HOMEWORK #2, Due Monday, Jan 26.

SOLUTIONS

1. Use the equations at the bottom of slide 7, which are:

   ➔ $$P_D = |V_1||V_2|B\sin\theta_{12}$$
   $$Q_D = -|V_2|^2 B + |V_1||V_2|B\cos\theta_{12}$$

2. Now, just bring the right hand side of these 2 equations over to the left-hand side, and you have the 2 equations that correspond to G(y,p)=0.

   ➔ $$P_D - |V_1||V_2|B\sin\theta_{12} = 0$$
   $$Q_D + |V_2|^2 B - |V_1||V_2|B\cos\theta_{12} = 0$$

3. Solve these equations to get the corresponding power flow solution (but you do not need Newton-Raphson to do this – you can just use the equation at the bottom of slide 10). Use V1=1, PD=0.4, pf=0.97 lagging, B=1 as the operating conditions.

   ➔ Solving using below code, we get |V2|=0.9220.

```
beta=.25;
B=2;
v1=1.0;
pdn=[0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.85 0.9];
v2n=sqrt((v1^2-(2/B)*beta.*pdn - sqrt(v1^4-
(4/B)*pdn.*((1/B)*pdn+beta*v1^2)))/2);
pdp=[0.85 0.8 0.75 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0];
v2p=sqrt((v1^2-(2/B)*beta.*pdp + sqrt(v1^4-
(4/B)*pdp.*((1/B)*pdp+beta*v1^2)))/2);
pd4=[pdn pdp];
v24=[v2n v2p];
```

   Then we obtain θ from

   $$P_D = |V_1||V_2|B\sin\theta_{12}$$
   $$\sin\theta_{12} = P_D/|V_1||V_2|B = 0.4/(1)(0.9220)(2) = 0.2169$$
   $$\Rightarrow \theta_{12} = \arcsin(0.2169) = 0.2186 rad = 12.527 \deg rees$$

4. Now you need to replace the value specified in the equations for PD (assuming that the initial load is 0.4) with 0.4*lambda. This gives you the equations in the form of slide 49: 0=G(theta,V,lambda). Note, however, that G is really two equations: G1 and G2. Again, use V1=1.0, pf=0.97 lagging, and B=1.0.

   ➔With pf=0.97 lagging, then Beta=0.25, and Q_D=Beta*P_D. Since P_D=0.4λ, then Q_D=0.25*0.4λ=0.1λ. Substituting these into equations of #2 above, we get:

   $$0.4\lambda - |V_1||V_2|B\sin\theta_{12} = 0$$
   $$0.1\lambda + |V_2|^2 B - |V_1||V_2|B\cos\theta_{12} = 0$$

   With B=2 and $|V_1|$=1, and dropping the subscripts from θ_{12}, the subscript and absolute value sign from |V2|, we get

$$G_1() = 0.4\lambda - 2V\sin\theta = 0$$

$$G_2() = 0.1\lambda + 2V^2 - 2V\cos\theta = 0$$

5. Now you need to formulate the equations on the slide 55. This is a matter of taking derivatives and then evaluating those derivatives at the solution that you obtained above. Note, however, that each element in the matrix of slide 55 actually represents 2 elements. That is:

   | dG1/dtheta dG1/dV dG1/dlambda|
   | dG2/dtheta dG2/dV dG2/dlambda|
   | 0 0 1 |

   ➔we obtain $\begin{bmatrix} -2V\cos\theta & -2\sin\theta & 0.4 \\ 2V\sin\theta & 4V - 2\cos\theta & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$

6. Evaluate each of the above matrix elements at the solution obtained in step 3.

   $$➔ \begin{bmatrix} -2V\cos\theta & -2\sin\theta & 0.4 \\ 2V\sin\theta & 4V - 2\cos\theta & 0.1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1.8001 & -0.4337 & 0.4 \\ 0.3999 & 1.7356 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Then solve these equations for the tangent vector. You can do this by inverting the above matrix (use matlab or a calculator to do this) and then multiply the right-hand-side by this inverted matrix.

   $$➔ \begin{bmatrix} -1.8001 & -0.4337 & 0.4 \\ 0.3999 & 1.7356 & 0.1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

   $$\begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} -1.7997 & -0.4339 & 0.4 \\ 0.4 & 1.7348 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2500 \\ -0.1152 \\ 1 \end{bmatrix}$$

