Understanding the Inductances

We have identified six different inductances (or reactances) for characterizing machine dynamics. These are:

L_d, L_q (synchronous), L'_d, L'_q (transient), L''_d, L''_q (subtransient)

As we saw, the transient and subtransient inductances may be expressed in terms of the synchronous inductances L_d and L_q, the rotor winding self inductances L_F, L_D, L_G, and L_Q, and the mutual inductances L_{AG} and L_{AQ}, through the following relations that we developed:

$$L'_d = L_d - \frac{L_{AD}^{\,2}}{L_F} \qquad\qquad L'_q = L_q - \frac{L_{AQ}^{\,2}}{L_G}$$

$$L''_d = L_d - \frac{L_D + L_F - 2L_{AD}}{\dfrac{L_F L_D}{L_{AD}^{\,2}} - 1} \qquad\qquad L''_q = L_q - \frac{L_Q + L_G - 2L_{AQ}}{\dfrac{L_G L_Q}{L_{AQ}^{\,2}} - 1}$$

There are two pictures that may be helpful in understanding these inductances. One comes from Kundur's book, Fig. 3.27, pg. 109, which characterizes the "phase a" stator current for a three-phase fault occurring at the machine terminals. This picture is given below[1].

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[1] Kundur describes, p. 109, that this figure neglects stator transients, which can be equivalently expressed as omission of the "transformer" (as opposed to the "speed") voltage terms; he makes an important statement: by neglecting the transformer terms, "we are not assuming that λ_d and λ_q remain constant; on the contrary, we are assuming that they change instantly following a perturbation." Boldea in his book, "Synchronous Generators," 2nd edition, 2005, Section 5.11, p. 233, "Neglecting the stator flux transients," addresses this issue; he provides additional insight in his 2010 book, "Electric machines: steady state, transients, and design with Matlab," p. 446. VMAF address the same issue in Section 7.3.1 "Network in the transient state." We address this issue further in the notes called "Simplified models."
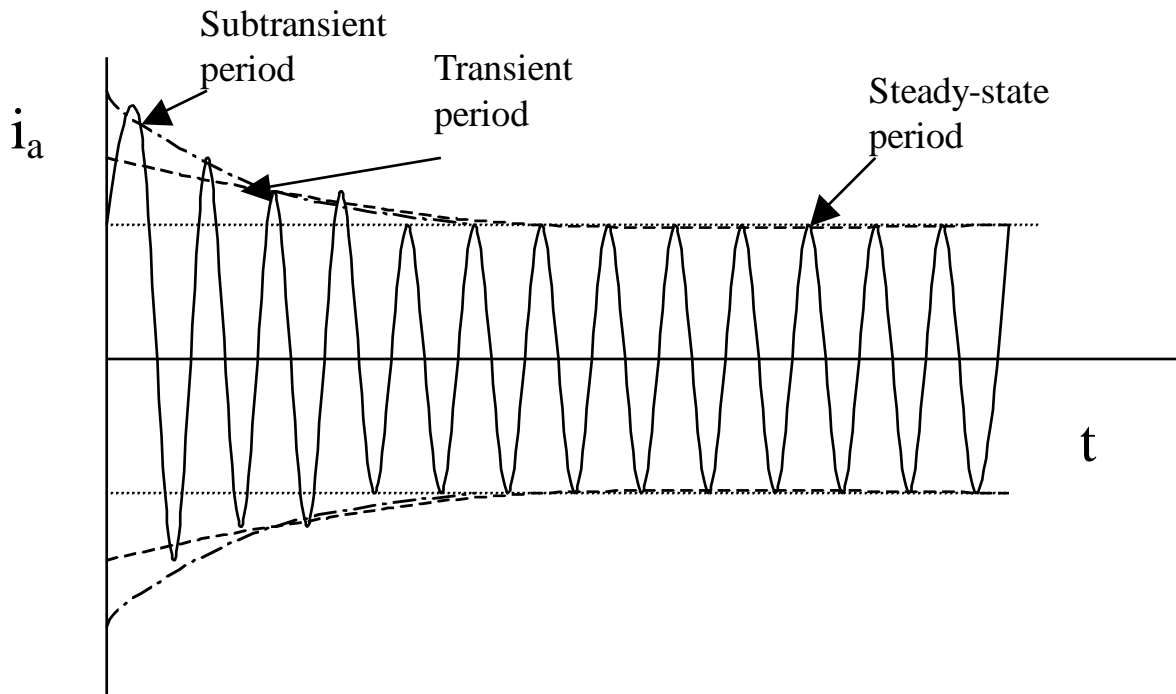
Note here that the subtransient period lasts for only a few cycles, during which the amplitude decays rapidly; the transient period spans a longer time frame, during which the amplitude decays more slowly; in the steady-state period, the amplitude is constant. The subtransient, transient, and synchronous inductances correspond to the subtransient, transient, and steady-state time periods, respectively.

