Equal Area Criterion

1.0 Development of equal area criterion

As in previous notes, all powers are in per-unit.

I want to show you the equal area criterion a little differently than the book does it.

Let's start from Eq. (2.43) in the book.

$$\frac{2H}{\omega_{\mathrm{Re}}}\frac{d^2\delta}{dt^2} = P_m - P_e = P_a \tag{1}$$

Note in (1) that the book calls ω_{Re} as ω_R; this needs to be 377 rad/sec (for a 60 Hz system).

We can also write (1) as

$$\frac{2H}{\omega_{\mathrm{Re}}}\frac{d\omega}{dt} = P_m - P_e = P_a \tag{2}$$

Now multiply the left-hand-side by ω and the right-hand side by dδ/dt (recall ω= dδ/dt) to get:

$$\frac{H}{\omega_{\mathrm{Re}}}\left\{2\omega\frac{d\omega}{dt}\right\} = \left[P_m - P_e\right]\frac{d\delta}{dt} \tag{3}$$

Note:

$$\frac{d\left(\omega(t)^2\right)}{dt} = 2\omega(t)\frac{d\omega(t)}{dt} \tag{4}$$

Substitution of (4) into the left-hand-side of (3) yields:

$$\frac{H}{\omega_{\mathrm{Re}}}\left\{\frac{d\omega^2}{dt}\right\} = \left[P_m - P_e\right]\frac{d\delta}{dt} \tag{5}$$

Multiply by dt to obtain:

$$\frac{H}{\omega_{\mathrm{Re}}} d\omega^2 = \left[P_m - P_e\right]d\delta \tag{6}$$

Now consider a change in the state such that the angle goes from δ_1 to δ_2 while the speed goes from ω_1 to ω_2. Integrate (6) to obtain:

$$\frac{H}{\omega_{\mathrm{Re}}}\int_{\omega_1^2}^{\omega_2^2} d\omega^2 = \int_{\delta_1}^{\delta_2}\left[P_m - P_e\right]d\delta \tag{7}$$

Note the variable of integration on the left is ω^2. This results in

$$\frac{H}{\omega_{\mathrm{Re}}}\left[\omega_2^2-\omega_1^2\right]=\int_{\delta_1}^{\delta_2}\left[P_m-P_e\right]d\delta \tag{8}$$

The left-hand-side of (8) is proportional to the change in kinetic energy between the two states, which can be shown more explicitly by substituting $H=W_k/S_B=(1/2)J\omega_R^2/S_B$ into (8), for H:

$$\frac{1}{2}\frac{J\omega_R^2}{S_B\omega_{\mathrm{Re}}}\left[\omega_2^2-\omega_1^2\right]=\int_{\delta_1}^{\delta_2}\left[P_m-P_e\right]d\delta \tag{8a}$$

$$\frac{\omega_R^2}{S_B\omega_{\mathrm{Re}}}\left[\frac{1}{2}J\omega_2^2-\frac{1}{2}J\omega_1^2\right]=\int_{\delta_1}^{\delta_2}\left[P_m-P_e\right]d\delta \tag{8b}$$

Returning to (8), let ω_1 be the speed at the initial moment of the fault ($t=0^+$, $\delta=\delta_1$), and ω_2 be the speed at the maximum angle reached ($\delta=\delta_r$), as shown in Fig. 1 below.

Note that the fact that we identify a maximum angle $\delta=\delta_r$ indicates an implicit assumption that the performance is stable. Therefore the following development *assumes stable performance*.

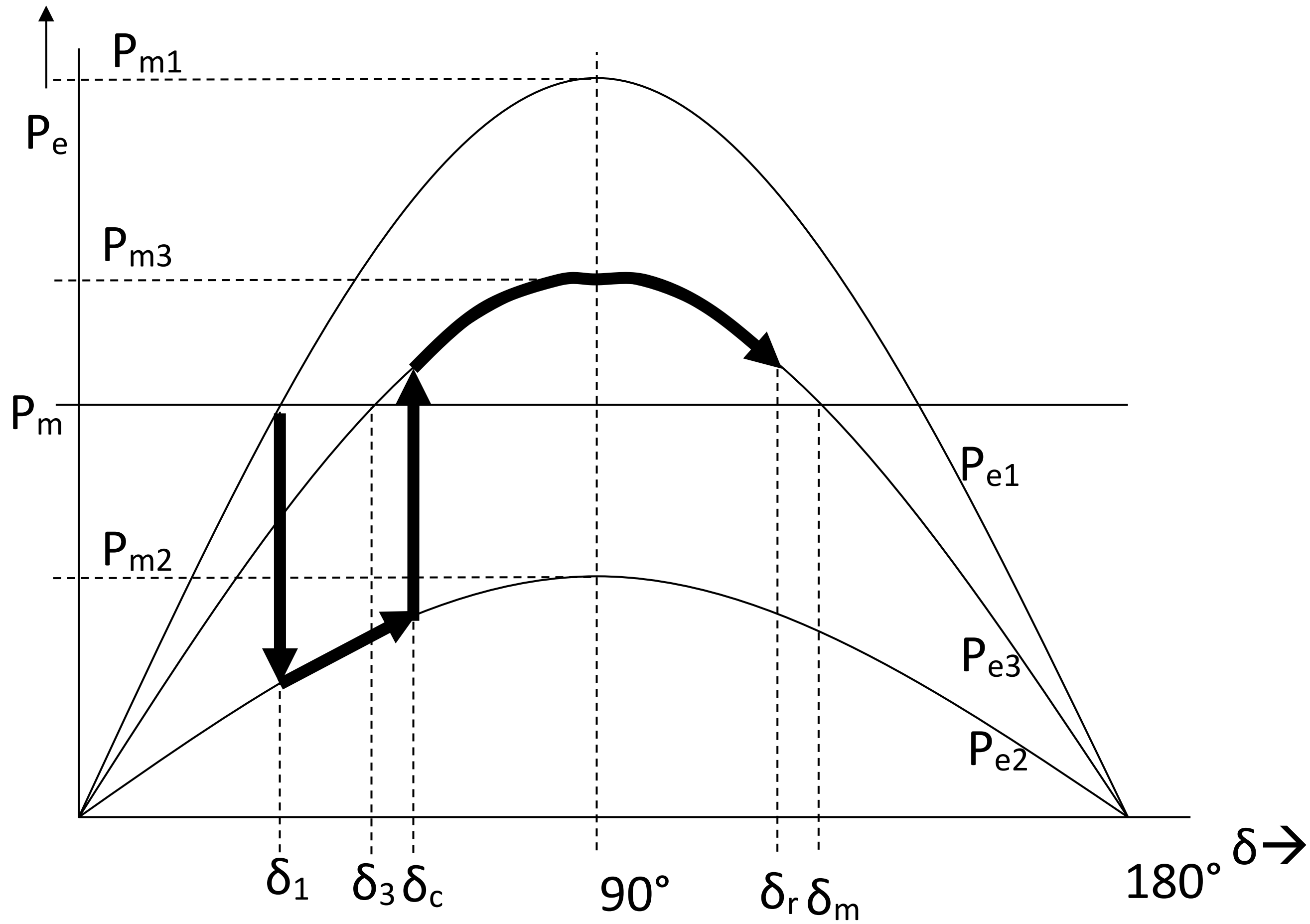

Fig. 1

Since speed is zero at t=0, it remains zero at $t=0^+$. Also, since δ_r is the maximum angle, the speed is zero at this point as well. Therefore, the angle and speed for the two points of interest to us are (note the dual meaning of δ_1: it is lower variable of integration; it is initial angle):

$\delta=\delta_1$ $\delta=\delta_r$

$\omega_1=0$ $\omega_2=0$

Therefore, (8) becomes:

$$\frac{H}{\omega_{\mathrm{Re}}}\left[\omega_2^2-\omega_1^2\right]=0=\int_{\delta_1}^{\delta_r}\left[P_m-P_e\right]d\delta \tag{9a}$$

We have developed a criterion under the assumption of stable performance, and that criterion is:

$$\int_{\delta_1}^{\delta_r}\left[P_m-P_e\right]d\delta=0 \tag{9b}$$

Recalling that $P_a=P_m-P_e$, we see that (9b) says that for stable performance, the integration of the accelerating power from initial angle to maximum angle must be zero. Recalling again (8b), which indicated the left-hand-side was proportional to the change in the kinetic energy between the two states, we can say that (9b) indicates that the accelerating energy must exactly counterbalance the decelerating energy.

