

CONCEPTS OF SYNCHRONOUS MACHINE STABILITY AS AFFECTED BY EXCITATION CONTROL

F. P. deMello C. Concordia
General Electric Company
Schenectady, N. Y.

ABSTRACT

The phenomena of stability of synchronous machines under small perturbations is explored by examining the case of a single machine connected to an infinite bus through external reactance.

The analysis develops insights into effects of thyristor-type excitation systems and establishes understandings of the stabilizing requirements for such systems. These stabilizing requirements include the voltage regulator gain parameters as well as the transfer function characteristics for a machine speed derived signal superposed on the voltage regulator reference for providing damping of machine oscillations.

INTRODUCTION

The phenomenon of stability of synchronous machine operation has received a great deal of attention in the past and will receive increasing attention in the future. As economies in system design are achieved with larger unit sizes and higher per unit reactance generating and transmission equipment designs, more emphasis and reliance is being placed on controls to provide the required compensating effects with which to offset the reductions in stability margins inherent from these trends in equipment design¹.

Concurrent with these trends are improvements in calculating methods and computing capability which permit predicting complex dynamic effects^{2,3,4,5}, providing the means for designing control equipment with the proper characteristics.

Among several aspects of stability of synchronous machine operation, an important one is the mode of small perturbation stability referred to as steady state, dynamic or conditional stability. Increasing attention has been focused recently on the effects of excitation control on the damping of oscillations which characterize the phenomena of stability. In particular, it has been found useful and practical to incorporate transient stabilizing signals derived from speed, terminal frequency or power^{6,7} superposed on the normal voltage error signal of voltage regulators to provide for additional damping to these oscillations.

This paper deals with an analysis of the phenomena of stability of synchronous machines under small perturbations by examining the case of a single machine connected to a large system through external impedance. The object of this analysis is to develop insights into effects of excitation systems and to establish an under-

standing of the stabilizing requirements for such systems. By examining a wide range of system and machine parameters and probing into causes and effects, a logical set of guide rules and concepts is developed to explain the nature of the problem and to arrive at a set of recommendations for stabilizing through excitation control. In order to limit the scope of this paper, the excitation system investigated is one which could be characterized by a small time constant of between 0.03 and 0.05 secs. which is typical of thyristor type systems.

The method of analysis of course can easily be extended to excitation systems with different dynamic characteristics. It is believed that from this analysis covering a wide range of conditions for the single machine case, one can project recommendations for stabilizing machines in multimachine systems.

DYNAMICS OF A SINGLE MACHINE CONNECTED TO A LARGE SYSTEM - FUNDAMENTAL CONCEPTS

Block Diagram Relations

The phenomena of stability and damping of synchronous machines for the mode of small perturbations can be examined with the aid of block diagrams relating the pertinent variables of electrical torque, speed, angle, terminal voltage, field voltage and flux linkages. The relations in the block diagrams discussed in this paper apply to a 2 axis machine representation with a field circuit in the direct axis but without amortisseur effects. Although a more rigorous representation should include amortisseur or solid iron eddy current effects in both axes, this simpler representation is sufficient to establish the basic effects and develop concepts.

The basic phenomenon in question is the stability of the torque-angle loop, i.e., the behavior of the rotor angle and speed following a small disturbance such as a mechanical torque disturbance. Although the whole subject can be and has been explored by various stability analysis techniques such as Routh's criterion⁸, eigenvalue analyses⁴, etc., considerable value is found in analyzing the phenomena in the light of elementary servomechanism and frequency response theory, thereby developing an insight into the basic elements that cause various effects.

We will attempt to relate the familiar concepts of small perturbation stability of a single machine supplying an infinite bus through external impedance to the elements and relations shown by

the block diagram of Fig. 1. These relations and block diagram have been treated previously in ref. 9. The parameters in these relations are listed in Appendix A. They are derived by small perturbation analysis on the fundamental synchronous machine equations, and hence are functions of machine and system impedances as well as operating point.

The parameters of Fig. 1 are defined as follows:

$$K_1 = \left. \frac{\Delta T_e}{\Delta \delta} \right|_{E'_q} = \text{change in electrical torque for a change in rotor angle with constant flux linkages in the d axis.}$$

$$K_2 = \left. \frac{\Delta T_e}{\Delta E'_q} \right|_{\delta} = \text{change in electrical torque for a change in d-axis flux linkages with constant rotor angle.}$$

$$K_3 = \text{impedance factor} = \frac{X'_d + X_e}{X'_d + X_e} \text{ for the case where the external impedance is a pure reactance } X_e.$$

$$K_4 = \frac{1}{K_3} \frac{\Delta E'_q}{\Delta \delta} = \text{demagnetizing effect of a change in rotor angle.}$$

$$K_5 = \left. \frac{\Delta e_t}{\Delta \delta} \right|_{E'_q} = \text{change in terminal voltage with change in rotor angle for constant } E'_q.$$

$$K_6 = \left. \frac{\Delta e_t}{\Delta E'_q} \right|_{\delta} = \text{change in terminal voltage with change in } E'_q \text{ for constant rotor angle.}$$

T'_{do} = field open circuit time constant

$T'_{dz} = K_3 T'_{do}$ = effective field time constant under load.

It is important to recognize that, with the exception of K_3 which is only a function of the ratio of impedances, all other parameters change with loading, making the dynamic behavior of the machine quite different at different operating points. Since these parameters change in rather complex manner, it is difficult to reach general conclusions based on parameter values for one operating point only.

Performance with Constant d-axis Flux Linkages

Returning to the block diagram of Fig. 1, let us first look at the portion shown in Fig. 2, which is pertinent for the condition of constant flux linkages in the d-axis.

This portion shows a torque-speed-angle loop

which gives rise to a natural period of oscillation $\omega_n = \sqrt{\frac{377 K_1}{M}}$ rads/sec. Neglecting damping due to

electrical load characteristics, we note that this torque-speed-angle loop is a pure oscillator with zero damping much like a spring-mass system. The lighter the machine (smaller M) and the higher the transient synchronizing power coefficient K_1 , the higher is the oscillation frequency. From the relations in Appendix A, we note that K_1 is decreased by increasing system and machine reactances. It is also affected by machine loading in a somewhat peculiar manner due to transient saliency effects.