The second helpful picture is Figure 15 out of Kimbarks' volume III (pg 26). This figure is key to understanding the inductances, what they represent, and their relative magnitudes (remembering their relative magnitudes is especially helpful in identifying inappropriate data).

Kimbark's picture identifies the flux paths for various reactances of a salient pole machine. The figures in this picture are drawn for the situation (the "thought-experiment") that we considered in deriving the inductances. In this situation, the rotor circuits were all closed but not excited (e.g., $v_F=0$), the rotor is turned forward at synchronous speed, and the stator circuits are energized at $t=0$ with a phase that forces the stator MMF wave to be in phase with either the direct-axis or the quadrature axis. Kimbark's picture shows six different figures (illustrating the machine flux paths), one for each of the six reactances that we have identified: L_d, L_q, L'_d, L'_q, L''_d, and L''_q.

Recall two facts:
1. Inductance is inversely proportional to reluctance: L=N/R, where R is reluctance of the flux path.
2. The reluctance of air is much greater than that of a ferromagnetic material.

So let's use the pictures to identify the flux path for each of the reactances.

There are five key ideas to be grasped in using these pictures. These ideas help us to understand the flux paths for each different condition. Once we understand the flux paths for each different condition, it will be possible to reason about reluctance, and thus the inductance, of each flux path. Note that the constant flux linkage theorem (CFLT), i.e., no instantaneous change in flux linkage, is central to the thinking of ideas 4 and 5 below.
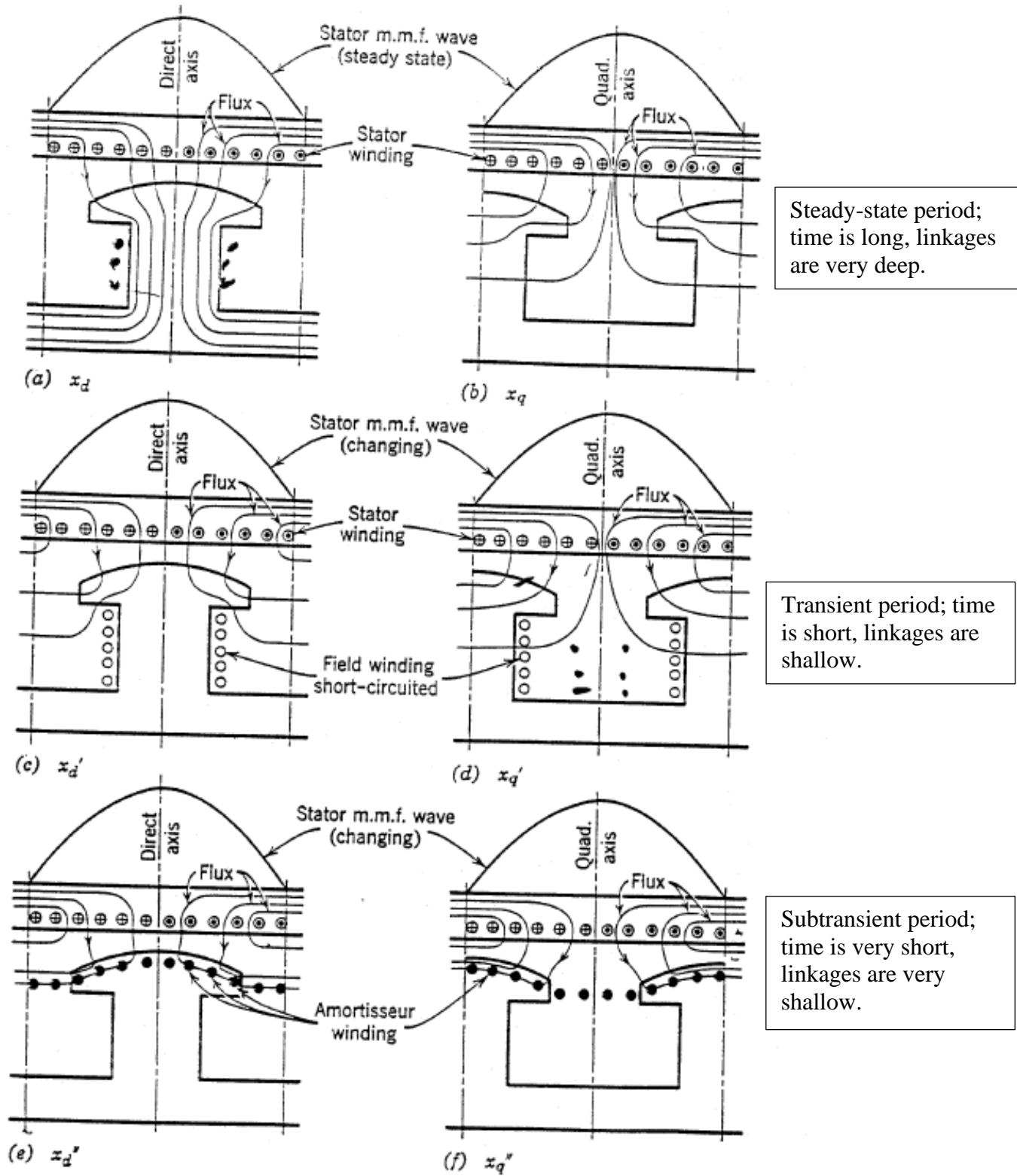