Inspection of Fig. 1 indicates that the integration of (9b) includes a discontinuity at the moment when the fault is cleared, at angle $\delta=\delta_c$. Therefore we need to break up the integration of (9b) as follows:

$$\int_{\delta_1}^{\delta_c} \left[P_m - P_{e2}\right] d\delta + \int_{\delta_c}^{\delta_r} \left[P_m - P_{e3}\right] d\delta = 0 \tag{10}$$

Taking the second term to the right-hand-side:

$$\int_{\delta_1}^{\delta_c} \left[P_m - P_{e2}\right] d\delta = -\int_{\delta_c}^{\delta_r} \left[P_m - P_{e3}\right] d\delta \tag{11}$$

Carrying the negative inside the right integral:

$$\int_{\delta_1}^{\delta_c} \left[P_m - P_{e2}\right] d\delta = \int_{\delta_c}^{\delta_r} \left[P_{e3} - P_m\right] d\delta \tag{12}$$

Observing that these two terms each represent areas on the power-angle curve, we see that we have developed the so-called *equal-area criterion* for stability. This criterion says that stable performance requires that the accelerating area be equal to the decelerating area, i.e.,

$$A_1 = A_2 \tag{13}$$

where

$$A_1 = \int_{\delta_1}^{\delta_c} \left[P_m - P_{e2}\right] d\delta \tag{13a}$$

$$A_2 = \int_{\delta_c}^{\delta_r} \left[P_{e3} - P_m\right] d\delta \tag{13b}$$

Figure 2 illustrates.

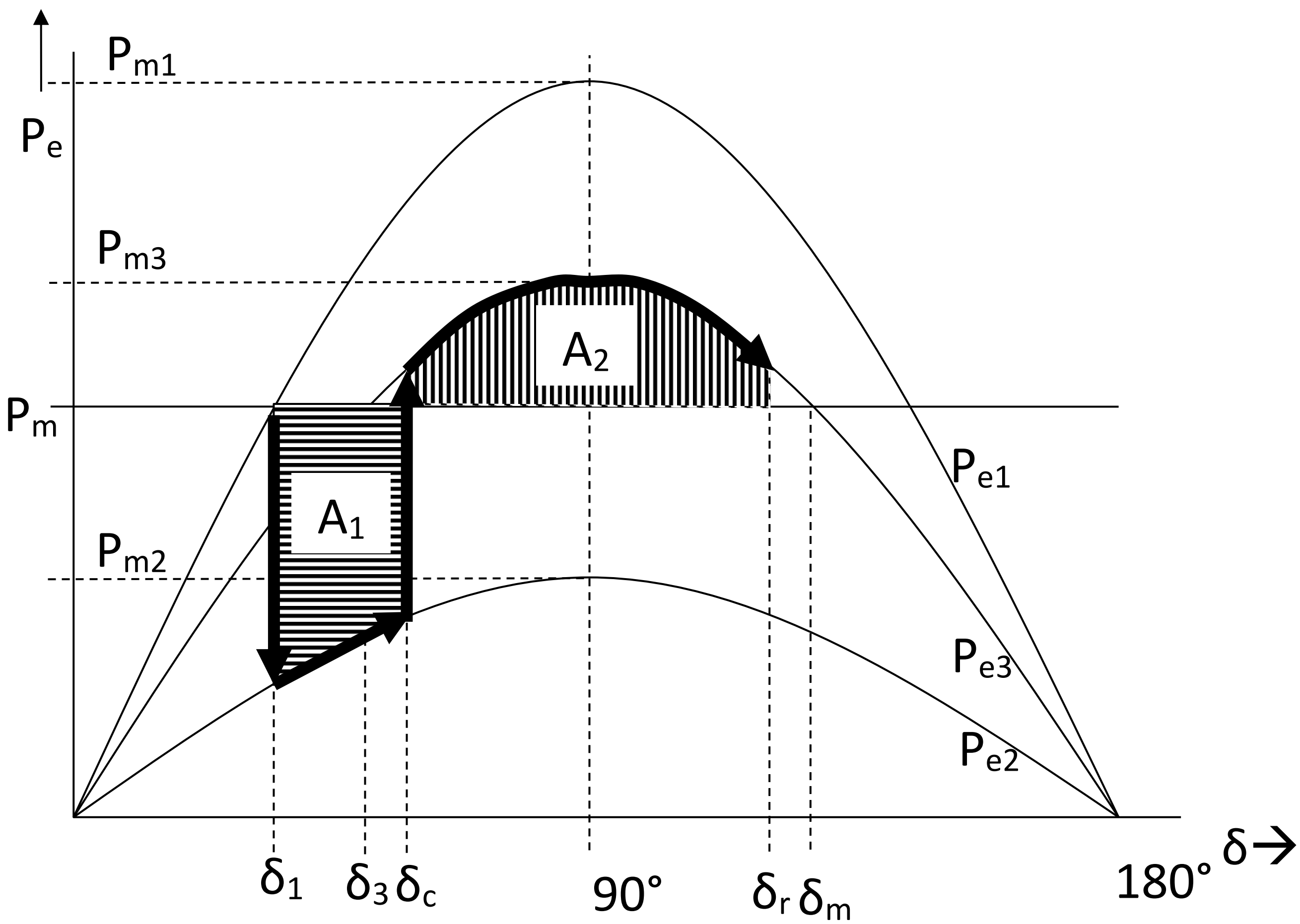

Fig. 2

Figure 2 indicates a way to identify the maximum swing angle, δ_r. Given a particular clearing angle δ_c, which in turn fixes A_1, the machine angle will continue to increase until it reaches an angle δ_r such that A_2=A_1.

2.0 Stability performance

In the notes called "PowerAngleTimeDomain.pdf," on pp. 10-21, we considered stability performance in terms of what causes increased acceleration, or, decreased deceleration. We can consider similarly here, in terms of A_1 and A_2.

Stability performance become more severe, or moves closer to instability, when A_1 increases, or if *available A_2* decreases. We consider A_2 as being bounded on the right by δ_m, because, as we have seen in previous notes, δ cannot exceed δ_m because $\delta>\delta_m$ results in more accelerating energy, not more decelerating energy. Thus we speak of the "available A_2" as being the area within P_{e3}-P_m bounded on the left by δ_c and on the right by δ_m.

Contributing factors to making stability performance worse by increasing A_1, and/or decreasing available A_2, are summarized in the following four bullets and corresponding illustrations.

