(10) Problem 1:

WR^2 has units of lb(m) * ft^2. To convert this to inertia in units of slug * ft^2, we divide by 32 ft/sec²:

$$\Rightarrow J(slug * ft^2) = \frac{WR^2}{32.2}$$

Given that 1 m = 3.281 ft, 1 kg = 0.0685 slug,

$$J(kg * m^2) = \frac{WR^2}{32.2} * \frac{1}{.0685\ slug} * \frac{1\ m^2}{3.281^2}$$

$$= .0421\ WR^2$$

If we want joules (W_K), then

$$W_K = \frac{1}{2} J \omega_R^2 = \frac{1}{2}(.0421) WR^2 \omega_R^2$$

$$= .02105\ WR^2 \omega_R^2 \text{ in joules}$$

or, given that $\omega_R = \frac{2\pi n_R}{60}$

$$W_K = .02105\ WR^2 \left[\frac{4\pi^2}{3600}\right] n_R^2 = 2.31 \cdot 10^{-4} WR^2 n_R^2$$

(in joules)

In Mjoules (or MW-sec)

$$\underline{W_K = 2.31 \cdot 10^{-10} WR^2 n_R^2}$$

2. (a) $\omega_R = \frac{2}{P}\omega_{Re} = \frac{2}{4}(377) = 188.5$ rad/sec

(6) $W_K = [2.105\cdot10^{-8}][WR^2]\omega_R^2 = [2.105\cdot10^{-8}][859{,}000][188.5]^2 = 642.5$ Mj

Alternatively, $n_R = \frac{188.5}{2\pi}\times 60 = 1800$ rpm

$\Rightarrow$ $W_K = (2.31\cdot10^{-10})(WR^2)\,n_R^2 = (2.31\cdot10^{-10})(859{,}000)[1800]^2 = 642.9$ Mj

$\Rightarrow$ $W_K \cong 643$ Mj $= \underline{643 \text{ MW-sec}}$

(b) $MVA_{RATED} = \frac{P_{RATED}}{pf_{RATED}} = \frac{85}{.85} = 100$ MVA

(6) $H_{MACH} = \frac{W_K}{MVA_{RATED}} = \frac{643}{100} = \underline{6.43 \text{ sec}}$

(6) (c) $M = \frac{(2H)(MVA_{RATED})}{\omega_m} = \frac{2(6.43)(100)}{188.5} = 6.82 \frac{\text{Mj-sec}^2}{\text{mech rad}}$

Alternatively,

$M = \frac{2W_K}{\omega_m} = \frac{2(643)}{188.5} = \underline{6.82 \frac{\text{Mj-sec}^2}{\text{mech rad}}}$

3. (5) (a) First, we need to convert the H-constants to the system base.

$H_1 = 2.0\left[\frac{500}{100}\right] = 10.0$; $H_2 = 2.5\left[\frac{300}{100}\right] = 7.5$

MACH 1 : $\frac{2(10)}{377}\ddot{\delta}_1 = P_{m_1} - P_{e_1} \Rightarrow .0531\,\ddot{\delta}_1 = P_{m_1} - P_{e_1}$

MACH 2 : $\frac{2(7.5)}{377}\ddot{\delta}_2 = P_{m_2} - P_{e_2} \Rightarrow .0398\,\ddot{\delta}_2 = P_{m_2} - P_{e_2}$

where P_{m_i} and P_{e_i} are given in pu on 100 MVA base.

(5) (b) Add the two swing equations:

$$\frac{2}{\omega_{re}}\left[H_1\ddot{\delta}_1 + H_2\ddot{\delta}_2\right] = \underbrace{(P_{m_1}+P_{m_2})}_{P_m} - \underbrace{(P_{e_1}+P_{e_2})}_{P_e}$$

If $\delta_1 = \delta_2$, i.e., if the generators are coherent (swing together), then

$\frac{2(H_1+H_2)}{\omega_{re}}\ddot{\delta} = P_m - P_e \Rightarrow \frac{2(17.5)}{377}\ddot{\delta} = P_m - P_e \Rightarrow \underline{.0928\,\ddot{\delta} = P_m - P_e}$

