90° ahead of the d-axis, whereas A&F have the d-axis 90° ahead of the q-axis.
- The constants applied to each row differ. This causes Krause's transformation to not be power invariant or orthogonal.

3.0 Krause's generalization of transformations

Krause also provides a table that is useful in seeing the additional transformations that one can use, adapted here as Table 1 below.

Table 1

Reference frame speed	Interpretation
ω (unspecified)	Stationary circuit variables referred to the arbitrary reference frame.
0	Stationary circuit variables referred to the stationary reference frame.
ω_r	Stationary circuit variables referred to a reference frame fixed in the rotor.
ω_e	Stationary circuit variables referred to the synchronously rotating reference frame.

Krause goes on to develop a generalized approach to transforming quantities from any one reference frame to another. If $\underline{K}_S^x$ transforms to reference frame x and $\underline{K}_S^y$ transforms to reference frame y, then ${}^x\underline{K}_S^y$ transforms variables in reference frame y to variables in reference frame x according to

$$ {}^x\underline{K}_S^y = \underline{K}_S^y \left(\underline{K}_S^x\right)^{-1} \tag{6} $$

4.0 Transformations for DFIGs

The following discussion is adapted from [3]. There are three main transformations of interest in designing control strategies for DFIGs: stationary reference frame, synchronous reference frame, and rotor reference frame.

4.1 Stationary reference frame

The stationary reference frame can be thought of as projecting three-phase voltages (or currents) varying in time along the a, b, and c phase axes onto two-phase voltages varying in time along a pair of orthogonal axes d^s and q^s fixed on the stator.

We assume for convenience, without loss of generality, that the q^s axis is collinear with the a-phase axis, as shown in Fig. 4.

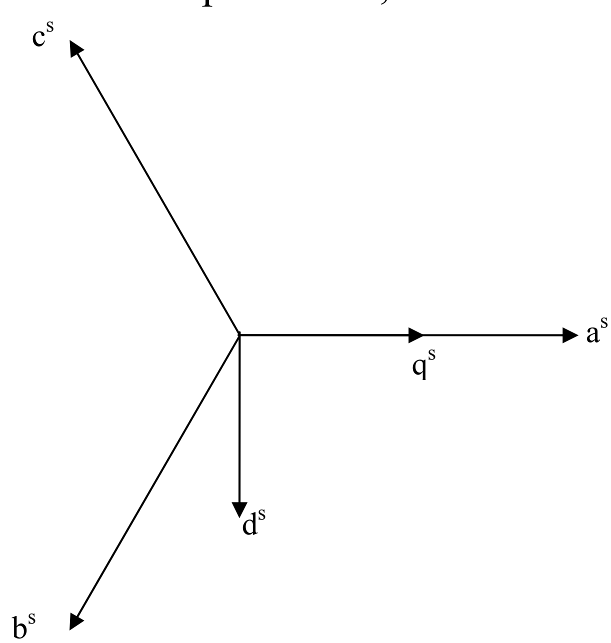

Fig. 4

The transformation to the stationary reference frame is given by (7)

$$\begin{bmatrix} v_q^s \\ v_d^s \\ v_0^s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{-\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \tag{7}$$

If the a-b-c voltages are balanced, then $v_0^S=0$.

The inverse transformation is given by (8).

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{-\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} v_q^s \\ v_d^s \\ v_0^s \end{bmatrix} \tag{8}$$

This reference frame is useful for integrating the DFIG quantities with the power system network.

4.2 Synchronous reference frame

The synchronous reference frame corresponds to a rotating reference frame moving at synchronous speed, denoted by ω_e. Figure 5 illustrates.

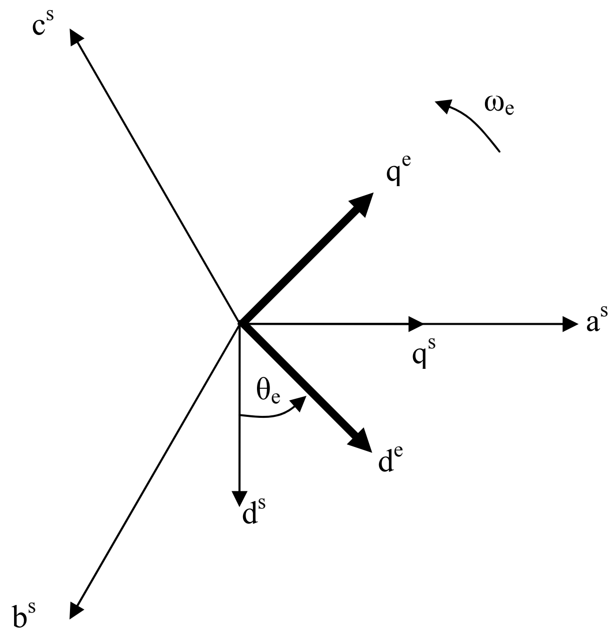

Fig. 5

Projection of a vector expressed in the stationary reference frame q^s-d^s into the synchronous reference frame q^e-d^e is obtained via

$$\begin{bmatrix} v_q^e \\ v_d^e \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} v_q^s \\ v_d^s \end{bmatrix} \tag{9}$$

