4.32 Incorporate the load equations for the system of one machine against an infinite bus

(a) Neglecting damper effects.

(b) Neglecting $\dot{\lambda}_d$ and $\dot{\lambda}_q$ for a machine with solid round rotor.

(c) Neglecting damaper effects and the terms $\dot{\lambda}_d$ and $\dot{\lambda}_q$.

Solution

(a) <u>Neglecting damper effects</u>: This is model 4, IEEE designation 1.0, called the E'q model. From (4.213) - (4.215)

$$\dot{\Lambda}_d = -\frac{r}{L'_d}\Lambda_d + \frac{r}{L'_d}E'_q - \omega\Lambda_q - V_d \tag{1}$$

$$\dot{\Lambda}_q = \omega\Lambda_d - \frac{r}{L_d}\Lambda_q - V_q \tag{2}$$

$$\dot{E}'_q = -\frac{L_d}{L'_d\tau'_{d0}}E'_q + \frac{L_d - L'_d}{L'_d\tau'_{d0}}\Lambda_d + \frac{1}{\tau'_{d0}}E_{FD} \tag{3}$$

From (4.149)

(Note that the left-hand-sides of the below two equations should be upper case).

$$v_d = -V_\infty \sin(\delta-\alpha) + R_e I_d + L_e\dot{I}_d + \omega L_e I_q \tag{4}$$

$$v_q = V_\infty \cos(\delta-\alpha) + R_e I_q + L_e\dot{I}_q - \omega L_e I_d \tag{5}$$

Also, from (4.199)

$$\begin{aligned} I_d &= \frac{1}{L'_d}\Lambda_d - \frac{L_{AD}}{\sqrt{3}L'_d L_F}\lambda_F \\ &= \frac{1}{L'_d}\Lambda_d - \frac{1}{L'_d}E'_q \end{aligned} \tag{6}$$

$$I_q = \frac{1}{L_q}\Lambda_q \tag{7}$$

$$\begin{aligned} V_d = -V_\infty \sin(\delta-\alpha) + \frac{R_e}{L'_d}\Lambda_d - \frac{R_e}{L'_d}E'_q + \frac{L_e}{L'_d}\dot{\Lambda}_d \\ - \frac{L_e}{L'_d}\dot{E}'_q + \frac{\omega L_e}{L_q}\Lambda_d \end{aligned} \tag{8}$$

$$\begin{aligned} V_q = V_\infty \cos(\delta-\alpha) + \frac{R_e}{L_q}\Lambda_q + \frac{L_e}{L_q}\dot{\Lambda}_q - \frac{\omega L_e}{L'_d}\Lambda_d \\ + \frac{\omega L_e}{L'_d}E'_q \end{aligned} \tag{9}$$

From (1) and (8)

$$\begin{aligned}\dot{\Lambda}_d &= -\frac{r}{L'_d}\Lambda_d + \frac{r}{L'_d}E'_q - \omega\Lambda_q + V_\infty \sin(\delta-\alpha) - \frac{R_e}{L'_d}\Lambda_d \\ &\quad + \frac{R_e}{L'_d}E' - \frac{L_e}{L'_d}\dot{\Lambda}_d + \frac{L_e}{L'_d}\dot{E}'_q - \frac{\omega L_e}{L_q}\Lambda_q\end{aligned}$$

$$\begin{aligned}\left(1+\frac{L_e}{L'_d}\right)\dot{\Lambda}_d - \frac{L_e}{L'_d}\dot{E}' &= -\frac{r+R_e}{L'_d}\Lambda_d + \frac{r+R_e}{L'_d}E'_q \\ &\quad -\omega\left(1+\frac{L_e}{L_q}\right)\Lambda_q + V_\infty\sin(\delta-\alpha)\end{aligned} \tag{A}$$