In most practical cases K_1 is positive giving rise to oscillations. It is possible in very unusual cases of very long ties and relative high loading on these ties to have K_1 go negative. In these cases, even if we were to maintain constant flux linkages in the d-axis, the machine would be unstable in a real root stability sense, i.e., the machine angle would run away monotonically and exponentially with time. Although with special stabilizing in the control of excitation, it is possible to render some of these systems stable, they are primarily of academic rather than practical interest.

Damping Torques, Synchronizing Torques - Damping Ratio

Before introducing the effects of field losses and excitation control, it is desirable to develop a few simple concepts relating the behavior of the second order system of Fig. 3 which introduces a damping term D.

In this system, damping is provided by developing a negative torque proportional to and in time phase with speed (p ω). The resulting characteristic equation is

$$S^2 + D/M S + \frac{K_1 377}{M} = 0 \quad (1)$$

giving rise to damped oscillations with frequency

$$\omega_n \sqrt{1 - \zeta^2} \text{ and damping ratio } \zeta$$

where

$$\omega_n = \sqrt{K_1 377 / M} \text{ and } \zeta = \frac{1}{2} \frac{D}{K_1 M 377}$$

One form of instability is approached as the damping ratio goes to zero. For normal values of damping ratios, the frequency of oscillation is not materially different from the value for zero damping ratio. For conceivable ranges of inertia, impedances and loading values, this frequency of oscillation will be in the neighborhood of 0.5 to 2 c.p.s. with the possibility of extreme values of up to 4 c.p.s. at the high end and 0.1 c.p.s. at the low end.

At this point it is appropriate to introduce the concept of synchronizing and damping torques. At any given oscillation frequency, braking torques are developed in phase with the machine rotor angle and in phase with the machine rotor speed. The former are termed synchronizing torques and the latter damping torques. The torque oscillations developed by any particular means can be broken down into these components for an insight as to their effect on stability.

Stability can be endangered by a lack of either or both synchronizing and damping torques. The traditional stability criterion with which industry is most acquainted concerns the tests for positive synchronizing torques which determine whether or not forces will be set up to restore the rotor angle of the machine following an arbitrarily small displacement of this angle. No correspondingly simple guide rules have been developed to test for the possibility of another form of instability which would arise for cases where damping torques became negative. This latter form of instability is becoming increasingly important as machine stable operation is more than ever dependent on automatic control of excitation.

Performance with Constant Field Voltage

Introducing now the effects of field losses, we refer to the block diagram of Fig. 4 which adds the armature reaction terms expressed as a demagnetizing influence with increasing rotor angle through the block K_4 .

Examine now the contribution of torque due to this effect shown by the dashed lines in Fig. 4.

The effect of this component of torque can be described by the expression

$$\left. \frac{\Delta T}{\Delta \delta} \right|_{\text{due to } \Delta E'_q} = - \frac{K_2 K_4 K_3}{1 + ST'_{do} K_3} \quad (2)$$

These coefficients are always positive, hence we note that at steady state or zero oscillation frequencies, this demagnetizing component of torque is $-(K_2 K_4 K_3) \Delta \delta$ which is opposed in sign to the transient synchronizing component discussed in the previous section, $+K_1 \Delta \delta$.

The familiar steady-state stability criterion with constant field voltage defines the stability limit as the condition for which the steady-state synchronizing power coefficient $K_1 - K_2 K_4 K_3$ is zero.

We note that at very high oscillation frequencies, $\omega \gg \frac{1}{T'_{do}}$ (note $\frac{1}{T'_{do}} = \frac{1}{T_{dz}}$), the

phase angle of the torque component in question given by (2) is plus 90° which means that at these high frequencies, these torques are almost entirely

damping torques. The magnitude of the torques, of course, gets attenuated with increasing frequency. For typical machine and system parameters resulting in oscillation frequencies of about 1 c.p.s., the field would only contribute a damping ratio of between 0.03 and 0.05.

In terms of time domain behavior, the series of traces in Fig. 5 help describe the significance of the various effects discussed so far. The plots are the time response of machine angle following a small step change in mechanical torque.

Fig. 5(a) shows the case of constant E'_d and $K_1 > 0$ establishing the characteristic oscillations with zero damping. As mentioned before, damping due to load characteristics is neglected in these discussions.

Fig. 5(b) shows the effect of the field demagnetizing resulting in the same type of oscillations with slight damping superposed on a monotonic drift as the machine establishes new steady-state flux level conditions. This case is for $K_1 - K_2 K_4 K_3 > 0$, i.e., the machine has steady-state stability or stability under close manual control in the traditional sense (positive zero frequency synchronizing power coefficient).

Fig. 5(c) illustrates the angle response for the case where the machine is operating with constant field voltage beyond the steady-state limit. Here we continue to see the oscillations; however, they are superimposed on a monotonically unstable component. For such an operating point, stable operation can only be achieved by superimposing the effects of excitation control which cancel out the unstable monotonic component. Operation in this region under regulator control has been referred to as operation with dynamic stability or conditional stability. Providing stability by excitation control requires cancelling out the monotonic instability without deteriorating damping or giving rise to negative damping. These effects will be discussed in the next section.

Finally, Fig. 5(d) shows the behavior for the rare case where K_1 is very small or negative and the system manifests instability without oscillations even under assumption of constant direct axis flux linkages, E'_q .

EFFECTS OF VOLTAGE REGULATORS

Block Diagram Relations

The effect of the voltage regulator on damping and synchronizing torques will now be analyzed by referring to the block diagram of Fig. 6 where the additional necessary relations between E'_d and e_t , δ and e_t , and E'_d and e_t have been included.

In this treatment, we are considering a simple voltage regulator - excitation system

$$\text{i.e., } \frac{\Delta E'_{fd}}{\Delta e_t} = \frac{-K_c}{1 + ST_c} \quad (3)$$

with a gain K_e and small time constant T_e as might be typical of thyristor type exciters.

By means of small perturbation analysis, the terminal voltage deviations Δe_t can be related to changes in angle $\Delta \delta$ through the proportionality factor K_5 and to changes in flux linkages ΔE_f by the proportionality factor K_6 .