8. Then take a "step" using an appropriately chosen step size per the equation on slide 56.

   ➔ A reasonable step size is σ=0.1 or 0.2. We will choose 0.2. Then the predicted point becomes:

   $$\begin{bmatrix} \theta^{(2,p)} \\ V^{(2,p)} \\ \lambda^{(2,p)} \end{bmatrix} = \begin{bmatrix} \theta^{(1)} \\ V^{(1)} \\ \lambda^{(1)} \end{bmatrix} + \sigma \begin{bmatrix} d\theta \\ dV \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0.2186 \\ 0.9220 \\ 1 \end{bmatrix} + 0.2 * \begin{bmatrix} 0.2500 \\ -0.1152 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2686 \\ 0.8990 \\ 1.2 \end{bmatrix}$$

9. Beginning from your predicted point that you identified in step 8 of #2a, develop equations for approach a, solve them, and identify the resulting corrected point in terms of voltage and power.

➔We need to develop the following equations:

$$\underline{0} = \underline{G}(\underline{y}^{(i+1)}, \lambda^{(i+1)}) \quad ➔ \quad \begin{aligned} &0.4\lambda - 2V\sin\theta = 0 \\ &0.1\lambda + 2V^2 - 2V\cos\theta = 0 \end{aligned}$$

$$\left\{\underline{y}^{(i+1)} - \underline{y}^{(i+1,p)}\right\} \bullet \underline{t} = 0 \quad ➔ \quad [\theta - 0.2686 \quad V - 0.899 \quad \lambda - 1.2] \bullet \begin{bmatrix} 0.2500 \\ -0.1152 \\ 1 \end{bmatrix} = 0$$

Performing the multiplication associated with the last equation results in

$$0.25\theta - 0.0712 - 0.1152V + 0.1036 + \lambda - 1.2 = 0 \Rightarrow 0.25\theta - 0.1152V + \lambda - 1.1676 = 0$$

Writing the three equations together

$$0.4\lambda - 2V\sin\theta = 0$$

$$0.1\lambda + 2V^2 - 2V\cos\theta = 0$$

$$0.25\theta - 0.1152V + \lambda - 1.1676 = 0$$

Define:

$$\underline{x} = \begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}, \quad \underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ f_3(\underline{x}) \end{bmatrix} = \begin{bmatrix} 0.4\lambda - 2V\sin\theta \\ 0.1\lambda + 2V^2 - 2V\cos\theta \\ 0.25\theta - 0.1152V + \lambda - 1.1676 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underline{0}$$

Writing $\underline{f}(\underline{x}) = \underline{0}$ as a Taylor series, we get

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} + \frac{1}{2}\underline{f}''(\underline{x}^{(0)})(\Delta\underline{x}^{(0)})^2 + \ldots = \underline{0}$$

We will make a guess at the solution, and hopefully our guess will be a good one such that $\Delta\underline{x}^{(0)}$ is small, then the higher order terms in the above relation are also small and we can write

$$\underline{f}(\underline{x}^{(0)} + \Delta\underline{x}^{(0)}) = \underline{f}(\underline{x}^{(0)}) + \underline{f}'(\underline{x}^{(0)})\Delta\underline{x}^{(0)} = \underline{0}$$

The above relation can be manipulated to get

$$\Delta\underline{x}^{(0)} = -\left\{\underline{f}'(\underline{x}^{(0)})\right\}^{-1} \underline{f}(\underline{x}^{(0)}) = -\underline{J}^{-1}\underline{f}(\underline{x}^{(0)})$$

where $\underline{J}$ is the Jacobian evaluated at our guess. Then we will correct our guess according to

$$\underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)})$$

Once the "mismatch" vector $\underline{f}(\underline{x})$ is within a small tolerance of the zero vector $\underline{0}$, we will assume that we have obtained the solution.

The relation will be

$$\underline{x}^{(1)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) = \begin{bmatrix} \theta^{(0)} \\ V^{(0)} \\ \lambda^{(0)} \end{bmatrix} - \begin{bmatrix} -2V\cos\theta & -2\sin\theta & 0.4 \\ 2V\sin\theta & 4V-2\cos\theta & 0.1 \\ 0.25 & -0.1152 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.4\lambda - 2V\sin\theta \\ 0.1\lambda + 2V^2 - 2V\cos\theta \\ 0.25\theta - 0.1152V + \lambda - 1.1676 \end{bmatrix}$$

Now we will use as our "initial guess" the predicted solution, which is $\begin{bmatrix} \theta^{(2,p)} \\ V^{(2,p)} \\ \lambda^{(2,p)} \end{bmatrix} = \begin{bmatrix} 0.2686 \\ 0.8990 \\ 1.2 \end{bmatrix}$