1. P_m increases ➔ A_1 increases, available A_2 decreases; both effects are extended by increase in δ_c

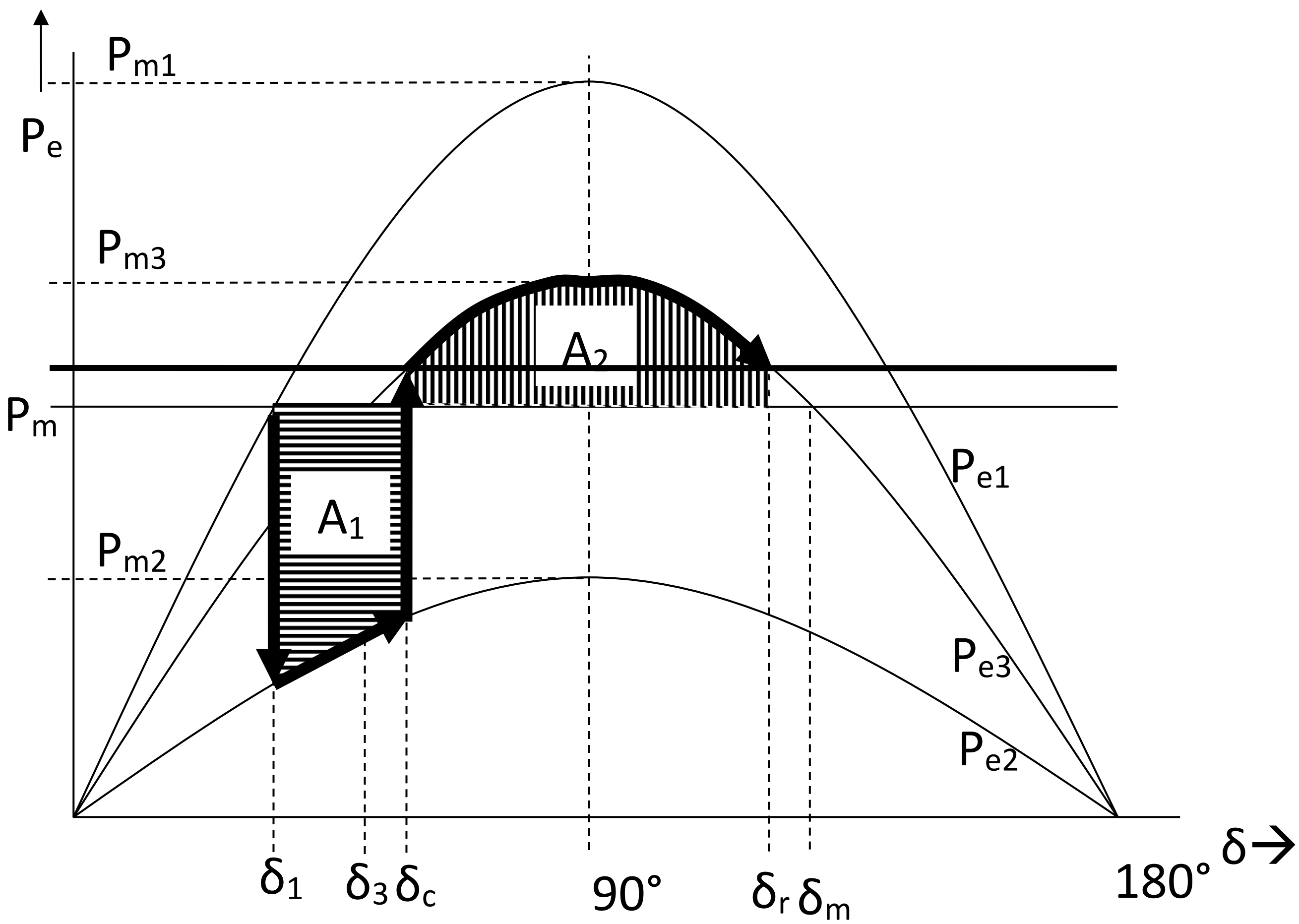

Fig. 3

2. P_{e2} decreases ➔ A_1 increases; A_2 would decrease as well due to increase in δ_c.

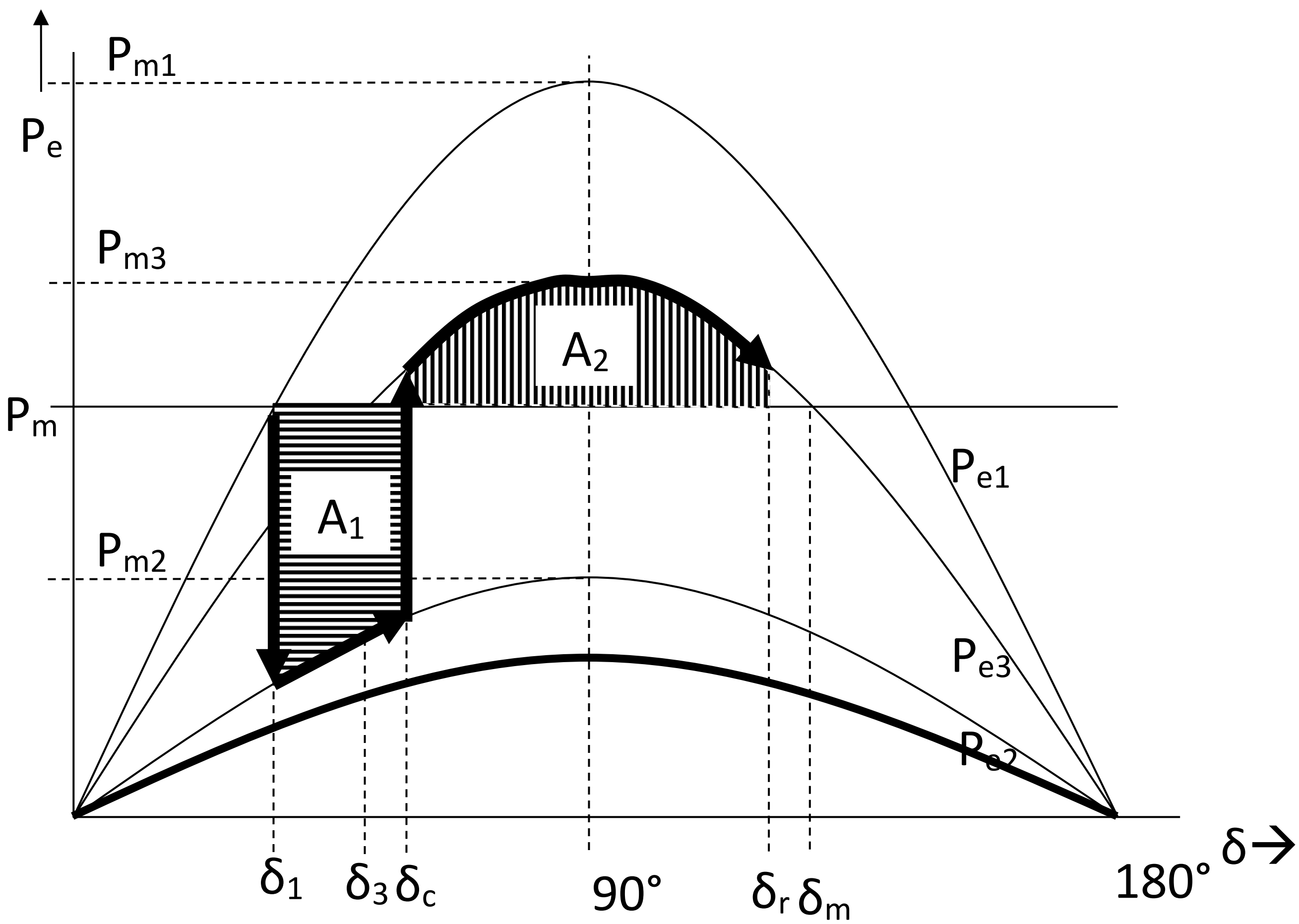

Fig. 4