Steady-state period; time is long, linkages are very deep.

Transient period; time is short, linkages are shallow.

Subtransient period; time is very short, linkages are very shallow.

Fig. 15. Flux paths for various reactances of a salient-pole synchronous machine. Direct-axis synchronous reactance. (b) Quadrature-axis synchronous reactance. (c) Direct-axis transient reactance. (d) Quadrature-axis transient reactance. (e) Direct-axis subtransient reactance. (f) Quadrature-axis subtransient reactance. (From Ref. 34 by permission.)

1. <u>Location of the Stator MMF wave crest relative to the rotor</u>:
   In the picture, compare column 1 (Figs. a, c, e) to column 2 (Figs. b, d, f) and note the difference in the location of the MMF wave crest.
   - For the purpose of understanding the direct axis inductances, the direct axis flux paths are best visualized when the MMF wave crest is in phase with the direct axis.
   - For the purpose of understanding the quadrature axis inductances, the quadrature axis flux paths are best visualized when the MMF wave crest is in phase with the quadrature axis.

   The conditions for which the MMF wave crest is in phase with either direct or quadrature axes are obtained by applying appropriate a-b-c voltages, as we saw in our "thought experiment" (see notes "SubtransTransLT") used to derive these inductances, according to:
   - d-axis quantities are obtained from letting $v_a \sim \cos\theta$
   - q-axis quantities are obtained from letting $v_a \sim \sin\theta$

2. <u>Whether flux *can* link with field winding or not</u>:
   - Any flux in the d-axis *CAN* link with the field winding: see fig. (a)
   - Any flux in the q-axis *CANNOT* link with the field winding: see fig. (b), (d) and how flux lines are orthogonal to d-axis winding.