From the relations in Appendix A, it can be seen that K_6 is always positive although its magnitude gets reduced with loading (large machine internal angles) and with small external impedances. The constant K_5 , on the other hand, can have either sign and considerable range in magnitude depending on the impedances and the operating condition. This variation in the parameter K_5 has considerable bearing on the effects on damping of voltage regulators. Before developing insights into these effects, let us analyze regulator gain constraints from other considerations.

Regulator Gain Constraints

One important criterion of voltage regulator performance is its operation with the machine on open circuit for which case machine angle has no significance. The regulator-machine loop is therefore described by the block diagram of Fig. 7.

For well damped performance of this regulating loop¹⁰, it is desirable to maintain the crossover frequency less than $\frac{1}{2T_e}$ which would mean that the gain K_e should be approximately less than $\frac{T_{do}'}{2T_e}$.

For an exciter time constant of 0.05 sec. and open circuit field time constant T_{do}' of 5 sec., we note that it would be desirable to hold the regulator gain to less than 50 due to these open-circuit performance considerations.

It should be noted that the gain we speak of can be interpreted also as the transient regulator gain. Since it may be desirable, from a steady-state regulation or droop consideration, to have considerably higher regulator steady state or d.c. gain, the desired transient gain reduction can be

obtained with a series lag-lead network $\frac{1 + T_1 s}{1 + T_2 s}$ where $T_1/T_2 = \frac{\text{transient gain}}{\text{steady state gain}}$ or alternately with rate feedback from exciter voltage.

By selecting $\frac{1}{T_1}$ as considerably lower than the crossover frequency, the effects of concern are not significantly different if the assumption is made that the regulator is represented by $\frac{K_{e1}}{1 + ST_e}$

rather than by $\frac{K_{e2} (1 + ST_1)}{(1 + ST_2)(1 + ST_e)}$ where

$K_{e1} = K_{e2} T_1/T_2$ is the transient gain.

One more point of interest is in order. The

regulating loop, as described in Fig. 7, also has significance with the machine under load. This loop by itself describes the closed-loop effects of voltage as affected by flux changes only, i.e., with no changes in angle. Under loaded conditions, this loop is given by the block diagram of Fig. 8.

The machine function $\frac{\Delta e_t}{\Delta E_{fd}}$ is now

$$\frac{\Delta e_t}{\Delta E_{fd}} = \frac{K_5 K_3}{1 + ST_{do}' K_3} \tag{4}$$

which can also be expressed as

$$\frac{\Delta e_t}{\Delta E_{fd}} = \frac{K_6 \frac{T_{dz}'}{T_{do}'}}{1 + ST_{dz}'} \tag{5}$$

Disregarding the effect of K_6 which can vary from about 1 to a considerably lower value (occasionally for very high external reactances combined with heavy shunt resistance loads K_6 can be slightly greater than the open circuit value of 1), we note that the effect of the shorter field time constant under load T_{dz}' is offset by the lower gain $\frac{T_{dz}'}{T_{do}'}$ so that the closed loop response of this

voltage component regulating loop under load conditions is not materially different from the closed loop response on open circuit.

As a first approximation using familiar frequency response analysis techniques¹⁰, this closed loop response between the terminal voltage component due to flux changes and change in voltage reference can be described by two time constants

in series, one equal to $\frac{T_{do}'}{K_e K_6}$ and the other equal to T_e .

$$\frac{\Delta e_t}{\Delta e_{t \text{ ref}}} \Big|_{\delta = \text{constant}} \approx \frac{K_e K_6}{(1/K_3 + K_e K_6) \left(1 + S \frac{T_{do}'}{K_e K_6}\right) (1 + ST_e)}$$

$$\approx \frac{1}{\left(1 + S \frac{T_{do}'}{K_e K_6}\right) (1 + ST_e)} \tag{6}$$

for large K_e

Similarly

$$\frac{\Delta E'_q}{\Delta \delta_{t \text{ ref}}} \approx \frac{K_e}{(1/K_3 + K_e K_6) \left(1 + S \frac{T'_{do}}{K_e K_6}\right) (1 + ST_e)} \quad (7)$$

$b = \text{constant}$

$$\approx \frac{1}{K_6 \left(1 + S \frac{T'_{do}}{K_e K_6}\right) (1 + ST_e)} \quad (7)$$

for large K_e

Voltage Regulator Effects on Machine Stability

Comparing the block diagram of Fig. 6 with that of Fig. 4, we note first that, with the voltage regulator in operation, the demagnetizing component $K_4 \Delta \delta$ causes a greatly attenuated effect relative to that which it had with no voltage regulator. That is, if we compare the expressions for change in electrical torque for a change in angle due to this demagnetizing action, we have

$$\frac{\Delta T}{\Delta \delta} = \frac{-K_4 K_2 K_3}{1 + ST'_{do} K_3} \quad (8)$$

in the case of no voltage regulator, and we previously described its effects in terms of field damping and negative synchronizing torques.

For the case with voltage regulator, this particular component of torque becomes approximately

$$\frac{\Delta T}{\Delta \delta} \approx \frac{-K_4 K_2}{K_e K_6 \left(1 + S \frac{T'_{do}}{K_e K_6}\right)} \quad (9)$$

When comparing expressions (8) and (9), we note that at low frequencies (9) is considerably

smaller than (8) by the factor $\frac{1}{K_e K_6 K_3}$, while

at very high frequencies, the two expressions approach each other. The significance of these facts is that the steady-state negative component of synchronizing power coefficient is practically eliminated with a voltage regulator with fairly high gain. Meanwhile, since the effective time constant has been reduced from T'_{do} to $\frac{T'_{do}}{K_e K_6}$, the damping component of torque due to this effect is correspondingly reduced, i.e., the phase lag approaches 90° for considerably higher oscillation frequencies.

Now that we have disposed of the component of torque produced by the K_4 branch in Fig. 6 as rather negligible and contributing only slightly to damping, let us analyze the contribution coming

through the branch K_5 accounting for the effect of angle on terminal voltage. This terminal voltage in turn affects the flux and torque through the closed voltage regulator loop.

The component in question is described by the block diagram of Fig. 9.