3. t_c increases ➔ A_1 increases, available A_2 decreases

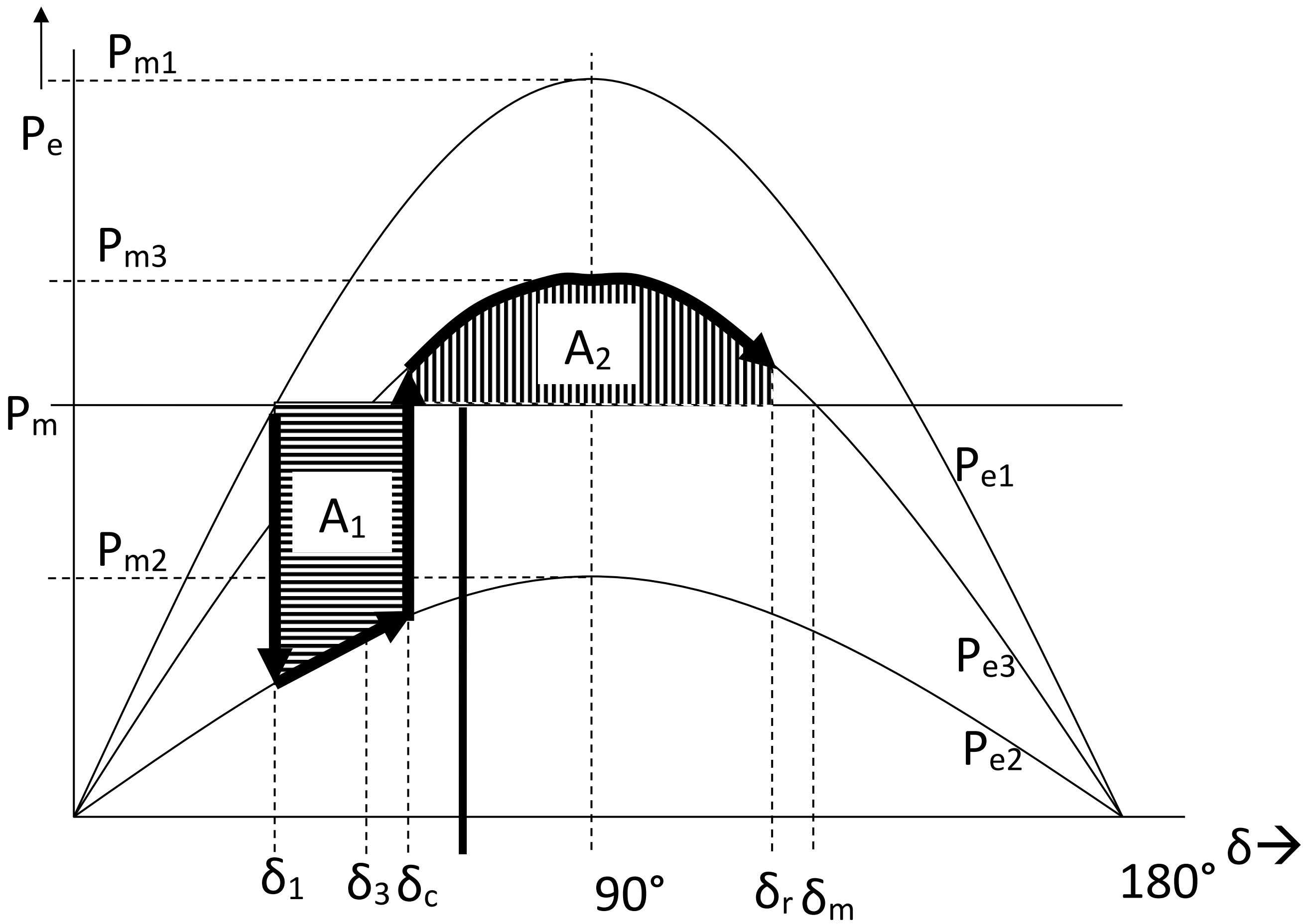

Fig. 5

4. P_{e3} decreases → available A_2 decreases.

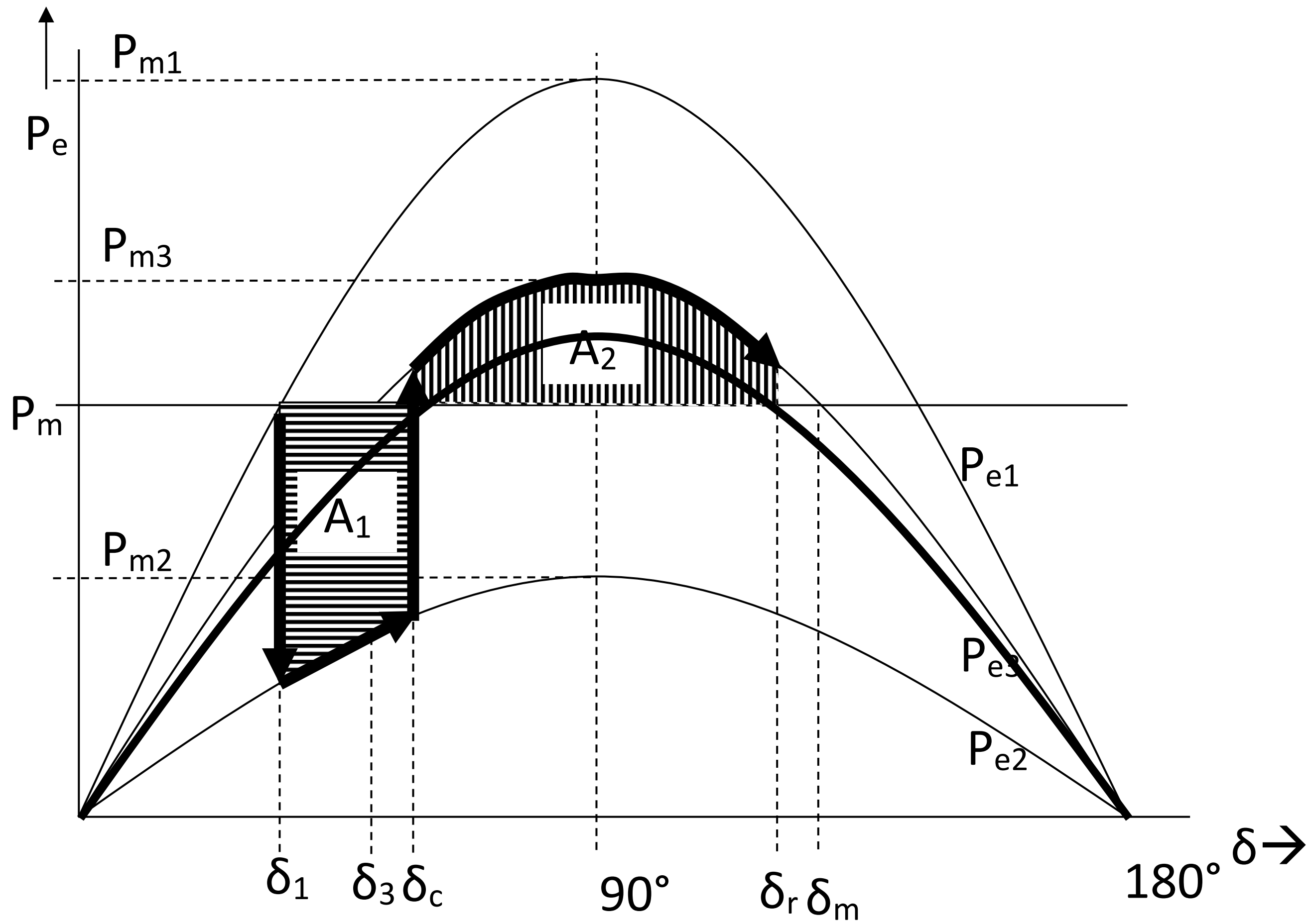

Fig. 6

3.0 Instability and critical clearing angle/time

Instability occurs when *available* $A_2 < A_1$. This situation is illustrated in Fig. 7.