3. <u>Whether we analyze with or without damper windings</u>:
   This is equivalent to whether we are analyzing after damper winding dynamics have died, or before.
   - L_d, L'_d, L_q, and L'_q assume no damper windings (after damper winding transients have decayed): see figs (a-d).
   - L''_d and L''_q assume damper windings (before damper winding transients have decayed): see figs (e-f).

4. <u>Whether d-axis flux in **F or G field windings** MUST be zero (CFLT)</u>:
   - L_d and L_q characterize the steady-state conditions; here the flux linkage has had time to build up and therefore it links with the F and G windings, respectively – see Figs. (a) and (b). The flux path for assessment of these inductances is "deep" into the rotor circuit.

- L'$_d$ and L''$_d$ are both computed at t=0+. Recall that at t=0, the flux linkages in the F-winding are zero (no circuit is energized) and because the flux linkages cannot change instantaneously (due to CFLT), these lines of flux CANNOT link with the F winding – see Figs. (c) and (e). The flux path for assessment of these inductances is not as "deep" into the rotor circuit as in Fig. (a).
- L'$_q$ and L''$_q$ are both computed at t=0+, and because the flux linkages cannot change instantaneously (due to CFLT), these lines of flux CANNOT link with the G winding. The flux path for these is not as "deep" into the rotor circuit as in Fig. (b). <u>Special note</u>: The figures were drawn assuming no G-winding (since it is a salient pole machine). Therefore the lines of flux for Fig. (d) (almost like those of Fig. (b)) do appear to exist in the interpolar region where the G-winding would be located. If we did include the effects of the G-winding representation, then Fig. (d) would appear a little more like Fig. (f).

5. <u>Whether d-axis, q-axis flux in **damper windings** MUST be zero (CFLT)</u>:
- For L$_d$ and L'$_d$, the d-axis flux has had time to link with the D-winding. See figs. (a) and (c).
  For L$_q$ and L'$_q$, the q-axis flux has had time to link with the Q-winding. See figs (b) and (d)
  For both d and q synchronous and transient reactances, the flux paths go relatively "deep" into the rotor (i.e., beyond the damper winding).
- For L''$_d$, which is computed at t=0+, the d-axis flux has not had time to link with D-winding. See fig (e).
  For L''$_q$, which is computed at t=0+, the q-axis flux has not had time to link with Q-winding. See fig (f).
  For both d and q subtransient reactances, the flux paths into the rotor are quite shallow.

The above 5 points should allow us to understand the flux paths for the different conditions illustrated in the different pictures.

From the pictures, we can visually observe whether each flux path appears to be more in the iron (giving low reluctance and high inductance) or air (giving high reluctance and low inductance) and develop "rules" for comparing different data.

Let's just consider the salient-pole (hydro) machine first.

1. <u>Comparison of d-axis inductances</u>: Observing figs. (a, c, and e), we conclude that the percentage of the path in air (and thus the reluctance) gets increasingly greater with each figure (effects of CFLT), and so the inductance gets smaller with each figure, implying $L_d > L'_d > L''_d$ ➔ $X_d > X'_d > X''_d$.

2. <u>Comparison of the q-axis inductances</u>: Observing figs. (b, d, and f), we conclude that the percentage of the path in air (and thus the reluctance) is the same for the steady state and transient periods (because no G-winding), but increases for the subtransient period (because of CFLT), implying that $L_q = L'_q > L''_q$ ➔ $X_q = X'_q > X''_q$. Thus, a useful way to recognize hydro data is that $X_q = X'_q$.

3. <u>Comparison of synchronous inductances</u>: Observing figs. (a, b), we conclude that the steady-state quadrature flux path occurs largely in the interpolar regions and the steady-state direct axis flux path occurs largely in the polar regions, implying that the quadrature flux has significantly more air in its path than the direct axis flux, and therefore quadrature flux path reluctance (inductance) is higher (lower) than direct axis flux path reluctance, so $L_d > L_q$ ➔ $X_d > X_q$.