4. (a) $\omega_R = \frac{2}{p}\omega_{Re} = \frac{2}{32}(377) = \underline{\underline{23.56 \text{ rad/sec}}}$

5 (b) $\frac{2H}{\omega_{Re}} = \frac{2(2)}{377}\ddot{\delta} = P_m - P_e \Rightarrow \underline{\underline{0.0106\,\ddot{\delta} = P_m - P_e}}$

5 (c) We need the initial angle to solve this problem. To get this, we could use $P_{e1} = \frac{|E||V|}{X}\sin\delta$, but we only know P_{e1}, i.e., we do not know $|E|$, $|V|$, or X. So let's just let $\delta(0) = \delta_0$.

During this fault, $P_{e2} = 0 \Rightarrow P_{a2} = P_m = 1.0$

$\Rightarrow .0106\,\ddot{\delta} = 1.0$ OR $.0106 \frac{d\omega}{dt} = 1.0$

Recall that initial speed must be zero since the machine is initially at steady state.

$\Rightarrow .0106\, d\omega = dt$

Integrate:

$$.0106 \int_0^{\omega(t)} d\omega = \int_0^t dt \Rightarrow .0106\,\omega(t) = t$$

OR $.0106 \frac{d\delta}{dt} = t \Rightarrow .0106\, d\delta = t\, dt$

Integrate again:

$$.0106 \int_{\delta_0}^{\delta(t)} d\delta = \int_0^t t\, dt \Rightarrow .0106[\delta(t) - \delta_0] = \frac{t^2}{2}$$

$$\Rightarrow \delta(t) = \frac{1}{2(.0106)} t^2 + \delta_0 = 47.17t^2 + \delta_0$$

At 3 cycles, $t = \frac{3}{60} = .05$ sec

$\Rightarrow \delta(.05) = 47.17(.05)^2 + \delta_0 = 0.118 + \delta_0$

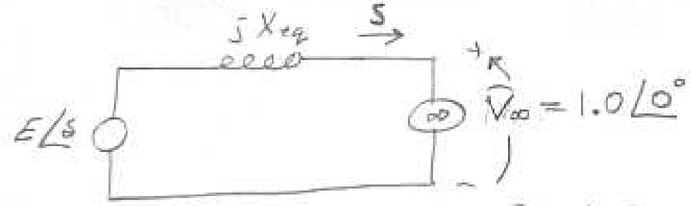
This is in radians. In degrees, we get

$\underline{\underline{\delta(.05) = 6.76° + \delta_0}}$

5\. (a) $X_{eq} = X'_d + X_T + X_{12} // (X_{13} + X_{23})$

(5) $= 0.3 + 0.1 + \dfrac{(0.2)(0.3)}{0.2+0.3} = 0.52 \text{ pu}$

So the system looks like:



Note we assumed $\bar{V}_\infty$ is the reference.

It was given that $P = 1.0$, where $S = P + jQ$,

and $pf = 0.95$ where $\tan\theta = \dfrac{Q}{P}$

$\Rightarrow |Q| = P\tan\theta = P\tan[\cos^{-1}(pf)] = 1.0\tan[\cos^{-1}(.95)] = 0.328$

Lagging $\Rightarrow Q > 0 \Rightarrow Q = 0.328 \text{ pu} \Rightarrow S = 1.0 + j0.328.$

$S = \bar{V}_\infty \bar{I}^* \Rightarrow \bar{I} = \left[\dfrac{S}{\bar{V}_\infty}\right]^* = \dfrac{1.0 - j0.328}{1.0\angle 0°} = 1.0 - j0.328$

$\Rightarrow \bar{I} = 1.0526\angle -18.195°$

Then $\bar{E} = \bar{V}_\infty + \bar{I}(jX_{eq}) = 1.0\angle 0° + (1.0526\angle -18.195°)(0.52\angle 90°)$