From (2) and (9), and from (3):

$$\begin{aligned}\dot{\Lambda}_q &= \omega\Lambda_d - \frac{r}{L_q}\Lambda_q - V_\infty\cos(\delta-\alpha) - \frac{R_e}{L_q}\Lambda_q - \frac{L_e}{L_q}\dot{\Lambda}_q \\ &\quad + \frac{\omega L_e}{L'_d}\Lambda_d - \frac{\omega L_e}{L'_d}E'_q\end{aligned}$$

$$\begin{aligned}\left(1+\frac{L_e}{L_q}\right)\dot{\Lambda}_q &= \omega\left(1+\frac{L_e}{L'_d}\right)\Lambda_d - \frac{\omega L_e}{L'_d}E'_q - \frac{r+R_e}{L_q}\Lambda_q \\ &\quad - V_\infty\cos(\delta-\alpha)\end{aligned} \tag{B}$$

$$\dot{E}'_q = -\frac{L_d}{L'_d\tau'_{d0}}E'_q + \frac{L_d-L'_d}{L'_d\tau'_{d0}}\Lambda_d + \frac{1}{\tau'_{d0}}E_{FD} \tag{C}$$

From (4.218) - (4.220)

$$\tau_{\mathrm{j}}\dot{\omega} = T_m - \frac{1}{L'_d}E'_q\Lambda_q + \left(\frac{1}{L'_d}-\frac{1}{L_q}\right)\Lambda_d\Lambda_q - D\omega \tag{D}$$

$$\dot{\delta} = \omega - 1 \tag{E}$$

$$\begin{bmatrix} (1+L_e/L'_d) & -L_e/L'_d & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & (1+L_e/L_q) & 0 & 0 \\ 0 & 0 & 0 & \tau_{\mathrm{j}} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix}\dot{\Lambda}_d \\ \dot{E}'_q \\ \dot{\Lambda}_q \\ \dot{\omega} \\ \dot{\delta}\end{bmatrix} =$$

$$\begin{bmatrix} -(r+R_e)/L'_d & (r+R_e)/L'_d & -\omega(1+L_e/L_q) & 0 & 0 \\ (L_d-L'_d)/L'_d\tau'_{d0} & -L_d/L'_d\tau'_{d0} & 0 & 0 & 0 \\ \omega(1+L_e/L'_d) & -\omega L_e/L'_d & -(r+R_e)/L_q & 0 & 0 \\ (1/L'_d - 1/L_q)\Lambda_q & (1/L'_d)\Lambda_q & 0 & -D & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}\Lambda_d \\ E'_q \\ \Lambda_q \\ \omega \\ \delta\end{bmatrix} + \begin{bmatrix} V_\infty\sin(\delta-\alpha) \\ (1/\tau'_{d0})E_{FD} \\ -V_\infty\cos(\delta-\alpha) \\ T_m \\ -1 \end{bmatrix}$$

(b) Neglecting $\dot{\lambda}_d$ and $\dot{\lambda}_q$ for a machine with solid round rotor. This is model 6, IEEE designation 1.1, called the two-axis model for a cylindrical rotor machine. This model is described on pp 138-140 of the text. Note that the text's description of this model uses notation that indicates the Q-circuit is present. However, for this model, it is the G-circuit that is actually present (to account for transient effects). The Q-circuit, as we have defined it (to account for subtransient effects), is not present.

From (4.287) - (4.289)

$$\tau'_{q0}\dot{E}'_d = -E'_d - \left(x_q - x'_q\right)I_q \qquad (1)$$

$$\tau'_{d0}\dot{E}'_q = E_{FD} - E = E_{FD} - E'_q + \left(x_d - x'_d\right)I_d \qquad (2)$$

From (4.149) and neglecting $\dot{I}_d$ and $\dot{I}_q$

$$V_d = -V_\infty \sin\left(\delta - \alpha\right) + R_e I_d + \omega L_e I_q \qquad (3)$$

$$V_q = V_\infty \cos\left(\delta - \alpha\right) + R_e I_q - \omega L_e I_d \qquad (4)$$

From (4.277) - (4.278)

$$E'_d = V_d + rI_d + x'_d I_q \qquad (5)$$

$$E'_q = V_q + rI_q - x'_d I_d \qquad (6)$$

$$E'_d = -V_\infty \sin\left(\delta - \alpha\right) + R_e I_d + \omega L_e I_q + rI_d + x'_d I_q$$

$$E'_q = V_\infty \cos\left(\delta - \alpha\right) + R_e I_q - \omega L_e I_d + rI_q - x'_d I_d$$