The inverse transformation from the synchronous reference frame q^e-d^e to the stationary reference frame q^s-d^s is given by

$$\begin{bmatrix} v_q^s \\ v_d^s \end{bmatrix} = \begin{bmatrix} \cos\theta_e & \sin\theta_e \\ -\sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} v_q^e \\ v_d^e \end{bmatrix} \tag{10}$$

4.3 Rotor reference frame

An important concept underlying the rotor reference frame is that in the induction machine, the rotor is also a three-phase system. Therefore, it too will have an a-phase axis. The rotor reference frame will maintain it's d-axis collinear with the axis of the rotor a-phase. The speed of rotation of this reference frame is ω_r. Figure 6 illustrates.

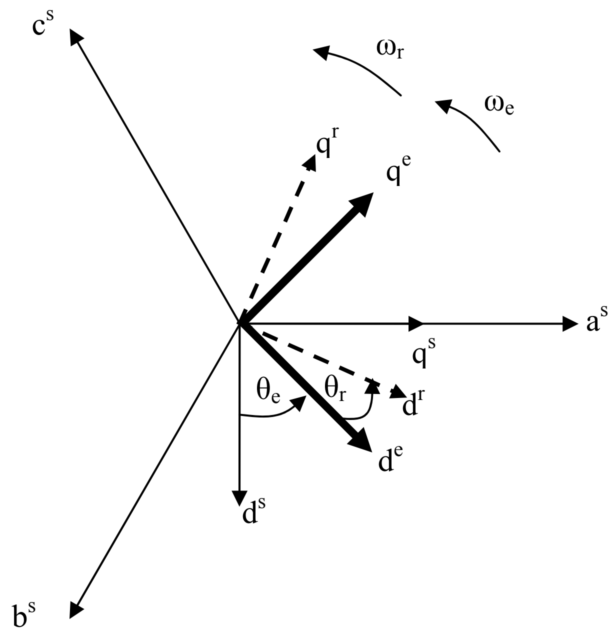

Fig. 6

Projection of a vector expressed in the stationary reference frame q^s-d^s into the rotor reference frame q^r-d^r is obtained via

$$\begin{bmatrix} v_q^r \\ v_d^r \end{bmatrix} = \begin{bmatrix} \cos(\theta_e + \theta_r) & -\sin(\theta_e + \theta_r) \\ \sin(\theta_e + \theta_r) & \cos(\theta_e + \theta_r) \end{bmatrix} \begin{bmatrix} v_q^s \\ v_d^s \end{bmatrix} \tag{11}$$

The inverse transformation from the synchronous reference frame q^e-d^e to the stationary reference frame q^s-d^s is given by

$$\begin{bmatrix} v_q^s \\ v_d^s \end{bmatrix} = \begin{bmatrix} \cos(\theta_e + \theta_r) & \sin(\theta_e + \theta_r) \\ -\sin(\theta_e + \theta_r) & \cos(\theta_e + \theta_r) \end{bmatrix} \begin{bmatrix} v_q^e \\ v_d^e \end{bmatrix} \tag{12}$$

We may also project from the synchronous rotor frame to the rotor reference frame according to:

$$\begin{bmatrix} v_q^r \\ v_d^r \end{bmatrix} = \begin{bmatrix} \cos\theta_r & -\sin\theta_r \\ \sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v_q^e \\ v_d^e \end{bmatrix} \tag{13}$$

And the inverse transformation is

$$\begin{bmatrix} v_q^e \\ v_d^e \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{bmatrix} \begin{bmatrix} v_q^r \\ v_d^r \end{bmatrix} \tag{14}$$

[1] P. Krause, O. Wasynczuk, and S. Sudhoff, "Analysis of Electric Machinery," 1995, IEEE Press.
[2]J. Marques, H. Pinheiro, H. A. Gründling, J. R. Pinheiro and H. L. Hey, "A Survey On Variable-Speed Wind Turbine System," Proc. of ????, pp. 732-738.
[3] M. Simoes and F. Farret, "Alternative Energy Systems: Design and Analysis with Induction Generators," second edition, CRC Press, 2008.