$= 1.0 + 0.547\angle 71.81° = \underline{1.2812\angle 23.95° = \bar{E}}$

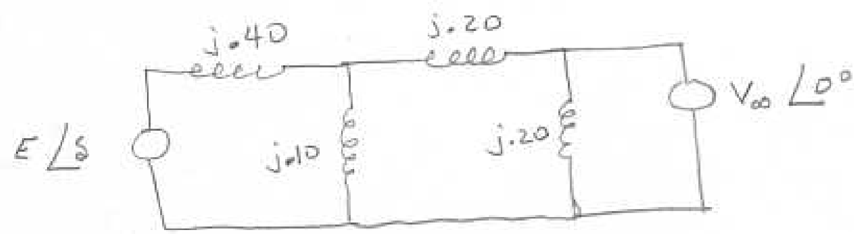
(5) (b) $P_e = \dfrac{|E||V_\infty|}{X_{eq}}\sin\delta \Rightarrow P_e = \dfrac{(1.2812)(1.0)}{0.52}\sin\delta$

$\Rightarrow \underline{P_e = 2.464\sin\delta}$

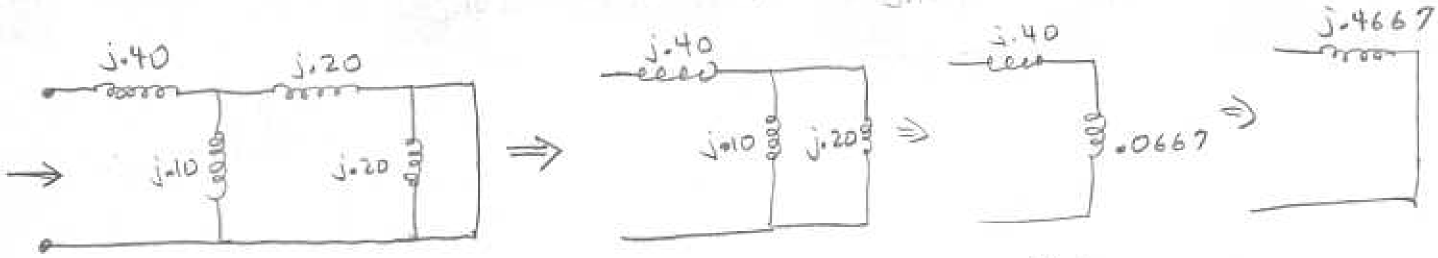
(18)

6\. From the previous problem, we know that the pre-fault power-angle curve is $P_{e_1} = 2.464\sin\delta$. We need to find the fault on curve P_{e2} and the post-fault curve P_{e2}.

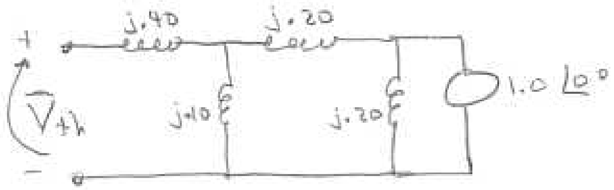
<u>Fault-on (fault at Bus 3)</u>: The network looks like:



We need the power-angle expression, but to get that, we must simplify the network. Let's obtain the Thevenin equivalent as seen "looking right" from the generator internal source voltage. To do this, we "idle" all sources (short the voltage sources and open the current sources). This results in



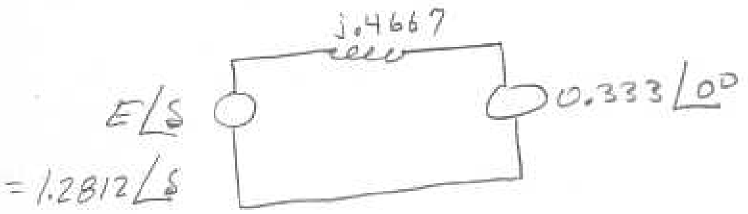
We still need V_{thev}, obtained from the following



Using voltage division, we find

$$\bar{V}_{th} = 1.0\angle 0^\circ \left[\frac{j.10}{j.10 + j.20}\right] = 0.333\angle 0^\circ$$