$$\hat{R}I_d + \hat{X}I_q = E'_d + V_\infty \sin\left(\delta - \alpha\right)$$

$$-\hat{X}I_d + \hat{R}I_q = E'_q - V_\infty \cos\left(\delta - \alpha\right)$$

$$\left(\hat{R}^2 + \hat{X}^2\right)I_d = \hat{R}E'_d - \hat{X}E'_q + V_\infty\left[\hat{R}\sin\left(\delta - \alpha\right) + \hat{X}\cos\left(\delta - \alpha\right)\right] \qquad (7)$$

$$\left(\hat{R}^2 + \hat{X}^2\right)I_q = \hat{X}E'_d + \hat{R}E'_q + V_\infty\left[\hat{X}\sin\left(\delta - \alpha\right) - \hat{R}\cos\left(\delta - \alpha\right)\right] \qquad (8)$$

$$I_d = \frac{\hat{R}}{\hat{R}^2 + \hat{X}^2}E'_d - \frac{\hat{X}}{\hat{R}^2 + \hat{X}^2}E'_q + \frac{V_\infty}{\hat{R}^2 + \hat{X}^2}\left[\hat{R}\sin\left(\delta - \alpha\right) + \hat{X}\cos\left(\delta - \alpha\right)\right] \qquad (9)$$

$$I_q = \frac{\hat{X}}{\hat{R}^2 + \hat{X}^2}E'_d + \frac{\hat{R}}{\hat{R}^2 + \hat{X}^2}E'_q + \frac{V_\infty}{\hat{R}^2 + \hat{X}^2}\left[\hat{X}\sin\left(\delta - \alpha\right) + \hat{R}\cos\left(\delta - \alpha\right)\right] \qquad (10)$$

$$\tau'_{q0}\dot{E}'_d = -\left[1 + \frac{\hat{X}\left(x_q - x'_q\right)}{\hat{R}^2 + \hat{X}^2}\right]E'_d - \frac{\hat{R}\left(x_q - x'_q\right)}{\hat{R}^2 + \hat{X}^2}E'_q$$

$$-\frac{\left(x_q - x'_q\right)V_\infty}{\hat{R}^2 + \hat{X}^2}\left[\hat{X}\sin\left(\delta - \alpha\right) - \hat{R}\cos\left(\delta - \alpha\right)\right] \qquad (11)$$

$$\tau'_{d0}\dot{E}'_q = E_{FD} + \frac{\hat{R}\left(x_q - x'_q\right)}{\hat{R}^2 + \hat{X}^2}E'_q - \left[1 + \frac{\hat{X}\left(x_d - x'_d\right)}{\hat{R}^2 + \hat{X}^2}\right]E'_q$$

$$+\frac{\left(x_d - x'_d\right)V_\infty}{\hat{R}^2 + \hat{X}^2}\left[\hat{R}\sin\left(\delta - \alpha\right) + \hat{X}\cos\left(\delta - \alpha\right)\right] \qquad (12)$$

$$\dot{\delta} = \omega - 1$$

(c) Neglecting damaper effects and the terms $\dot{\lambda}_d$ and $\dot{\lambda}_q$: This is model 7, with IEEE designation 1.0, called the one-axis model.

$$\tau'_{d0}\dot{E}'_q = E_{FD} - E = E_{FD} - E'_q + (x_d - x'_d)I_d \tag{1}$$

From the machine equations

$$V_d = -rI_d - \dot{\lambda}_d - \omega L_q I_q = -rI_d - x_q I_q \quad \text{neglecting } \dot{\lambda}_d \tag{2}$$

$$V_q = -rI_q + x'_d I_d + E'_q \tag{3}$$

Then

$$V_d = -V_\infty \sin(\delta - \alpha) + R_e I_d + X_e I_q \tag{4}$$

$$V_q = +V_\infty \cos(\delta - \alpha) + R_e I_q - X_e I_d \tag{5}$$

From (2) and (4)

$$-rI_d - x_q I_q = -V_\infty \sin(\delta - \alpha) + R_e I_d + X_e I_q$$

$$(r + R_e)I_d + (x_q + X_e)I_q = V_\infty \sin(\delta - \alpha) \tag{6}$$

and, from (3) and (5)

$$E'_q + x'_d I_d - rI_q = V_\infty \cos(\delta - \alpha) + R_e I_q - X_e I_d$$

$$(r + R_e)I_q - (x'_d + X_e)I_d = E'_q - V_\infty \cos(\delta - \alpha) \tag{7}$$