The exact expression for this component of torque due to a change in angle and its effect on voltage is

$$\frac{\Delta T}{\Delta \delta} = - \frac{K_2 K_e K_5}{(1/K_3 + K_e K_6) + S(T_e/K_3 + T'_{do}) + S^2 T'_{do} T_e} \quad (10)$$

Using the approximate expressions for the closed loop which apply for the usual range of constants in question, this becomes

$$\frac{\Delta T}{\Delta \delta} \approx \frac{-K_2 K_5}{K_6 \left(1 + S \frac{T'_{do}}{K_e K_6}\right) (1 + ST_e)} \quad (11)$$

Finally, including the previous component produced by the K_4 branch, the demagnetizing effect of angle, we have the complete expression for changes in torque due to changes in angle via their effect on E'_q

$$\frac{\Delta T}{\Delta \delta} = \frac{-K_2 \left\{ \left[\frac{K_e K_5}{K_6} + K_4 \right] + ST_e K_4 \right\}}{1/K_3 + K_e K_6 + S(T_e/K_3 + T'_{do}) + S^2 T'_{do} T_e} \quad (12)$$

due to $\Delta E'_q$

Examine now the synchronizing and damping torque contributions from this expression for different oscillation frequencies:

Synchronizing Torques

This torque is the component in time phase with angle. Neglecting the demagnetizing effect through K_4 , i.e., using the simpler expression (10), we note that this synchronizing torque component T_S at oscillation frequency ω is:

$$\frac{-K_2 K_e K_5}{1/K_3 + K_e K_6 - \omega^2 T'_{do} T_e} \Delta \delta \quad (13)$$

$$\text{For very low frequencies } T_S = \frac{-K_2 K_e K_5 \Delta \delta}{1/K_3 + K_e K_6}$$

$$\approx \frac{-K_2 K_5 \Delta \delta}{K_6} \quad \text{for high values of } K_e.$$

We note that with K_5 positive this synchronizing component is negative. This is of no particular concern as for those cases where K_5

is positive (low to medium external impedance and low to medium loadings) K_1 is usually high so that the net synchronizing component $K_1 - \frac{K_2 K_5}{K_6}$ is still significantly greater than zero.

For K_5 negative, which occurs for moderate to high system transfer impedances, and heavy loadings, we note that the synchronizing torque component is positive at low frequencies. This is beneficial in those cases where the transient synchronizing coefficient K_1 is low or negative, or where the steady-state synchronizing coefficient without regulator $K_1 - K_2 K_4 K_3$ is negative.

As oscillation frequencies increase and ω^2 approaches $\frac{1/K_3 + K_6 K_e}{T_{do} T_e}$, we note from expression

(13) that the synchronizing torque becomes infinite, and for even larger frequencies it reverses sign. Actually before this can happen, the oscillation frequency becomes very high and instability develops because of negative damping.

Damping Torques

From expression (11), we see that the damping torque component T_D due to voltage regulator action is

$$\frac{K_2 K_e K_5 (T_e/K_3 + T'_{do}) \omega}{(1/K_3 + K_6 K_e - \omega^2 T_{do} T_e)^2 + (T_e/K_3 + T_{do})^2 \omega^2} \Delta \delta \quad (14)$$

This component gives positive damping whenever K_5 is positive, but for a large number of cases where K_5 is negative, it contributes negative damping. Further, we note that, with K_5 negative, increasing K_e increases the magnitude of negative damping causing instability. However, at the other extreme with zero regulator gain, we only get the small amount of field damping described before.

We thus have a conflicting problem. In those cases where K_5 is negative and which are generally the cases involving stability problems, a voltage regulator is of major help in providing synchronizing torques and curing that part of the stability problem. However, in so doing it destroys the natural damping of the machine which is small to start with. The recourse has been to have just enough regulator gain to provide synchronizing power coefficient without cancelling all of the inherent machine damping.

This can be a satisfactory solution in most cases; however, there can be instances where stability is provided by the regulator with very poor damping, making operation extremely oscillatory. In some special cases of very long lines requiring operation near the line limit, the solution is to have a fairly high regulator gain to provide the necessary synchronizing power coefficient. In these cases, one effective way to solve the damping problem is to provide a special

stabilizing signal derived from machine speed, terminal frequency, or power.

Recognizing that the problem of damping can be solved by auxiliary means removes some of the constraints placed on regulator gain. However, as was noted before, there remain the regulator gain constraints explained in the section titled, "Regulator Gain Constraints". Since there are fixed relationships between the variables, speed, terminal frequency and power which can be used to relate the requirements of one signal in terms of the requirements of another, we will confine our analysis to use of signals from machine rotor speed. It is recognized that the use of any particular signal in preference to another will depend on hardware and implementation considerations outside the scope of this paper.

STABILIZING SIGNALS FROM SPEED

Phase and Magnitude Relations

The stabilizing signal requirements to provide damping through transient manipulation of the voltage reference in response to speed can be derived as follows. Fig. 10 shows the elements of the system in question relating the effect of speed through the stabilizing function $G(S)$ through the voltage regulator loops affecting $\Delta E'_d$ which in turn produces a component of torque ΔT_{sig} .

The question is: What should be the nature of $G(S)$ to provide adequate damping over a wide range of machine and system parameters and operating conditions?

The over-all function between the component of torque produced by the stabilizing signal and speed is

$$\frac{\Delta T_{sig}}{p\delta} = \frac{G(S) K_2 K_e}{(1/K_3 + K_e K_6) + S(T_e/K_3 + T'_{do}) + S^2 T'_{do} T_e} \quad (15)$$

For the usual range of constants, this function can be approximated by

$$\frac{\Delta T_{sig}}{p\delta} \approx \frac{G(S) K_2 K_e}{(1/K_3 + K_e K_6) \left(1 + S \frac{T'_{do}}{K_e K_6}\right) (1 + S T_e)} \quad (16)$$

For large values of K_e this expression can be further approximated by

$$\frac{\Delta T_{sig}}{p\delta} \approx \frac{K_2}{K_6} \frac{G(S)}{\left(1 + S \frac{T'_{do}}{K_6 K_e}\right) (1 + S T_e)} \quad (17)$$

If we wish $\frac{\Delta T}{\Delta e_{ref}} \frac{sig}{p\delta}$ to provide pure damping throughout the frequency range, then $G(S)$ ideally should be a pure lead function with two zeros, i.e., $G(S) = K(1 + S T_{do}/K_0 K_e)(1 + S T_e)$. Such a function is not physically realizable and further we do not wish to have a permanent steady-state effect of speed on the voltage reference. Hence, the stabilizing function should be some compromise function which provides damping over the spectrum of expected frequencies of oscillation, i.e., a function with enough phase lead to compensate a significant part of the phase lag contributed by the machine and regulator.