4. <u>Comparison of transient inductances</u>: Observing figs. (c, d), we see that the d-axis path consists largely of the interpolar region whereas the q-axis path consists largely of the polar region, and we conclude that the transient quadrature flux has significantly less air in its path than the transient direct axis flux, and therefore quadrature flux path reluctance (inductance) is lower (higher) than direct axis flux path reluctance, so $L'_d < L'_q$ ➔ $X'_d < X'_q$.

5. <u>Comparison of subtransient inductances</u>: Observing figs. (e, f), we see a situation similar to the transient situation, i.e., the d-axis path consists largely of the interpolar region whereas the q-axis path consists largely of the polar region, and we conclude that the subtransient quadrature flux has significantly less air in its path than the direct axis flux, and therefore quadrature flux path reluctance (inductance) is lower (higher) than direct axis flux path reluctance, so $L''_d < L''_q$➜ $X''_d < X''_q$

In the case of the round rotor (steam-turbine) machine, number 1 above is the same. The remaining 4 are slightly different, due to the presence of the G-circuit, as follows:

2. <u>Comparison of the q-axis inductances</u>: Observing figs. b, the modified d, and f, we conclude that the percentage of the path in air (and thus the reluctance) gets increasingly greater with each figure (effects of CFLT), implying that $L_q > L'_q > L''_q$ ➜ $X_q > X'_q > X''_q$. So we see that whereas hydro machines have $X_q = X'_q$, the steam-turbine machines have $X_q > X'_q$.

3. <u>Comparison of synchronous inductances</u>: The difference in synchronous inductances for the salient pole machine occurred because of the differences between the polar region and the interpolar region, resulting in $X_d > X_q$. In the round rotor machine, these two regions appear virtually the same, so $L_d = L_q$➜ $X_d = X_q$. This is another useful way to distinguish hydro data from steam data.

4. <u>Comparison of transient inductances</u>: The same reasoning as in (3) above (for the round rotor machine) would lead us to conclude that $L'_d = L'_q$. This is not the case. The reason is the CFLT requires that the transient flux in each axis **not** link with the F and G-winding, respectively. This effect is more pronounced for the F-winding since it is a "stronger" (more turns) winding than the G-winding. Therefore, q-axis flux goes "deeper" into the rotor, i.e., more of the d-axis flux is forced into the air gap than for the q-axis flux. So $L'_d < L'_q$➜ $X'_d < X'_q$, which is the same for the salient pole machine.

5. <u>Comparison of subtransient inductances</u>: The reasoning of (4) above (for the round rotor machine) would lead us to conclude that $L''_d < L''_q$. This is not the case. The reason is that the damper winding "strengths" are about the same and their effect on each axis is quite similar. So here, we have that $L''_d = L''_q \rightarrow X''_d = X''_q$, although some variation from this rule may occur.

It is interesting to inspect the Diablo Canyon data to see if it conforms to the above rules (it is a steam-turbine machine so we use the second set of rules).

```
                          UT
                          NY   O
                          IP   W
              I       P    TE   N   X''D X''Q T''DT''Q
MD < NAME ><KV>D<MVA> <F> <#> <> < > <    >< >< >< >
MD DIABLO CYN 211340.  90   1   N PG+ 0.2810.281 430.124


              I        P  Q MVA                       T'D T'Q    SG10 SG12 D
MF < NAME ><KV>D<MWS ><%><%><   ><RA><X'D><X'Q><XD ><XQ >< >< ><XL >< >< >< >
MF DIABLO CYN 214650.01.01.0134000210.3460.9911.6931.6366.581.5 2280  769 4100.0
```

$X_d = 1.693$ $X'_d = 0.346$ $X''_d = 0.281$

$X_q = 1.636$ $X'_q = 0.991$ $X''_q = 0.281$

Rule 1: $X_d > X'_d > X''_d \rightarrow$ Yes.

Rule 2: $X_q > X'_q > X''_q \rightarrow$ Yes.

Rule 3: $X_d = X_q \rightarrow$ Almost.

Rule 4: $X'_d < X'_q \rightarrow$ Yes.

Rule 5: $X''_d = X''_q \rightarrow$ Yes.