From (6) and (7)

$$I_d = \frac{1}{\hat{R}^2 + (x_q + X_e)(x'_d + X_e)}\left\{\begin{array}{l} -(x_q + X_e)E'_q \\ +V_\infty\left[\hat{R}\sin(\delta - \alpha) + (x_q + X_e)\cos(\delta - \alpha)\right] \end{array}\right\} \tag{8}$$

$$I_q = \frac{1}{\hat{R}^2 + (x_q + X_e)(x'_d + X_e)}\left\{\begin{array}{l} \hat{R}E'_q \\ +V_\infty\left[(x'_d + X_e)\sin(\delta - \alpha) - \hat{R}\cos(\delta - \alpha)\right] \end{array}\right\} \tag{9}$$

$$\tau'_{d0}\dot{E}'_q = E_{FD} - \left[1 + \frac{(x_d - x'_d)(x_q + X_e)}{\hat{R}^2 + (x_q + X_e)(x'_d + X_e)}\right]E'_q \tag{A}$$

$$+ \frac{(x_d - x'_d)}{\hat{R}^2 + (x_q + X_e)(x'_d + X_e)}\left\{V_\infty\left[\hat{R}\sin(\delta - \alpha) + (x_q + X_e)\cos(\delta - \alpha)\right]\right\}$$

$$\tau_j\dot{\omega} = T_m - D\omega - \left[E'_q I_q - (L_d - L'_d)I_d I_q\right] \tag{B}$$

$$\dot{\delta} = \omega - 1$$

5.1 The synchronoous machine discussed in Examples 5.1 and 5.2 is operating at rated terminal voltage, and its output power is 0.80 pu. The angle between the q axis and the terminal voltage is 45°. Find the steady-state operating conditions: the d and q axis voltages, currents, flux linkages, and the angle ϕ.

Solution

$V_a = 1.0 \ pu$

$P = 0.8 \ pu$ on a 3 phase base

$\delta - \beta = 45^\circ$

$$V_q = V_a \cos(\delta - \beta) = 0.707$$
$$V_d = -V_a \sin(\delta - \beta) = -0.707$$
$$I_r = I \cos\phi = \frac{P}{V_t} = 0.8 \text{ pu}$$
$$\tan(\delta - \beta) = \frac{x_q I_r + r I_x}{V_a + r I_r - x_q I_x} = \tan \ 45^\circ = 1$$
$$x_q I_r + r I_x = V_a + r I_r - x_q I_x$$
$$(x_q - r) I_r + (x_q + r) I_x = 1.0$$

Then

$$(1.64 - 0.001096)(0.8) + (1.64 + 0.001096) I_x = 1.0 \quad \text{or}$$
$$1.311123 + 1.641096 \ I_x = 1.0$$
$$I_x = -0.189583$$

Now $I_a = \left(I_r^2 + I_x^2\right)^{1/2}$ or $I_a = 0.822157$

Then

$$F_p = \frac{0.8}{0.822157} = 0.973051 = \cos\phi$$
$$\phi = 13.331915^\circ$$
$$\delta - \beta + \phi = 58.331915^\circ$$
$$I_q = I_a \cos(\delta - \beta + \phi) = 0.431630$$
$$i_q = 0.747606$$
$$I_d = -I_a \sin(\delta - \beta + \phi) = -0.699741$$
$$i_d = -1.211986$$

$$E = V_q + r I_q - x_d I_d$$
$$= 0.707107 + (0.001096)(0.431630) - (1.70)(-0.699741)$$
$$= 1.897139 = E_{FD}$$
$$i_F = \frac{\sqrt{3} E}{L_{AD}} = \frac{\sqrt{3}(1.897139)}{1.55} = 2.11996$$
$$\lambda_d = L_d i_d + L_{AD} i_F = 1.7(-1.211986) + 1.55(2.119962) = 1.225564$$
$$\lambda_q = L_q i_q = 1.64(0.747606) = 1.226073$$
$$\lambda_F = L_{AD} i_d + L_F i_F = 1.55(-1.211986) + (1.651)(2.119962) = 1.621478$$
$$\lambda_D = \lambda_{AD} = L_{AD}(i_d + i_F) = 1.407362$$
$$\lambda_Q = \lambda_{AQ} = L_{AQ} i_q = 1.113932$$

