INTRODUCTION TO THE 1989 IEEE/PES SYMPOSIUM ON EIGENANALYSIS AND FREQUENCY DOMAIN METHODS FOR SYSTEM DYNAMIC PERFORMANCE

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Abstract - To make the Symposium papers more accessible to non-specialists, a tutorial review is made of the basic terminology and mathematics that underlie them. Featured topics includes s-domain treatment of elementary dynamic systems, fundamental aspects of eigenanalysis, participation factors, and singular values.

Keywords: dynamic system, transfer function, eigenvalue, participation factor, singular value.

I. INTRODUCTION

This symposium presents contemporary mathematical methods and tools that are becoming essential to the planning, analysis, and control of modern power systems. The increasing complexity of these systems has mandated much closer inspection of such issues as intrinsic stability, dynamic interactions, and model validity. Use of even well-established tools for this requires at least a basic familiarity with the underlying mathematics, however. The necessary concepts are fairly straightforward, and are fundamental material from differential equations and/or automatic control. Even so, there is a good deal of this material, and it is well-laced with special terms and notation. It is hoped that this Introduction will help the non-specialist to overcome this.

II. SOME FUNDAMENTAL NOMENCLATURE

The systems considered here are commonly represented in the form

$$\dot{x} = Ax + Bu \ , \tag{1A}$$

Basic model

$$y = Cx + Du \tag{1B}$$

where $\dot{x}$ denotes differentiation with respect to time. Variables u and y are respectively the input and the output of the system; x, the internal state of the system, is usually taken to be a vector of n elements (n being the order of the system differential equation). System matrices A,B,C,D are fixed under present assumptions, so the system itself is termed "LTI" (linear time-invariant). Were one or more of these matrices time-varying, the system would be "LTV".

Systems are also classified according to the number of (scalar) inputs and outputs. Thus, if both u and y contain just one element, the system is "SISO" (single-input, single-output). If u and/or y contain multiple elements then the system is MIMO, MISO, or SIMO.

III. DYNAMICS OF LOW-ORDER SYSTEMS

The papers presented in this symposium build, with various strategies and objectives, upon the same central aspect of dynamic systems analysis. This is the ability to predict the future time-domain behavior of an LTI system through relatively simple calculations in the "complex frequency" domain [1-3]. An elementary example of this is provided by a system obeying the first-order differential equation

$$\dot{x} = \sigma x + u(t) \tag{2A}$$

and with the output

$$y = Cx \ . \tag{2B}$$

All quantities are scalar in this very reduced case. Suppose that the system starts at rest, with initial condition $x(t_0)=x_0=0$ at time $t_0=0$. Then equation (2A) Laplace transforms as

$$sX(s) = \sigma X(s) + U(s) \tag{3A}$$

and

$$Y(s) = \frac{CU(s)}{s-\sigma} \triangleq G(s)U(s) \tag{3B}$$

where

$$G(s) = \frac{C}{s-\sigma} \tag{3C}$$

is the transfer function for the system. (The qualified equality "$\triangleq$" is used to indicate the introduction of newly defined terms.) Response to particular inputs u(t) can now be determined by calculating the associated U(s) and inverting G(s)U(s) to time domain. For example, a unit impulse produces U(s)=1 and

$$y(t) = C\exp(\sigma t) \ . \tag{4}$$

For u(t) a step of height $\bar{U}$, $U(s)=\bar{U}/s$ and

$$Y(S) = \frac{C\bar{U}}{s(s-\sigma)} \tag{5A}$$

$$= \frac{C\bar{U}/\sigma}{s} + \frac{-C\bar{U}/\sigma}{s-\sigma} \triangleq \frac{K_0}{s} + \frac{K_1}{s-\sigma} \ , \tag{5B}$$

$$y(t) = K_0 + K_1\exp(\sigma t) \tag{5C}$$

$$= (C\bar{U}/\sigma)[1 - \exp(\sigma t)] \ . \tag{5D}$$

Equation (5B) provides a partial-fraction expansion for Y(s). Each K_i weights a term of form $1/(s-\lambda_i)$, and is said to be the residue associated with λ_i. The λ_i are generally referred to (not quite