Remember: The inductance (reactance) with the smallest reluctance path will be largest. $X_d > X'_d > X''_d$
        $X_q = X'_q > X''_q$ (Salient pole)
        $X_q \geq X'_q > X''_q$ (Round rotor)

This is consistent with the figure given at the beginning of these notes (from Kundur, pg. 109) where one can observe that I''>I'>I where each current is determined by the ratio of a voltage to the appropriate reactance.

You can review some of the data in appendix C of your text to see if it conforms to the above rules.

And you can check Kundur, page 153, for some comparative data for both salient pole and round-rotor machines, which I have copied out below.

Table 4.2

SALIENT   Round

Parameter		Hydraulic Units	Thermal Units
Synchronous Reactance	X_d	0.6 - 1.5	1.0 - 2.3
	X_q	0.4 - 1.0	1.0 - 2.3
Transient Reactance	X_d'	0.2 - 0.5	0.15 - 0.4
	X_q'	-	0.3 - 1.0
Subtransient Reactance	X_d''	0.15 - 0.35	0.12 - 0.25
	X_q''	0.2 - 0.45	0.12 - 0.25
Transient OC Time Constant	T_{d0}'	1.5 - 9.0 s	3.0 - 10.0 s
	T_{q0}'	-	0.5 - 2.0 s
Subtransient OC Time Constant	T_{d0}''	0.01 - 0.05 s	0.02 - 0.05 s
	T_{q0}''	0.01 - 0.09 s	0.02 - 0.05 s
Stator Leakage Inductance	X_l	0.1 - 0.2	0.1 - 0.2
Stator Resistance	R_a	0.002 - 0.02	0.0015 - 0.005

Notes: 1. Reactance values are in per unit with stator base values equal to the corresponding machine rated values.
2. Time constants are in seconds.

Similar data is in Chapter 4 of VMAF, p. 135: Note Table 4.3 comes from Kimbark, see next page of these notes.

Table 4.3. Typical Synchronous Machine Time Constants in Seconds

Time constant	Turbogenerators			Waterwheel generators			Synchronous condensers		
	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High
τ'_{d0}	2.8	5.6	9.2	1.5	5.6	9.5	6.0	9.0	11.5
τ'_d	0.4	1.1	1.8	0.5	1.8	3.3	1.2	2.0	2.8
$\tau''_d = \tau''_q$	0.02	0.035	0.05	0.01	0.035	0.05	0.02	0.035	0.05
τ_a	0.04	0.16	0.35	0.03	0.15	0.25	0.10	0.17	0.30

Source: Reprinted by permission from *Power System Stability*, vol. 3, by E.W. Kimbark. © Wiley, 1956.

Table 4.4. Typical Turbogenerator and Synchronous Condenser Characteristics

Parameter	Generators		Synchronous condensers	
	Range	Recommended average	Range	Recommended average
Nominal rating	300–1000 MW	...	50–100 MVA	...
Power factor	0.80–0.95	0.90	...	...
Direct axis synchronous reactance x_d	140–180	160	170–270	220
Transient reactance x'_d	23–35	25	45–65	55
Subtransient reactance x''_d	15–23	20	35–45	40
Quadrature axis synchronous reactance x_q	150–160	155	100–130	115
Negative-sequence reactance x_2	18–20	19	35–45	40
Zero-sequence reactance x_0	12–14	13	15–25	20
Short circuit ratio	0.50–0.72	0.64	0.35–0.65	0.50
Inertia constant H, $\frac{(kW \cdot s)}{(kVA)}$ 3600 r/min	3.0–5.0	4.0	...	...
Inertia constant H, $\frac{(kW \cdot s)}{(kVA)}$ 1800 r/min	5.0–8.0	6.0	...	...

Source: From the 1964 National Power Survey made by the U.S. Federal Power Commission. USGPO.
Note: All reactances in percent on rated voltage and kVA base. kW losses for typical synchronous condensers in the range of sizes shown, excluding losses associated with step-up transformers, are in the order of 1.2–1.5% on rated kVA base. No attempt has been made to show kW losses associated with generators, since generating plants are generally rated on a net power output basis and losses vary widely dependent on the generator plant design.