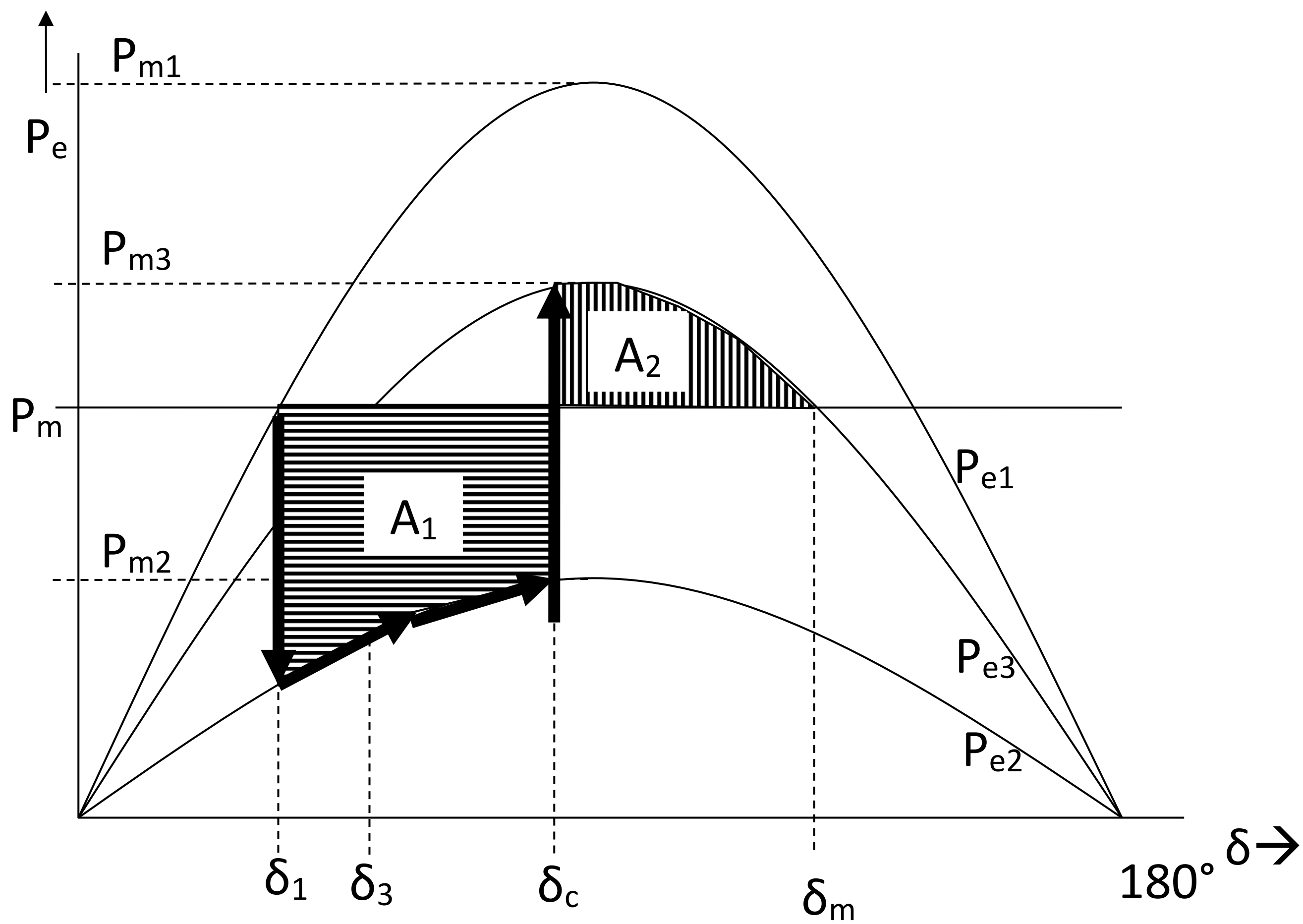

Fig. 7

Consideration of Fig. 7 raises the following question: Can we express the maximum clearing angle for marginal stability, δ_{cr}, as a function of P_m and attributes of the three power angle curves, P_{e1}, P_{e2}, and P_{e3}?

The answer is yes, by applying the equal area criterion and letting $\delta_c=\delta_{cr}$ and $\delta_r= \delta_m$. The situation is illustrated in Fig. 8.

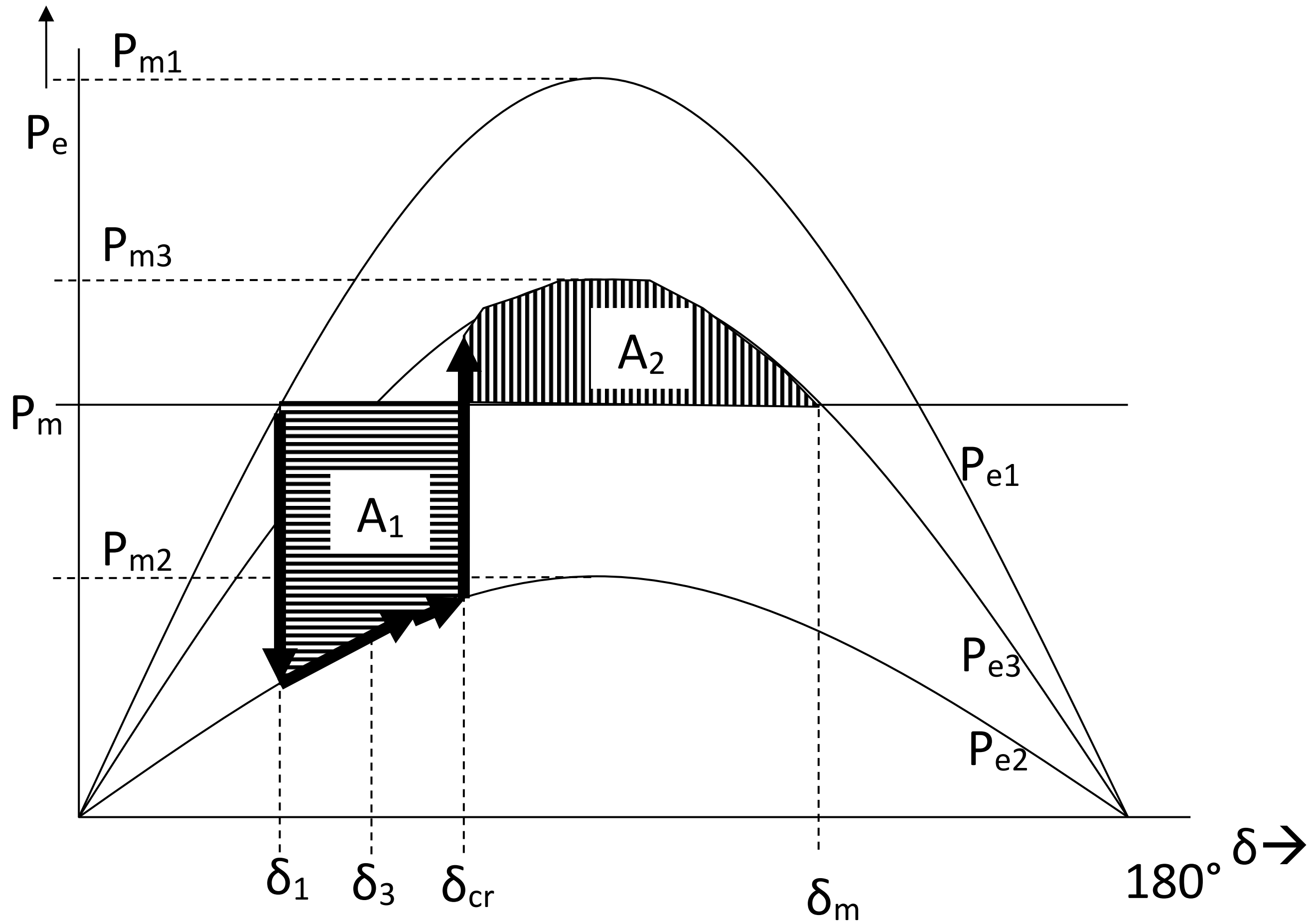

Fig. 8

Applying A_1=A_2, we have that

$$\int_{\delta_1}^{\delta_{cr}} \left[P_m - P_{e2}\right] d\delta = \int_{\delta_{cr}}^{\delta_m} \left[P_{e3} - P_m\right] d\delta \tag{14}$$

The approach to solve this is as follows (this is #7 in your homework #1):

1. Substitute $P_{e2}=P_{M2}\sin\delta$, $P_{e3}=P_{M3}\sin\delta$
2. Do some calculus and then some algebra.
3. Define $r_1=P_{M2}/P_{M1}$, $r_2=P_{M3}/P_{M1}$, which is the same as $r_1=X_1/X_2$, $r_2=X_1/X_3$.
4. Then you obtain:

$$\cos\delta_{cr} = \frac{\frac{P_m}{P_{M1}}\left(\delta_m - \delta_1\right) + r_2\cos\delta_m - r_1\cos\delta_1}{r_2 - r_1} \tag{15}$$

   And this is equation (2.51) in your text.