Reverting back to the concepts of synchronizing and damping components of torque, we note the following. If at a given frequency of oscillation ω , $G(j\omega)$ has more lead angle than the lag angle of the machine function

$$\frac{\Delta T}{\Delta e_{ref}} = \frac{K_2 K_e}{(1/K_3 + K_S K_0) + j\omega(T_e/K_3 + T'_{do}) - \omega^2 T_{do} T_e}$$

which for large K_e is approximately equal to

$$\frac{K_2}{K_G(1 + j\omega T'_{do}/K_0 K_e)(1 + j\omega T_e)} \quad (\text{this difference}$$

being less than 90°), then in addition to the damping component, we have a negative synchronizing component, the magnitude of these two components being related to the sine and cosine of this difference in angle. That is, let the lead angle of $G(S)$ be θ at a given frequency of oscillation ω , and the lag angle of the machine-regulator function

$\frac{\Delta T}{\Delta e_{ref}}$ be γ at this same frequency. Then the

relative magnitude of the synchronizing torque T_S to damping torque T_D at that frequency produced by the stabilizing signal is

$$\frac{T_S}{T_D} = \frac{\sin(\gamma - \theta)}{\cos(\gamma - \theta)} \quad (18)$$

Since it is very difficult to synthesize a function which will provide exact angle cancellation for all conditions, i.e., having $\theta = \gamma$ at all points, and further since the stability problem is often helped by providing some synchronizing component, we come to the following conclusions relating to the nature of the stabilizing signal function.

1. At low frequencies of oscillation which are indicative of weak synchronizing torques, one should strive to hold θ less than γ , or if this is not possible, at least keep θ close to γ . At these low frequencies, the machine

regulator function $\frac{\Delta T}{\Delta e_{ref}}$ has small lagging

phase angles, hence the stabilizing function should correspondingly have small leading

phase angles. As was mentioned before, a medium-high regulator gain provides synchronizing torques through the regulator by virtue of the effect of angle on terminal voltage K_S . Hence, what should be attempted at the low frequency range is to provide damping without appreciably hurting the synchronizing torque.

2. At the high extreme of the spectrum of frequencies of oscillation, there is no particular danger to subtracting from the synchronizing component. As a matter of fact, since the machine-regulator function's phase angle becomes increasingly lagging with increasing frequency whereas, because of hardware limitations, the stabilizing function ceases to provide increasing phase leads at these higher frequencies, it is natural to expect θ to be less than γ . However, in this range of frequencies ($\gamma - \theta$) should not be so large as to add significantly to the synchronizing component with its attendant effect on increasing the frequency of oscillation. As long as the difference between the machine regulator lag and the phase lead for the stabilizing function is within $\pm 30^\circ$ for the frequency spectrum of concern, acceptable results are obtained.

Stabilizing Signal Constraints

There are several constraints that have to be observed when prescribing a stabilizing signal design that will be physically realizable and desirable from other considerations. These considerations and constraints are:

1. The signal should not produce a steady-state offset of voltage reference with frequency, hence it must approach a pure rate signal at very low frequencies.
2. Lead functions can only be realized with lead-lag pairs, and there is a limit as to how small the lag time constant can be. It is conservative to assume that this lag time constant cannot be smaller than 0.05 sec.

SEARCH FOR UNIVERSALLY APPLICABLE STABILIZING FUNCTION

General Approach

The insights that have been developed into the problem of providing damping from stabilizing signals may convince one that there is no way of developing a universally applicable stabilizing function that would be optimal for a wide range of machine and system parameters, and operating conditions. However, even if not optimal, it would be highly desirable to have a signal transfer function which would provide damping in those cases where it was badly needed without deteriorating damping in those instances where there was no particular need for providing supplementary damping.

The single machine against infinite bus analysis outlined in this paper has been used to explore a variety of conditions of a single machine with different parameters and loading conditions. For each of these conditions there are any number of functions which provide adequate damping simply by having the right range of magnitude and phase at the particular frequency of oscillation of concern.

By examining the phase and magnitude of these functions as related to the frequency of oscillation that was pertinent for the particular case, we arrive at bands of phase angles and magnitudes as functions of frequency of oscillation. It turns out that this band falls within ranges for which a general function with a rather well defined range of parameters serves reasonably well.

Heuristic reasoning permits casting the results of this single machine analysis to the general multimachine case. It is known that in a multimachine situation, there is a wide band of modes of oscillation ranging from those between tightly coupled light machines to those between groups of machines separated by long ties. It is reasonable then to expect that to be effective in the general power system environment, the stabilizing signal on any given machine accommodate this wide range of possibilities. Further, from a practical standpoint, it is essential that the range of adjustment not be critical since systems change from month to month as do operating conditions.

Range of Single Machine Cases Studied

The parameters K_1, K_2, K_3, K_4, K_5 and K_6 , defined in Appendix A, were calculated for a number of cases.

Two system configurations were studied. One was described by a single machine supplying an infinite bus through an external impedance, Fig. 11(a). Table I describes all the different combinations of parameters and loading explored with this system configuration.

Table I

Single Machine against Infinite Bus through External Reactance - Terminal Voltage = 1.0

Machine Type	Inertia H	External Reactance X_e	Machine Loading P + jQ
Hydro, Steam	1.5 and 5	0.1, 0.4, 0.7, 1.0	0.1+j0, 0.5+j0, 1+j0.5, 1-j0.5

The machine constants that were used are:

	Hydro	Steam
X_d	1.14	1.6
X'_d	0.24	0.32
X_q	0.66	1.55
T'_{do}	12	6

The second configuration attempting to simulate the condition of a system supplying local load connected to a large system through a weak tie is shown on Fig. 11(b).

Table II lists the different combinations of parameters explored for this configuration.