Table 4.5. Typical Hydrogenerator Characteristics

Parameter	Small units	Large units
Nominal rating (MVA)	0–40	40–200
Power factor	0.80–0.95*	0.80–0.95*
Speed (r/min)	70–350	70–200
Inertia constant H, $\frac{(kW \cdot s)}{(kVA)}$	1.5–4.0	3.0–5.5
Direct axis synchronous reactance x_d	90–110	80–100
Transient reactance x'_d	25–45	20–40
Subtransient reactance x''_d	20–35	15–30
Quadrature axis synchronous reactance x_q	...	...
Negative-sequence reactance x_2	20–45	20–35
Zero-sequence reactance x_0	10–35	10–25
Short circuit ratio	1.0–2.0	1.0–2.0

Source: From the 1964 National Power Survey made by the U.S. Federal Power Commission. USGPO.
Note: All reactances in percent on rated voltage and kVA base. No attempt has been made to show kW losses associated with generators, since generating plants are generally rated on a net power output basis and losses vary widely dependent on the generator plant design.
*These power factors cover conditions for generators installed either close to or remote from load centers.

And From Kimbark, p. 40 (note time constant data at bottom of table is used in VMAF table, given on previous page).

TYPICAL CONSTANTS OF THREE-PHASE SYNCHRONOUS MACHINES
(Adapted from Refs. 15, 34, and 41)

	Turbo-generators (solid rotor)			Water-Wheel Generators (with dampers)†			Synchronous Condensers			Synchronous Motors (general purpose)		
	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High
Reactances in per unit												
x_d	0.95	1.10	1.45	0.60	1.15	1.45	1.50	1.80	2.20	0.80	1.20	1.50
x_q	0.92	1.08	1.42	0.40	0.75	1.00	0.95	1.15	1.40	0.60	0.90	1.10
x_d'	0.12	0.23.	0.28	0.20	0.37	0.50‡	0.30	0.40	0.60	0.25	0.35	0.45
x_q'	0.12	0.23	0.28	0.40	0.75	1.00	0.95	1.15	1.40	0.60	0.90	1.10
x_d''	0.07	0.12	0.17	0.13	0.24	0.35	0.18	0.25	0.38	0.20	0.30	0.40
x_q''	0.10	0.15	0.20	0.23	0.34	0.45	0.23	0.30	0.43	0.30	0.40	0.50
x_p	0.07	0.14	0.21	0.17	0.32	0.40	0.23	0.34	0.45			
x_2	0.07	0.12	0.17	0.13	0.24	0.35	0.17	0.24	0.37	0.25	0.35	0.45
x_0^*	0.01		0.10	0.02		0.21	0.03		0.15	0.04		0.27
Resistances in per unit												
r_a(d-c.)	0.0015		0.005	0.003		0.020	0.002		0.015			
r(a-c.)	0.003		0.008	0.003		0.015	0.004		0.010			
r_e	0.025		0.045	0.012		0.20	0.025		0.07			
Time constants in seconds												
T_{do}'	2.8	5.6	9.2	1.5	5.6	9.5	6.0	9.0	11.5			
T_d'	0.4	1.1	1.8	0.5	1.8	3.3	1.2	2.0	2.8			
$T_d'' = T_q''$	0.02	0.035	0.05	0.01	0.035	0.05	0.02	0.035	0.05			
T_a	0.04	0.16	0.35	0.03	0.15	0.25	0.1	0.17	0.3			

*x_0 varies from about 0.15 to 0.60 of x_d'', depending upon winding pitch.

†For water-wheel generators without damper windings,

$$x_d'' \approx 0.85 x_d', \qquad x_q'' = x_q' = x_q, \qquad x_2 = (x_d' + x_q)/2,$$

and x_0 is as listed.

‡For curves showing the normal value of x_d' of water-wheel-driven generators as a function of kilovolt-ampere rating and speed, see Ref. 54.