$$T_{e\phi} = i_q\lambda_d - i_d\lambda_q = 2.402222$$
$$T_e = 0.800741$$

$$\boldsymbol{V}_\infty = \boldsymbol{V}_a - \boldsymbol{Z}_e\boldsymbol{I}_a$$
$$\begin{aligned}V_\infty\angle 0^\circ &= V_a e^{j\beta} - \boldsymbol{Z}_e I_a e^{j(\beta-\phi)}\\ &= 1.0 - (0.400500\ \angle 87.1376)(0.822157\ \angle -13.3319)\\ &= 1.0 - 0.329273\ \angle 73.8056\\ &= 1.0 - (0.091833 + j0.316208) = 0.908167 - j0.316208\\ &= 0.961642\ \angle -19.197251^\circ\end{aligned}$$
$$\beta = -19.197251^\circ$$
$$\delta - \beta = 45$$
$$\delta = 25.802749$$

5.2 The same synchronous machine connected to the same transmission line, as in Examples 5.1 and 5.2, has a local load of unity power factor, which is represented by a resistance $R = 10$ pu. The infinite bus voltage is 1.0 pu. The power at the infinite bus is 0.9 pu at 0.9 PF lagging. Find the operating condition of tha machine.

Solution

Local Load: $R_L = 10$ pu

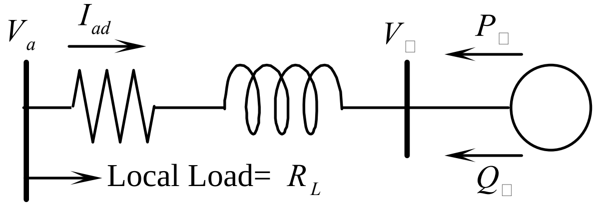
$R_L = 10$ pu

$V_\infty = 1.0$ pu

$P_\infty = -0.9$ pu @ 0.9 PF lagging

$F_p = 0.9 = \cos\phi_\infty$

$\phi_\infty = 25.841$



$F_Q = \sin\phi_\infty = 0.435890$

$S_\infty = -1.0$

$Q_\infty = -0.435890$

Let V_∞ be the phasor reference. $\boldsymbol{V}_\infty = 1.0\ \angle 0^\circ$

Then $\boldsymbol{V}_\infty\boldsymbol{I}_\infty^* = P_\infty + jQ_\infty = -0.9 - j0.435890$

$$\boldsymbol{I}_\infty = -0.9 + j0.435890 = -\boldsymbol{I}_{at}$$
$$\boldsymbol{I}_{at} = 0.9 - j0.435890 = 1.00\angle -25.8419^\circ$$
$$\begin{aligned}\boldsymbol{V}_t &= \boldsymbol{I}_{at}(R_e + jX_e) + \boldsymbol{V}_\infty\\ &= (1.0\angle -25.8419)(0.4005\ \angle 87.1376) + 1.0\angle 0\\ &= 1.192356 + j0.351282 = 1.243025\ \angle 16.415598 = V_t\angle\beta\end{aligned}$$

Now at the load

$$\boldsymbol{I}_{aL} = \frac{\boldsymbol{V}_t}{R_L} = \frac{\boldsymbol{V}_t}{10} = 0.1243025\ \angle 16.415598$$

The total current supplied by bus a is

$$\boldsymbol{I}_a = \boldsymbol{I}_{at} + \boldsymbol{I}_{aL} = 0.9 - j0.435890 + 0.119236 + j0.035128$$
$$= 1.019236 - j0.400762 = 1.09195 \angle -21.464666\,°$$
$$P + jQ = \mathbf{V}_t \boldsymbol{I}_a^* = 1.361355 \angle 37.880265 = 1.07451 + j0.835890$$

which is noted to be a machine overload.