Table II

Machine Type	Inertia H	External Impedance		Machine Loading P + jQ
		X_E	R_E	
Steam Hydro	1.5, 5.0	1.0	1.0	1.0+j0, 1.5+j0.3, 1.5-j0.3, 0.5+j0.3, 0.5-j0.3
		5.0	1.0	1.0+j0, 1.2+j0.1, 1.2-j0.1, 0.8+j0.3, 0.8-j0.3

Guide Rules for Stabilizing Function Phase and Magnitude Requirements

As developed under the section titled, "Phase and Magnitude Relations", the lead phase angle of the signal G(S) at a particular oscillation frequency, ω , required to provide pure damping at that frequency would be equal to the phase lag of the expression

$$\frac{\Delta T}{\Delta \theta_{t \text{ ref}}} = \frac{K_2 K_e}{1/K_3 + K_e K_6 + S (T_e/K_3 + T'_{do}) + S^2 T'_{do} T_e} \quad (19)$$

The phase angle of expression (19) as well as the reciprocal of its magnitude, A, for the frequency

of oscillation $\omega = \sqrt{\frac{K_1 377}{M}}$ rads was calculated

for the various cases listed above using the expressions for the coefficients K_1 to K_6 in Appendix A.

As far as a criterion for the required magnitude of the signal is concerned, we could arbitrarily select a magnitude of this signal that would result in a damping ratio of 0.5.

Referring to the torque-angle characteristic equation

$$S^2 + D/M S + \frac{K_1 377}{M} = 0 \quad (1)$$

where D describes the gain of the damping term

$\frac{\Delta T_D}{p\delta}$, we note that $\omega_n = \sqrt{\frac{K_1 377}{M}}$ and $\zeta = \frac{1}{2} \sqrt{\frac{D}{K_1 M 377}}$.

For a constant damping ratio, say $\zeta = 0.5$, we have $D = \sqrt{377 K_1 M} = M \frac{K_1 377}{H} = M \omega_n$.

The magnitude of the signal that would be required would then be $A \sqrt{K_1 M 377}$ or $AM\omega_n$ where A is the reciprocal of the magnitude of the function

$$\frac{\Delta T}{\Delta e_{t \text{ ref}}} @ \omega = \frac{\sqrt{K_1 377}}{H}$$

The voltage regulator transient K_2 gain was taken at a good compromise value of $4 T_{do}'$.

Fig. 12 is a scatter plot of the desired magnitude $AM\omega_n$ of the signal versus frequency of oscillation ω as dictated by the above criterion. Points for cases with the system configuration of Fig. 11(a) are marked with xx, whereas those with the system configuration of Fig. 11(b) are marked with dots.

A composite plot of phase angle dictated by the above idealized criterion for all cases incorporating the two configurations is shown on Fig. 13. Again xx are points corresponding to cases with the configuration of Fig. 11(d), whereas dots correspond to those for the configuration of Fig. 12(b).

It is interesting to note the ranges of the scatter plots for the two different configurations. Most stability problems are characterized by oscillations at the low end of the spectrum and at this end, the stabilizing function needs little phase lead. This confirms the results of several studies which were concerned with long distance transmission problems.

On Figs. 12 and 13 we have superposed plots of magnitude and phase, respectively, for two stabilizing functions discussed in the next section.

Stabilizing Functions Studied - Analog Computer Results

From the general shape of the scatter plots of the ideal stabilizing function by the above criterion, and guided by the understanding of the many effects developed in previous sections, several stabilizing functions were tried on the system of Fig. 1 simulated on an analog computer.

These analog computer studies confirmed with time domain results the general findings of the frequency response analysis. In particular, these results were valuable in obtaining a calibration on how far from the ideal values of $AM\omega_n$ and ϕ the magnitude and phase angle of a stabilizing function could be without adverse effects.

The nature of the scatter plots shows that ideally the function should have little phase

angle in the frequency range of 1 to 2 rads/sec. and then have a rapidly rising phase angle as the frequency of oscillation increases to about 15 rads/sec. or so.

The magnitude and phase angle of the function

$$\frac{\Delta e_{t \text{ ref}}}{p\delta} = \frac{60S (1 + S/8 + S^2/64)}{(1 + 3S) (1 + S/20 + S^2/400)}$$

is shown on Figs. 12 and 13, respectively. The function in question is made up of a rate term with long washout of 3 sec. in series with a complex pair of zeros ($\omega_z = 8, \zeta = 0.5$) and a complex pair of poles ($\omega_p = 20, \zeta = 0.5$).

On Figs. 12 and 13 we also have plotted the magnitude and phase of the function

$$\frac{\Delta e_{t \text{ ref}}}{p\delta} = \frac{60S (1 + S/8)^2}{(1 + 3S) (1 + S/20)^2}$$

which is similar to the previous function except for a damping ratio $\zeta = 1.0$ which turns the zeros and poles to real values.

Figs. 14 and 15 are sample results of the analog study illustrating some significant aspects of the problem. The traces are deviations in speed for a small mechanical torque disturbance. Where pertinent, values of $AM\omega_n / \phi$, as ideally desired by the above criterion, have been listed and noted alongside are the actual phase and magnitude values of the stabilizing functions S / ϕ at frequency ω . Parameters K_1 through K_6 have also been listed.

Among these parameters describing machine dynamics, the most significant ones are

$\omega = \sqrt{\frac{K_1 377}{M}}$, K_5 and K_6 . ω indicates the fre-

quency of oscillation of concern. K_5 by its sign and magnitude indicates the extent to which the voltage regulator per se helps or hinders damping. Finally, K_6 , as explained in the section titled, "Voltage Regulator Effects on Machine Stability", influences the phase lag contributed by the voltage-regulator loop.

In Fig. 14(a) and (b) are shown the effects of stabilizing with various strengths of the complex function $\frac{KS (1 + S/8 + S^2/64)}{(1 + 3S) (1 + S/20 + S^2/400)}$

for the case of a fully loaded overexcited steam unit supplying a stiff system ($X_d = 0.1$). Cases are shown for inertia constants of $H = 1.5$ and $H = 5.0$. All pertinent constants are listed in the captions.