Your text, section 2.8.2, illustrates application of (15) for the examples 2.4 and 2.5 (we also worked these examples in the notes called "ClassicalModel"). We will do a slightly different example here but using the same system.

Example: Consider the system of examples 2.3-2.5 in your text, but assume that the fault is

- At the machine terminals➔$r_1=P_{M2}/P_{M1}=0$.
- Temporary (no line outage)➔$r_2=P_{M3}/P_{M1}=1$.

The pre-fault swing equation, given by equation (22) of the notes called “ClassicalModel,” is

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t)=0.8-2.223\sin\delta \tag{16}$$

with H=5. Since the fault is temporary, the post-fault equation is also given by (16) above.

Since the fault is at the machine terminals, then the fault-on swing equation has $P_{e2}=0$, resulting in:

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t)=0.8 \tag{17}$$

With $r_1=0$ and $r_2=1$, the equation for critical clearing angle (15) becomes:

$$\cos\delta_{cr}=\frac{P_m}{P_{M1}}\left(\delta_m-\delta_1\right)+\cos\delta_m \tag{18}$$

Since pre and post-fault power-angle curves are identical in this case, we have $\delta_m=\pi-\delta_1$; substituting in (18) results in

$$\cos\delta_{cr} = \frac{P_m}{P_{M1}}(\pi - 2\delta_1) + \cos(\pi - \delta_1) \tag{19}$$

Recall the trig identity that $\cos(\pi-x)=-\cos(x)$. Then (19) becomes:

$$\cos\delta_{cr} = \frac{P_m}{P_{M1}}(\pi - 2\delta_1) - \cos\delta_1 \tag{20}$$

We can solve for δ_1 from the pre-fault swing equation, (with 0 acceleration) according to

$$\begin{aligned} &0 = 0.8 - 2.223\sin\delta_1 \\ &\Rightarrow \delta_1 = 0.3681 rad = 21.0925^\circ \end{aligned} \tag{21}$$

In this case, because pre-fault and post-fault power angle curves are same, δ_m is determined from δ_1 according to

$$\delta_m = 180 - \delta_1 = 180 - 21.0925 = 158.9075^\circ \tag{22}$$

This is illustrated in Fig. 9.

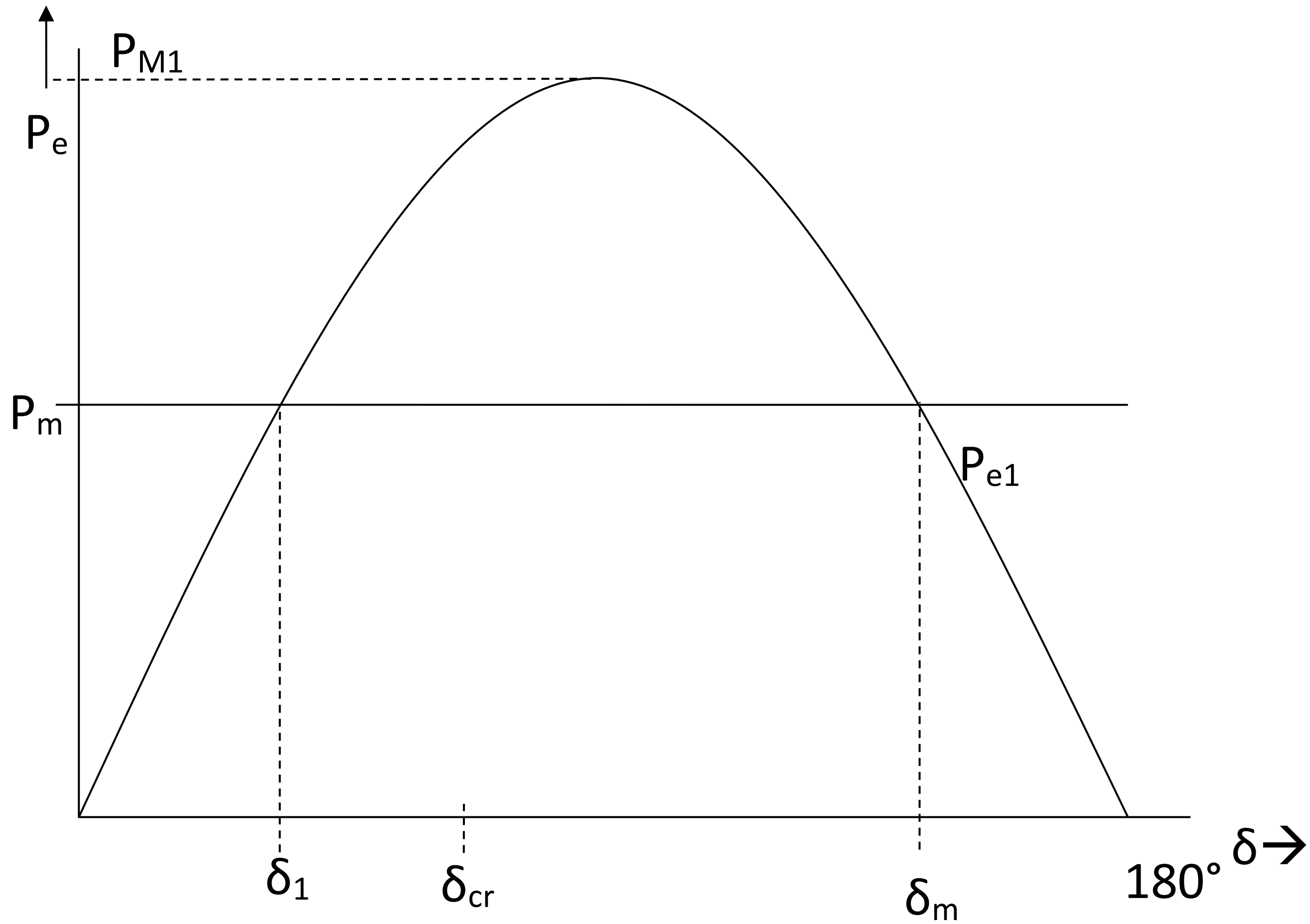

Fig. 9

From (16), we see that P_m=0.8 and P_{M1}=2.223, and (20) can be evaluated as

$$\cos\delta_{cr} = \frac{P_m}{P_{M1}}\left(\pi - 2\delta_1\right) - \cos\delta_1$$
$$= \frac{0.8}{2.223}\left(\pi - 2(0.3681)\right) - \cos(0.3681)$$
$$= 0.8656 - 0.9330 = -0.0674$$

Therefore δ_{cr}=1.6382rad=93.86°.

It is interesting to note that in this particular case, we can also express the clearing time corresponding to any clearing angle δ_c by performing two integrations of the swing equation. We start with the basic swing equation:

$$\frac{2H}{\omega_{\mathrm{Re}}}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{23}$$

For a fault at the machine terminals, P_e=0, so

$$\frac{2H}{\omega_{\mathrm{Re}}}\frac{d^2\delta}{dt^2} = P_m \Rightarrow \frac{d^2\delta}{dt^2} = \frac{\omega_{\mathrm{Re}}}{2H}P_m \tag{24a}$$

Thus we see that for the condition of fault at the machine terminals, the acceleration is a constant. This makes it easy to obtain t in closed form, as follows.