Power and reactive factors:

$$F_P = \cos 37.880265 = 0.789296$$
$$F_Q = \sin 37.880265 = 0.614013$$

$$I_r = I_a \cos\phi = 0.864432$$
$$I_x = -I_a \sin\phi = -0.672464$$
$$\delta - \beta = \tan^{-1}\frac{rI_x + x_q I_r}{V_a + rI_r - x_q I_x} = \tan^{-1}\frac{1.416932}{2.346814} = 31.122250\,°$$
$$\delta = 31.122 + 16.415 = 47.537$$
$$\delta - \beta + \phi = 69.0025$$

$$I_d = -I_a \sin(\delta - \beta + \phi) = -1.022470 \qquad i_d = -1.770970$$
$$I_q = +I_a \cos(\delta - \beta + \phi) = +0.392438 \qquad i_d = +0.679722$$
$$V_d = -V_a \sin(\delta - \beta) = -0.642477 \qquad v_d = -1.112803$$
$$V_q = +V_a \sin(\delta - \beta) = +1.064112 \qquad v_q = +1.843096$$
$$E_{FD} = E = V_q + rI_q - x_d I_d = 2.802741$$
$$i_F = \frac{\sqrt{3}E_{FD}}{L_{AD}} = 3.131929$$

d-axis flux linkages:

$$\lambda_d = L_d i_d + L_{AD} i_F = 1.843841$$
$$\lambda_F = L_{AD} i_d + L_F i_F = 2.425812$$
$$\lambda_D = \lambda_{AD} = (i_d + i_F) L_{AD} = 2.109487$$

q-axis flux linkages:

$$\lambda_q = L_q i_q = 1.114745$$
$$\lambda_Q = \lambda_{AQ} = i_q L_{AQ} = 1.012786$$

Torques:

$$T_{e\phi} = i_q \lambda_d - i_d \lambda_q = 1.253300 + 1.974179 = 3.227479$$
$$T_e = T_{e\phi}/3 = 1.075826$$

9.1 If the $\overline{\mathbf{Y}}$ matrix of the network, reduced to the generator nodes, is such that $\theta_{ij} = 90°, i \neq j$, derive the general form of the matrix $\overline{\mathbf{M}}$.

Solution

For $\theta_{ij} = 90°$, derive the general expression for $\overline{\mathbf{M}}$

$$\overline{\mathbf{M}} = \begin{bmatrix} Y_{11}e^{j\theta_{11}} & Y_{12}\,e^{j(\theta_{12}-\delta_{12})} & \cdots & Y_{1n}e^{j(\theta_{1n}-\delta_{1n})} \\ Y_{21}e^{j(\theta_{21}-\delta_{21})} & Y_{22}\,e^{j\theta_{22}} & \cdots & Y_{2\,n}e^{j(\theta_{2\,n}-\delta_{2n})} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{n1}e^{j(\theta_{n1}-\delta_{n1})} & Y_{n\,2}e^{j(\theta_{n\,2}-\delta_{n2})} & \cdots & Y_{nn}\,e^{j\theta_{nn}} \end{bmatrix}$$

For $\theta_{ij} = 90^\circ$, $e^{j(\theta_{ij}-\delta_{ij})} = \cos(\theta_{ij} - \delta_{ij}) + j\sin(\theta_{ij} - \delta_{ij})$

$$e^{j(\theta_{ij}-\delta_{ij})} = (\cos\theta_{ij}\cos\delta_{ij} + \sin\theta_{ij}\sin\delta_{ij}) + j(\sin\theta_{ij}\cos\delta_{ij} - \cos\delta_{ij}\sin\theta_{ij})$$

$$= \sin\delta_{ij} + j\cos\delta_{ij}$$

The diagonal terms for diagonal element *i* are:

$$Y_{ii}e^{j\theta_{ij}} = Y_{ii}(\cos\theta_{ii} + j\sin\theta_{ii}) = G_{ii} + jB_{ii}$$