Fig. 14(c) and (d) contains similar results for the case where the stabilizing function is composed of real poles and zeros

$$\frac{KS (1 + S/8)^2}{(1 + 3S) (1 + S/20)^2}$$

Table III lists the values of ω , AM_s/ϕ and S/θ for the various cases of Fig. 14.

Table III

Case	ω	AM_s/ϕ	S/θ for $K = 60$
a	11.27	81.4 98°	50 86°
b	6.17	81.8 80.3°	18 44.5°
c	11.27	81.4 98°	50 52°
d	6.17	81.8 80.3°	29 44°

These values show that for the light inertia cases (a) and (c), the first of the stabilizing functions is more effective than the second, and this is confirmed by the analog computer results.

Fig. 15 contains results of similar nature for the configuration of local load with high tie reactance ($R_g = 1$, $X_g = 5$) with a relatively high tie loading of $0.2 + j0.1$.

The two stabilizing functions are almost equally effective. This rather extreme loading case shows that it is possible to stabilize with a voltage regulator and supplementary speed signal even though the constant flux linkages synchronizing coefficient K_1 is negative.

In order to illustrate the effect of voltage regulator gain on damping, we have included in Fig. 16(a) and 16(b) results of angle deviations for a step change in mechanical torque for the case where no stabilizing is used. Fig. 16(c) is for a case where the parameter K_5 is positive - leading to some benefit in damping with voltage regulator gain, whereas Fig. 16(b) shows the adverse effects of voltage regulator gain on damping for the case where K_5 is negative.

CONCLUDING REMARKS

The small perturbation stability characteristics of a single machine supplying an infinite bus through external impedance have been explored by means of frequency response analyses giving insights into effects of machine and system parameters, voltage regulator gain, and stabilizing functions derived from speed and working through the voltage reference of the voltage regulator.

The study has explored a variety of machine loadings, machine inertias, and system external impedances with a determination of the oscillation and damping characteristics of voltage or speed following a small disturbance in mechanical torque. An attempt has been made to develop some unifying concepts that explain the stability phenomena of concern, and to predict desirable phase and magnitude characteristics of stabilizing functions.

It is evident that for every combination of machine and system parameters, and loading conditions, there are a variety of stabilizing signal transfer functions operating on speed which give essentially the right phase and magnitude relation

for the frequency of concern. It is a challenge, however, to find a universal function which would be adequate for the whole spectrum of possibilities. By examining this spectrum of possibilities over a credible band of machine and system parameter values, as well as loading conditions, it appears that a fair compromise function can be recommended that will be almost universally applicable.

In the general case of a machine in a power system, there are many modes of oscillation between machines and groups of machines; these modes can change from day to day with loading conditions and from year to year with changes in power system configuration. Accordingly, the search for a universally applicable stabilizing function has a great deal of merit.

For thyristor type excitation systems characterized by a time constant in the neighborhood of or smaller than 0.05 sec., certain general conclusions and recommendations can be made to cover the majority of applications. These are:

1. A transient voltage regulator gain of about 25 p.u. $\Delta E_{fd}/p.u.$ Δe_t is a good compromise for a machine with an open circuit time constant $T_{d0} = 6$. An examination of the block diagram relations and the constraints set by voltage regulator loop damping on open circuit convinces us that the most direct way of compensating for differences in machine open circuit time constant is to vary the transient voltage regulator gain in direct proportion to this time constant.
2. The transition from steady-state gain to transient gain can be obtained with rate feedback from exciter voltage or with a lag-lead network $\frac{1 + T_2 S}{1 + T_1 S}$ in series with the regulator where $T_2/T_1 = \frac{\text{transient gain}}{\text{steady-state gain}}$ and T_2 should be larger than 1 sec.
3. A stabilizing function operating on speed and developing a transient offset of voltage regulator reference can be selected to be close to universally applicable for a wide range of machine and system parameters. This function could be of the form

$$\frac{\Delta e_t \text{ ref (p.u.)}}{p\delta \text{ (p.u.)}} = \frac{KS(1 + ST_1)^2}{(1 + ST)(1 + ST_2)^2}$$

where

$$T = 2 \text{ to } 4 \text{ sec.}$$

$$T_1 = 0.1 \text{ to } 0.2 \text{ sec.}$$

$$T_2 = 0.05 \text{ sec.}$$

$$\text{and } K/T = 10 \text{ to } 40$$

This function usually provides a significant amount of damping to the rotor angle oscillations. It should not be concluded, however, that other functions are not equally or better suited for particular cases, or that stabilizing is mandatory in all cases. Normal damping forces provided by load characteristics and amortisseurs are entirely adequate in many instances.

4. There appears to be some correlation between the strength of the signal required and the inertia of the machine. From examination of several sets of runs where the only difference was inertia, we conclude that the strength of the signal should vary approximately as the square root of inertia.
5. The need for stabilizing increases with increasing dependence on the excitation to provide synchronizing power through a high response excitation system.

These conclusions and recommendations have been checked out in detailed digital multimachine simulations. Among several problems being presently studied are the dynamics under large disturbances and design requirements of signal limiting. Also being studied are the relative merits of stabilizing with terminal frequency or with a signal derived from electrical power instead of speed.

NOMENCLATURE

All quantities in per unit on machine base.

- i_d, i_q - Armature current, direct and quadrature axis components
- e_d, e_q - Armature voltage, direct and quadrature axis components
- e_t - Terminal voltage
- E'_q - Voltage proportional to direct axis flux linkages
- E_{fd} - Generator field voltage (one per unit is the value for 1 per unit terminal voltage on the air gap line, open circuit)
- X_e, X_E - Equivalent system reactances
- R_E - System shunt resistance
- S - Laplace operator, or magnitude of stabilizing signal
- δ - Angle between quadrature axis and infinite bus
- $p\delta$ - Per unit speed deviation from synchronous
- T_e - Electrical torque
- H - Inertia constant, seconds
- M - Inertia coefficient = $2H$, seconds

D - Damping coefficient

E - Infinite bus voltage

Subscript o means steady-state value

Prefix Δ indicates small change

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APPENDIX A

For a machine connected to an infinite bus with voltage E through an external impedance $r_e + jX_e$, Fig. 11(a), assuming as negligible amortisseur effects, armature resistance, armature p ψ terms and saturation, the following relationships apply.