To solve (24a), we recall that ω=dδ/dt, so that (24a) may be rewritten as

$$\frac{d\omega}{dt} = \frac{\omega_{Re}}{2H}P_m \tag{24b}$$

$$\Rightarrow d\omega = \frac{\omega_{Re}}{2H}P_m dt \tag{24c}$$

Then we may integrate (24c) from t=0 to t=t (on the right) and, correspondingly, ω=0 to ω (on the left), resulting in

$$\int_0^{\omega} d\omega = \int_o^t \frac{\omega_{Re}}{2H} P_m dt \tag{24d}$$

Integration of (24d) results in

$$\omega = \frac{\omega_{Re}}{2H} P_m t \tag{24e}$$

Again, recalling that ω=dδ/dt, we can express (24e) as

$$\frac{d\delta}{dt} = \frac{\omega_{Re}}{2H} P_m t \tag{24f}$$

which can be written as

$$d\delta = \frac{\omega_{Re}}{2H} P_m t dt \tag{25}$$

Then we may again integrate from t=0 to t=t (on the right) and, correspondingly, $\delta=\delta_1$ to δ (on the left), resulting in

$$\int_{\delta_1}^{\delta} d\delta = \int_o^t \frac{\omega_{Re}}{2H} P_m t dt \tag{26}$$

Now integrate the right-hand side of (26) from t=0 to t=t and the left-hand-side from corresponding angles δ_1 to δ, resulting in

$$\delta(t) - \delta_1 = \frac{\omega_{\mathrm{Re}}}{2H} P_m \frac{t^2}{2} \tag{27}$$

Solving for t yields:

$$t = \sqrt{\frac{4H}{\omega_{\mathrm{Re}} P_m}\left(\delta(t) - \delta_1\right)} \tag{28}$$

So we obtain the time t corresponding to any clearing angle δ_c, when fault is temporary (no loss of a component) and fault is at machine terminals, using (28), by setting $\delta(t)=\delta_c$.

Returning to our example, where we had P_m=0.8, H=5sec, δ_1=0.3681rad=21.09°, and δ_{cr}=1.6382rad=93.86°, we can compute critical clearing time t_{cr} according to

$$t_{cr} = \sqrt{\frac{4(5)}{(377)(0.8)}\left(1.6382 - 0.3681\right)} = 0.2902$$

The units should be seconds, and we can check this from (28) according to the following:

$$\sqrt{\frac{\sec}{(rad/\sec)(pu)}\left(rad\right)} = \sec$$

I have used my Matlab numerical integration tool to test the above calculation. I have run three cases:

t_c=0.28 seconds (16.8 cycles)

t_c=0.2902 seconds (17.412 cycles)

t_c=0.2903 seconds (17.418 cycles)

Results for angles are shown in Fig. 10, and results for speeds are shown in Fig. 11.

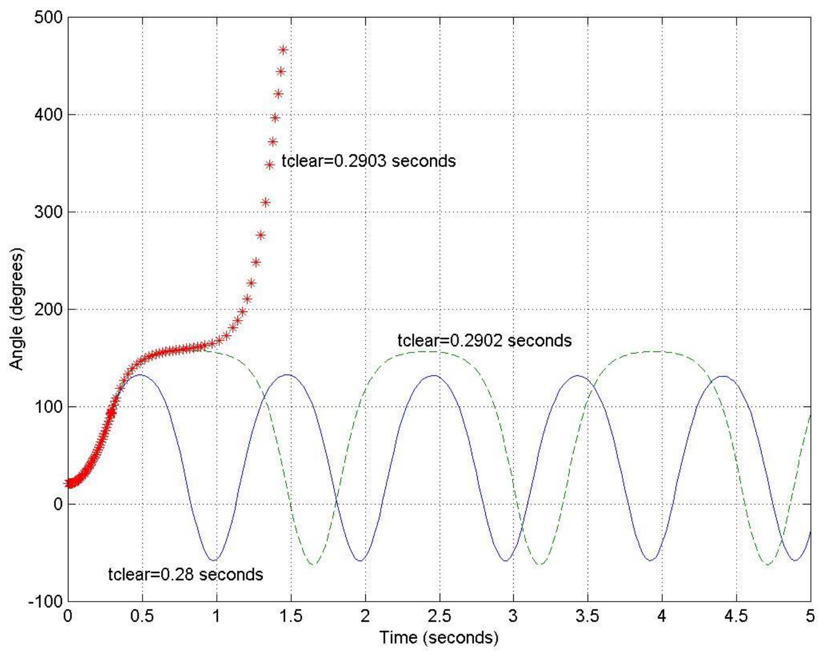

Fig. 11

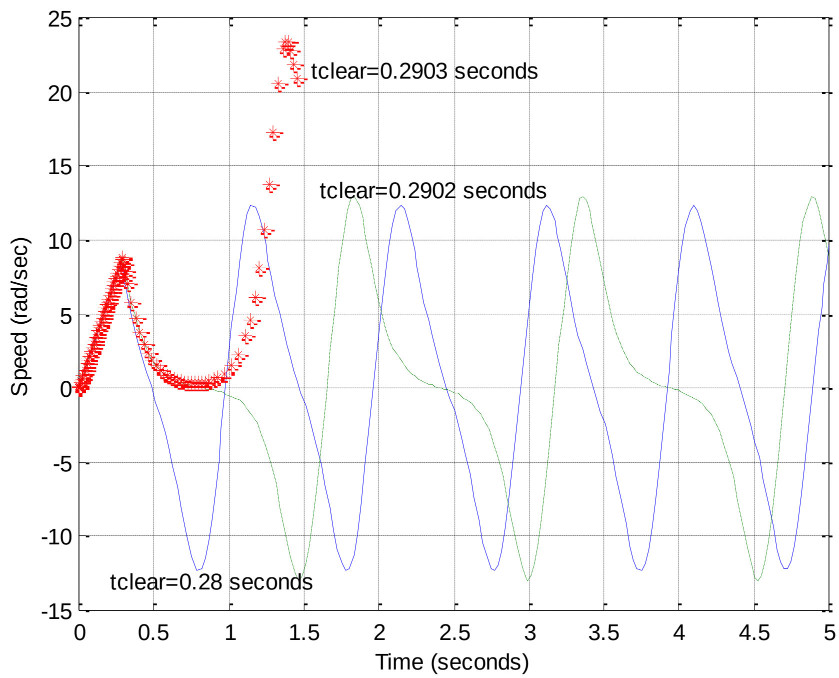

Fig. 12

Some interesting observations can be made for the two plots in Figs. 11 and 12.

<u>In the plots of angle</u>:

- The plot of asterisks has clearing time 0.2903 seconds which exceeds the critical clearing time of 0.2902 seconds by just a little. But it is enough; exceeding it by any amount at all will cause instability, where the rotor angle increases without bound.

- The plot with clearing time 0.28 seconds looks almost sinusoidal, with relatively sharp peaks. In contrast, notice how the plot with clearing time 0.2902 seconds (the critical clearing time) has very rounded peaks. This is typical: as a case is driven more closely to the marginal stability point, the peaks become more rounded.

<u>In the plots of speed</u>:

- The speed increases linearly during the first ~0.28-0.29 seconds of each plot. This is because the accelerating power is constant during this time period, i.e., $P_a=P_m$, since the fault is at the machine terminals (and therefore $P_e=0$).
- In the solid plot (clearing time 0.28 seconds), the speed passes straight through the zero speed axis with a constant deceleration; in this case, the “turn-around point” on the power-angle curve (where speed goes to zero) is a point having angle less than δ_m. But in the dashed plot (clearing time 0.2902 seconds), the speed passes through the zero speed axis with decreasing deceleration; in this case, the “turn-around point” on the power angle curve (where speed goes to zero) is a

point having angle equal to δ_m. This point, where angle equals δ_m, is the unstable equilibrium point. You can perhaps best understand what is happening here if you think about a pendulum. If it is at rest (at its stable equilibrium point), and you give it a push, it will swing upwards. The harder you push it, the closer it gets to its unstable equilibrium point, and the more slowly it decelerates as it "turns around." If you push it just right, then it will swing right up to the unstable equilibrium point, hover there for a bit, and then turn around and come back.