Then

$$\overline{\mathbf{M}} = \begin{bmatrix} G_{11} & B_{12}\sin\delta_{12} & B_{13}\sin\delta_{13} & \cdots & B_{1n}\sin\delta_{1n} \\ B_{21}\sin\delta_{21} & G_{22} & B_{23}\sin\delta_{23} & \cdots & B_{2n}\sin\delta_{2n} \\ B_{31}\sin\delta_{31} & B_{32}\sin\delta_{32} & G_{33} & \cdots & B_{3n}\sin\delta_{3n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n1}\sin\delta_{n1} & B_{n2}\sin\delta_{n2} & B_{n3}\sin\delta_{n3} & \cdots & G_{nn} \end{bmatrix} + j\begin{bmatrix} B_{11} & B_{12}\cos\delta_{12} & B_{13}\cos\delta_{13} & \cdots & B_{1n}\cos\delta_{1n} \\ B_{21}\cos\delta_{21} & B_{22} & B_{23}\cos\delta_{23} & \cdots & B_{2\,n}\cos\delta_{2\,n} \\ B_{31}\cos\delta_{31} & B_{32}\cos\delta_{32} & B_{33} & \cdots & B_{3\,n}\cos\delta_{3\,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ B_{n\,1}\cos\delta_{n\,1} & B_{n2}\cos\delta_{n2} & B_{n\,3}\cos\delta_{n\,3} & \cdots & B_{nn} \end{bmatrix}$$

9.2 For the conditions of Problem 9.1, obtain the real matrices for $\mathbf{I}_q$ and $\mathbf{I}_d$ in terms of $\mathbf{V}_q$ and $\mathbf{V}_d$. Compare with (9.40) for a two-machine system with $G_{12} = G_{21} = 0$.

Solution

For a two-machine system

$$\mathbf{H} = \begin{bmatrix} G_{11} & B_{12}\sin\delta_{12} \\ B_{21}\sin\delta_{21} & G_{22} \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} B_{11} & B_{12}\cos\delta_{12} \\ B_{12}\cos\delta_{21} & B_{22} \end{bmatrix}$$

$$\mathbf{I} = [\mathbf{HV}_q - \mathbf{SV}_d] + j[S\mathbf{V}_q + H\mathbf{V}_d]$$

$$\begin{bmatrix} I_{q1} \\ I_{q2} \end{bmatrix} = \begin{bmatrix} G_{11} & B_{12}\sin\delta_{12} \\ B_{21}\sin\delta_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_{q1} \\ V_{q2} \end{bmatrix} - \begin{bmatrix} B_{11} & B_{12}\cos\delta_{12} \\ B_{21}\cos\delta_{21} & B_{22} \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix}$$

$$\begin{bmatrix} I_{d1} \\ I_{d2} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12}\cos\delta_{12} \\ B_{21}\cos\delta_{21} & B_{22} \end{bmatrix} \begin{bmatrix} V_{q1} \\ V_{q2} \end{bmatrix} - \begin{bmatrix} G_{11} & B_{12}\sin\delta_{12} \\ B_{21}\sin\delta_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_{d1} \\ V_{d2} \end{bmatrix}$$

9.3 Repeat Example 9.3, using the synchronous machine model called the one-axis model (see Section 4.15.4).

Solution

One-Axis Model

$$\begin{cases} E'_{di} = -\left(x_{qi} - x'_i\right) I_{qi} \\ \tau'_{d0} \dot{E}'_{qi} = E_{FD} - E'_{qi} + \left(x_{di} - x'_i\right) I_{di} \\ \tau_{ji} \dot{\omega}_i = T_{mi} - \left(I_{di} E'_{di} + I_{qi} E'_{qi}\right) \\ \dot{\delta}_i = \omega_i - 1 \end{cases}$$

From (9.42), for a two-machine system

$$\begin{cases} \left[1 - \left(x_{q1} - x'_1\right) B_{11}\right] E'_{d1} - \left(x_{q1} - x'_1\right) F_{B-G}\left(\delta_{12}\right) E'_{d2} = -\left(x_{q1} - x'_1\right) G_{11} E'_{q1} \\ \qquad -\left(x_{q1} - x'_1\right) F_{G+B}\left(\delta_{12}\right) E'_{q2} \\ -\left(x_{q2} - x'_2\right) F_{B-G}\left(\delta_{21}\right) E'_{d1} + \left[1 - \left(x_{q2} - x'_2\right) B_{22}\right] E'_{d2} = -\left(x_{q2} - x'_2\right) F_{G+B}\left(\delta_{21}\right) E'_{q1} \\ \qquad -\left(x_{q2} - x'_2\right) G_{22} E'_{q2} \end{cases}$$

Thus, E'_{d1} and E'_{d2} can be determined in terms of E'_{q1} and E'_{q2}

The rest of the equations of (9.42) remain the same.