So our fault-on network appears as

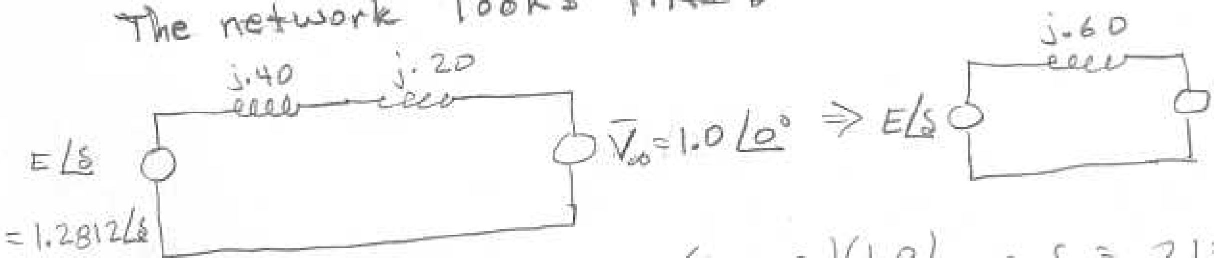


$$\Rightarrow P_{e_2} = \frac{(1.2812)(0.333)}{0.4667} \sin\delta$$

$$\Rightarrow \boxed{P_{e_2} = 0.9142 \sin\delta}$$

<u>Post Fault (after loss of lines 1-3, 2-3)</u>

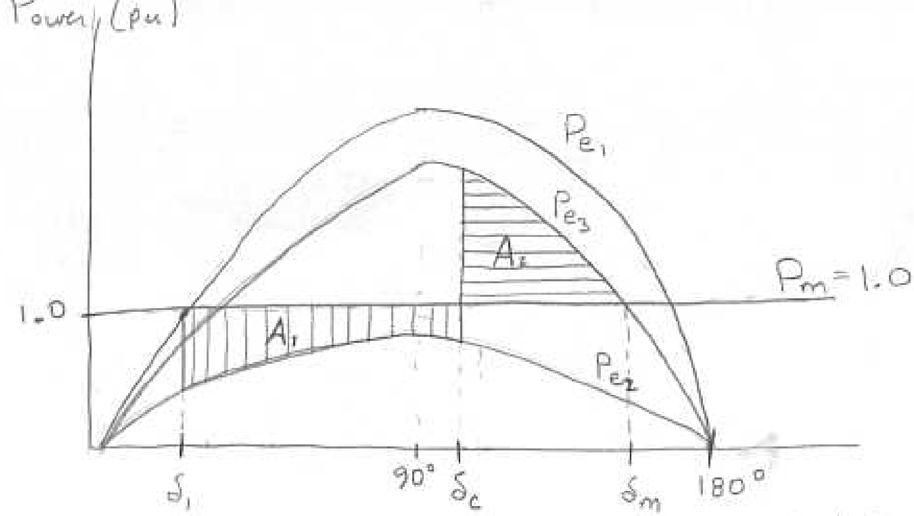
The network looks like:



$$\Rightarrow P_{e_3} = \frac{|E||V_\infty|}{X_{eq}} \sin\delta = \frac{(1.2812)(1.0)}{.60} \sin\delta = 2.135 \sin\delta$$

$$\Rightarrow \boxed{P_{e_3} = 2.135 \sin\delta}$$

So the power angle curves, the initial angle (at 23.95° from problem 4), the clearing angle, and the areas necessary for the clearing angle to be critical, are shown in the figure below...



For δ_c to be critical, then $A_1 = A_2$ with A_2 bounded by $\delta = \delta_m$.

$$A_1 = \int_{\delta_1}^{\delta_c} P_m - P_{e_2}\, d\delta = \int_{\delta_1}^{\delta_c} (1.0 - 0.9142 \sin\delta)\, d\delta$$

$$= \delta + .9142\cos\delta \Big|_{\delta_1}^{\delta_c} = \delta_c + .9142\cos\delta_c - \delta_1 - .9142\cos\delta_1$$

with $\delta_1 = 23.95° = .4178$ rad $\Rightarrow A_1 = \delta_c + .9142\cos\delta_c - .4178 - .9142\cos(23.95°)$

$$\Rightarrow A_1 = \delta_c + .9142\cos\delta_c - 1.25$$

$$A_2 = \int_{\delta_c}^{\delta_m} P_{e_3} - P_m\, d\delta = \int_{\delta_c}^{\delta_m} 2.135\sin\delta - 1.0\, d\delta = -2.135\cos\delta - \delta \Big|_{\delta_c}^{\delta_m}$$