- In the speed plot of asterisks, corresponding to clearing time of 0.2903 seconds, the speed increases, and then decreases to zero, where it hovers for a bit, and then goes back positive, i.e., it does not turn-around at all. This is equivalent to the situation where you have pushed the pendulum just a little harder so that it reaches the unstable equilibrium point, hovers for a bit, and then falls the other way.
- It is interesting that the speed plot of asterisks (corresponding to clearing time of 0.2903 seconds)

increases to about 24 rad/sec at about 1.4 second and then seems to turn around. What is going on here? To get a better look at this, I have plotted this to 5 seconds, as shown in Fig. 13.

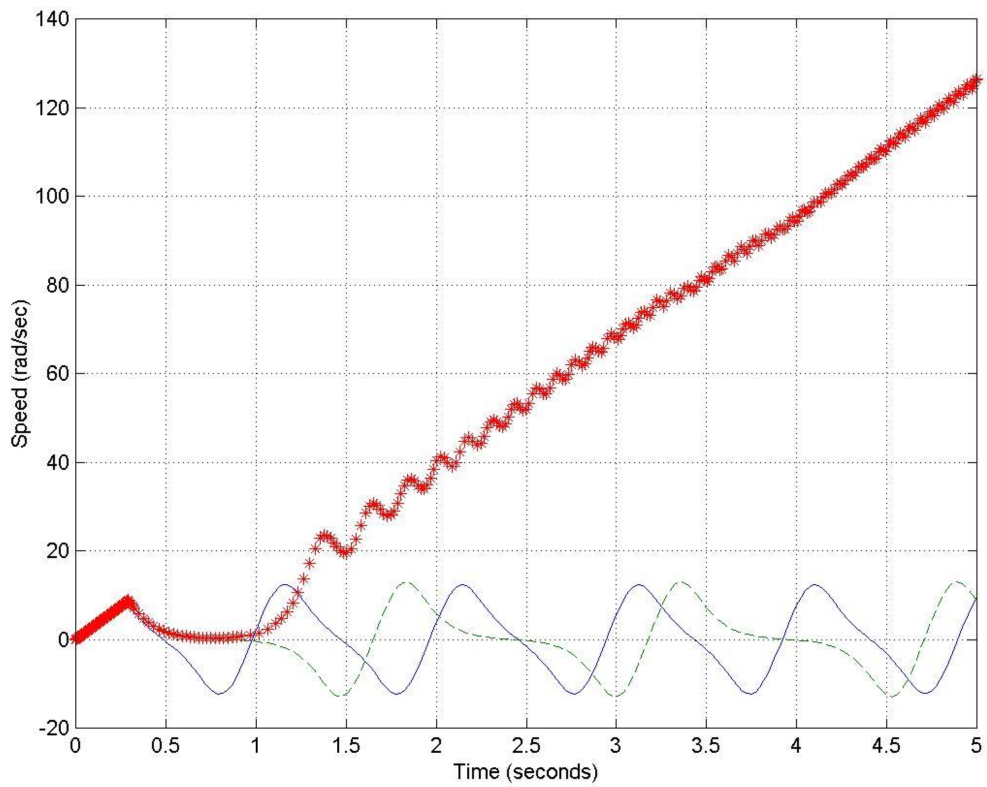

Fig. 13

In Fig. 13, we observe that the oscillatory behavior continues forever, but that oscillatory behavior occurs about a linearly increasing speed. This oscillatory behavior may be understood in terms of the power angle curve, as shown in Fig. 14.

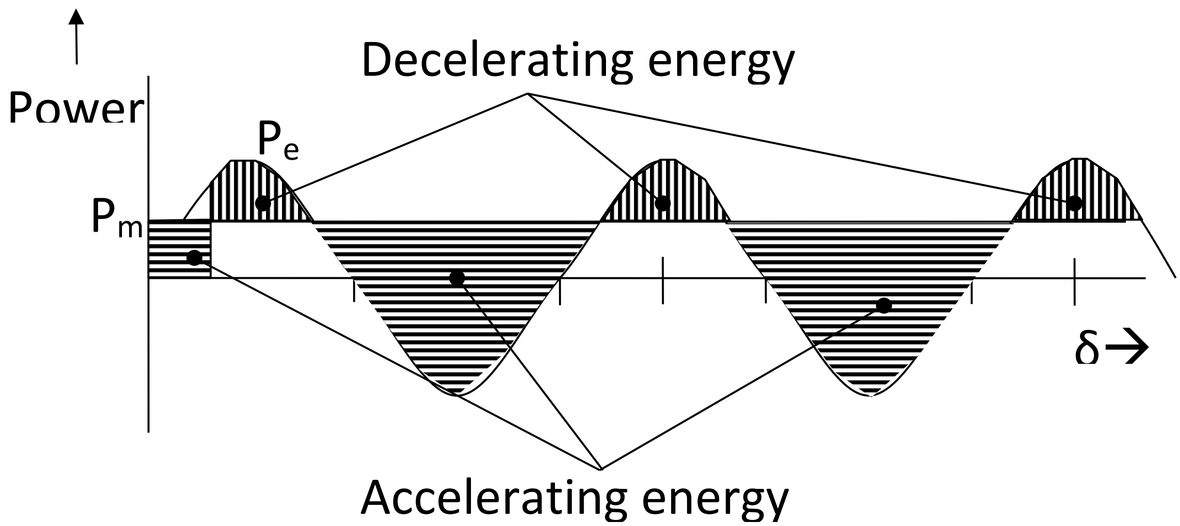

Fig. 14

We see that Fig. 14 indicates that the machine does in fact cycle between a small amount of decelerating energy and a much larger amount of accelerating energy, and this causes the oscillatory behavior. The fact that, each cycle, the accelerating energy is much larger than the decelerating energy is the reason why the speed is increasing with time.

You can think about this in terms of the pendulum: if you give it a push so that it “goes over the top,” if there are no losses, then it will continue to “go round and round.” In this case, however, the average velocity would not increase but would be constant. This is because our analogy of a “one-push” differs from the generator case, where the generator is being “pushed” continuously by the mechanical power into the machine.

You should realize that Fig. 14 fairly reflects what is happening in our plot of Fig. 13, i.e., it appropriately represents our model. However, it differs from what would actually happen in a synchronous machine. In reality, once the angle reaches 180 degrees, the rotor magnetic field would be reconfigured with respect to the stator magnetic field. This is called “slipping a pole,” and without out-of-step relaying (OOR), the unit will experience multiple pole slips in rapid succession thereafter. Most generators have out-of-step protection that is able to determine when this happens and would then trip the machine. We will study OOR at the end of the course.

4.0 A few additional comments

4.1 Critical clearing time

Critical clearing time, or critical clearing angle, was very important many years ago when protective relaying was very slow, and there was great motivation for increasing relaying speed. Part of that motivation came from the desire to lower the critical clearing time. Today, however, we use protection with the fastest clearing times and so there is typically no option to increase relaying times significantly.

Perhaps of most importance, however, is to recognize that critical clearing time has never been a good operational performance indicator because clearing time is not adjustable once a protective system is in place.

4.2 Small systems

What we have done applies to a one-machine-infinite bus system. It also applies to a 2-generator system (see problem 2.14 in the book which is your assigned #8 on HW1). It does not apply to multimachine systems, except in a conceptual sense.

4.2 Multimachine systems

We will see that numerical integration is the main way we have of analyzing multimachine systems. We will take a brief look at this in the next set of notes.