The above expression was coded in the following, using Matlab. Note that the stopping criterion is that the maximum absolute value of $\underline{f}(\underline{x})$ must be less than tol, which was set to 0.00001.

```
xold=[0.2686; 0.8990; 1.2];
f=[10;10;10];
tol=0.00001;
count=0;
while max(abs(f)) > tol,
   t=xold(1);
   v=xold(2);
   lam=xold(3);
   J=[-2*v*cos(t) -2*sin(t)   0.4;
     2*v*sin(t)  4*v-2*cos(t)  0.1;
     0.25        -0.1152     1.0];
   f=[0.4*lam-2*v*sin(t);
     0.1*lam+2*v^2-2*v*cos(t);
     0.25*t-0.1152*v+lam-1.1676];
   xnew=xold-inv(J)*f;
   xold=xnew;
   count=count+1;
end
count, xnew, f,
```

The above converged on the third iteration to the following solution:

$$\begin{bmatrix} \theta^{(1)} \\ V^{(1)} \\ \lambda^{(1)} \end{bmatrix} = \begin{bmatrix} 0.2718 \\ 0.8962 \\ 1.2029 \end{bmatrix}$$

10. Repeat #9 except implement approach b.

➔Noting that the largest element in the tangent vector is dλ, we choose λ for the continuation parameter. We then need to develop the following equations:

$$\underline{0} = \underline{G}(\underline{y}^{(i+1)}, \lambda^{(i+1)}) \quad \rightarrow \quad \begin{aligned} 0.4\lambda - 2V\sin\theta &= 0 \\ 0.1\lambda + 2V^2 - 2V\cos\theta &= 0 \end{aligned}$$

$$\left\{ y_k^{(i+1)} - \eta \right\} = 0 \quad \rightarrow \quad \lambda - 1.2 = 0$$

Writing the three equations together

$$0.4\lambda - 2V\sin\theta = 0$$
$$0.1\lambda + 2V^2 - 2V\cos\theta = 0$$
$$\lambda - 1.2 = 0$$

Define:

$$\underline{x} = \begin{bmatrix} \theta \\ V \\ \lambda \end{bmatrix}, \qquad \underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \\ f_3(\underline{x}) \end{bmatrix} = \begin{bmatrix} 0.4\lambda - 2V\sin\theta \\ 0.1\lambda + 2V^2 - 2V\cos\theta \\ \lambda - 1.2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \underline{0}$$

The Newton-Raphson procedure is

$$\underline{x}^{(1)} = \underline{x}^{(0)} - \underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) = \begin{bmatrix} \theta^{(0)} \\ V^{(0)} \\ \lambda^{(0)} \end{bmatrix} - \begin{bmatrix} -2V\cos\theta & -2\sin\theta & 0.4 \\ 2V\sin\theta & 4V - 2\cos\theta & 0.1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0.4\lambda - 2V\sin\theta \\ 0.1\lambda + 2V^2 - 2V\cos\theta \\ \lambda - 1.2 \end{bmatrix}$$

We again use as our "initial guess" the predicted solution, which is $\begin{bmatrix} \theta^{(2,p)} \\ V^{(2,p)} \\ \lambda^{(2,p)} \end{bmatrix} = \begin{bmatrix} 0.2686 \\ 0.8990 \\ 1.2 \end{bmatrix}$.

The following Matlab code was used, again with stopping criterion that the maximum absolute value of f(x) must be less than tol, which was set to 0.00001.

```
xold=[0.2686; 0.8990; 1.2];
f=[10;10;10];
tol=0.00001;
count=0;
while max(abs(f)) > tol,
   t=xold(1);
   v=xold(2);
   lam=xold(3);
   J=[-2*v*cos(t) -2*sin(t)   0.4;
     2*v*sin(t)  4*v-2*cos(t) 0.1;
     0           0            1.0];
   f=[0.4*lam-2*v*sin(t);
     0.1*lam+2*v^2-2*v*cos(t);
     lam-1.2];
   xnew=xold-inv(J)*f;
   xold=xnew;
   count=count+1;
end
count, xnew, f,
```

The above converged on the third iteration to the following solution:

$$\begin{bmatrix} \theta^{(1)} \\ V^{(1)} \\ \lambda^{(1)} \end{bmatrix} = \begin{bmatrix} 0.2710 \\ 0.8966 \\ 1.2 \end{bmatrix}$$