$$= -2.135\cos\delta_m - \delta_m + 2.135\cos\delta_c + \delta_c$$

But note from the picture that $\delta_m = 180 - \delta_1$

$\Rightarrow \delta_m = 180 - 23.95° = 156.05° = 2.72$ rad

So $A_2 = -2.135\cos(156.05) - 2.72 + 2.135\cos\delta_c + \delta_c$

$$= -.7688 + 2.135\cos\delta_c + \delta_c$$

For δ_c critical, $A_1 = A_2$

$\Rightarrow \cancel{\delta_c} + .9142\cos\delta_c - 1.25 = -.7688 + 2.135\cos\delta_c + \cancel{\delta_c}$

$\Rightarrow -1.253\cos\delta_c = .4812 \Rightarrow \cos\delta_c = -.384 \Rightarrow \underline{\underline{\delta_c = 112.6°}}$

18 7. We can use the picture drawn in the problem 5 solution to write that

7/7

$$\int_{\delta_1}^{\delta_c} P_m - P_{e2}\, d\delta = \int_{\delta_c}^{\delta_m} P_{e3} - P_m\, d\delta$$

where $P_{e2} = P_{M_2}\sin\delta$ and $P_{e3} = P_{M_3}\sin\delta$ so that

$$\int_{\delta_1}^{\delta_c} P_m - P_{M_2}\sin\delta\, d\delta = \int_{\delta_c}^{\delta_m} P_{M_3}\sin\delta - P_m\, d\delta$$

$$\Rightarrow \left[P_m\delta + P_{M_2}\cos\delta\right]\Big|_{\delta_1}^{\delta_c} = \left[-P_{M_3}\cos\delta - P_m\delta\right]\Big|_{\delta_c}^{\delta_m}$$

$$\Rightarrow \cancel{P_m\delta_c} + P_{M_2}\cos\delta_c - P_m\delta_1 - P_{M_2}\cos\delta_1 = -P_{M_3}\cos\delta_m - P_m\delta_m + \cancel{P_{M_3}\cos\delta_c} + \cancel{P_m\delta_c}$$

Eliminate $P_m\delta_c$ term from both sides, and then move all δ_c terms to LHS!

$$P_{M_2}\cos\delta_c - P_{M_3}\cos\delta_c = P_m\delta_1 - P_m\delta_m + P_{M_2}\cos\delta_1 - P_{M_3}\cos\delta_m$$

$$\Rightarrow \cos\delta_c = \frac{P_m\left[\delta_1 - \delta_m\right] + P_{M_2}\cos\delta_1 - P_{M_3}\cos\delta_m}{P_{M_2} - P_{M_3}}$$

Now divide top and bottom by P_{M_1}

$$\Rightarrow \cos\delta_c = \frac{\frac{P_m}{P_{M_1}}\left[\delta_1 - \delta_m\right] + \frac{P_{M_2}}{P_{M_1}}\cos\delta_1 - \frac{P_{M_3}}{P_{M_1}}\cos\delta_m}{\frac{P_{M_2}}{P_{M_1}} - \frac{P_{M_3}}{P_{M_1}}}$$

NOTE: $\dfrac{P_m}{P_{M_1}} = \dfrac{\frac{EV_\infty}{X_1}\sin\delta_1}{\frac{EV_\infty}{X_1}} = \sin\delta_1$ $\qquad \dfrac{P_{M_3}}{P_{M_1}} = \dfrac{EV_\infty/X_3}{EV_\infty/X_1} = \dfrac{X_1}{X_3} = r_2$

$$\frac{P_{M_2}}{P_{M_1}} = \frac{EV_\infty/X_2}{EV_\infty/X_1} = \frac{X_1}{X_2} = r_1$$

$$\Rightarrow \cos\delta_c = \frac{\left[\delta_1 - \delta_m\right]\sin\delta_1 + r_1\cos\delta_1 - r_2\cos\delta_m}{r_1 - r_2}$$

Multiply top + bottom by $-1 \Rightarrow$ $\boxed{\cos\delta_c = \dfrac{\left[\delta_m - \delta_1\right]\sin\delta_1 - r_1\cos\delta_1 + r_2\cos\delta_m}{r_2 - r_1}}$