$$e_t^2 = e_d^2 + e_q^2$$

$$e_d = \psi_q = -X_q i_q$$

$$e_q = \psi_d = E'_q - X'_d i_d$$

$$E_q = E'_q + (X_q - X'_d) i_d$$

$$T_e = E_q i_q$$

$$i_d = \frac{[E_q - E \cos \delta]}{[r_e^2 + (X_e + X_q)^2]} \frac{[X_e + X_q]}{r_e} - E \sin \delta \frac{r_e}{[r_e^2 + (X_e + X_q)^2]}$$

$$i_q = \frac{[E_q - E \cos \delta]}{[r_e^2 + (X_e + X_q)^2]} \frac{r_e}{r_e} + E \sin \delta \frac{[X_e + X_q]}{[r_e^2 + (X_e + X_q)^2]}$$

$$E'_q = X_{ad} I_{fd} - (X_d - X'_d) i_d$$

$$T_{do} \frac{dE'_q}{dt} = E_{fd} - X_{ad} I_{fd}$$

$$T_m - T_e = M \frac{d(p\delta)}{dt}$$

Expressing all equations in small oscillation form and preserving the basic variables Δe_t , $\Delta E'_q$ and $\Delta \delta$ one can derive the following relations.

$$\Delta e_t = K_5 \Delta \delta + K_6 \Delta E'_q$$

$$\Delta E'_q = \frac{K_3 \Delta E_{fd}}{1 + ST_{do}' K_3} - \frac{K_3 K_4 \Delta \delta}{1 + ST_{do}' K_3}$$

$$\Delta T_e = K_1 \Delta \delta + K_2 \Delta E'_q$$

where

$$K_1 = \frac{E_{qo} E_o}{A} [r_e \sin \delta_o + (X_e + X'_d) \cos \delta_o] + \frac{i_{qo} E_o}{A} [(X_q - X'_d) (X_e + X_q) \sin \delta_o - r_e (X_q - X'_d) \cos \delta_o]$$

$$K_2 = \left[\frac{r_e E_{qo}}{A} + i_{qo} \left(1 + \frac{(X_e + X_q) (X_q - X'_d)}{A} \right) \right]$$

$$K_3 = \left[1 + \frac{(X_e + X_q) (X_d - X'_d)}{A} \right]^{-1}$$

$$K_4 = \frac{E_o (X_d - X'_d)}{A} [(X_e + X_q) \sin \delta_o - r_e \cos \delta_o]$$

$$K_5 = \frac{e_{do}}{e_{to}} X_q \left(\frac{r_e E_o \sin \delta_o + (X_e + X'_d) E_o \cos \delta_o}{A} \right) + \frac{e_{qo}}{e_{to}} X'_d \left(\frac{r_e E_o \cos \delta_o - (X_e + X_q) E_o \sin \delta_o}{A} \right)$$

$$K_6 = \frac{e_{qo}}{e_{to}} \left(1 - \frac{X'_d (X_e + X_q)}{A} \right) + \frac{e_{do}}{e_{to}} X_q \frac{r_e}{A}$$

$$A = [r_e^2 + (X_e + X'_d) (X_q + X_e)]$$

The steady-state operating values of δ_o , E_{qo} , E_o , e_{do} and e_{qo} are derived from the standard machine vector diagram. Expressed as function of steady-state terminal voltage e_{to} , and steady-state real and reactive load currents I_{po} and I_{qo} , these values are

$$E_{qo} = \sqrt{(e_{to} + I_{qo} X_q)^2 + (I_{po} X_q)^2}$$

$$E_o = \sqrt{(e_{to} - I_{po} r_e - I_{qo} X_e)^2 + (I_{po} X_e - I_{qo} r_e)^2}$$

$$\sin \delta_o = \frac{e_{to} I_{po} (X_q + X_e) - r_e X_q (I_{po}^2 + I_{qo}^2)}{E_{qo} E_o}$$

$$\cos \delta_o = \frac{e_{to} (e_{to} - I_{qo} (X_q - X_e) - I_{po} r_e)}{E_{qo} E_o}$$

$$- \frac{X_e X_q (I_{po}^2 + I_{qo}^2)}{E_{qo} E_o}$$

$$i_{q0} = \frac{I_{d0}(e_{to} + I_q X_q) - I_{q0} I_{d0} X_q}{E_{q0}}$$

$$i_{d0} = \frac{I_{d0}^2 X_q + I_{q0}(e_{to} + I_{q0} X_q)}{E_{q0}}$$

$$e_{q0} = \left(\frac{e_{to} + I_{q0} X_q}{E_{q0}} \right) e_{to}$$

$$e_{d0} = i_{q0} X_q$$

For the special case of zero external resistance, the expressions become:

$$K_1 = \frac{X_q - X'_d}{X_e + X'_d} i_{q0} E_0 \sin \delta_0 + \frac{E_{q0} E_0 \cos \delta_0}{X_e + X_q}$$

$$K_2 = \frac{E_0 \sin \delta_0}{X_e + X'_d}$$

$$K_3 = \frac{X'_d + X_e}{X_d + X_e}$$

$$K_4 = \frac{X_d - X'_d}{X_e + X'_d} E_0 \sin \delta_0 = \frac{1}{K_3} \frac{\Delta E_0}{E_0}$$

$$K_5 = \frac{X_q}{X_e + X_q} \frac{e_{d0}}{e_{to}} E_0 \cos \delta_0 - \frac{X'_d}{X_e + X'_d} \frac{e_{q0}}{e_{to}} E_0 \sin \delta_0$$

$$K_6 = \frac{X_e}{X_e + X'_d} \frac{e_{q0}}{e_{to}}$$

The configuration of Fig. 11(b) is converted to the form of Fig. 11(a) by making a thevenin equivalent where

$$r_e = \frac{R_E X_E^2}{R_E^2 + X_E^2}$$

and

$$X_e = \frac{R_E^2 X_E}{R_E^2 + X_E^2}$$

$$E_{q0} = \frac{X_d - X'_d}{X_e + X'_d} E_0 \sin \delta_0$$

$$E_{q0} = \frac{X_d - X'_d}{X_e + X'_d} E_0 \sin \delta_0$$

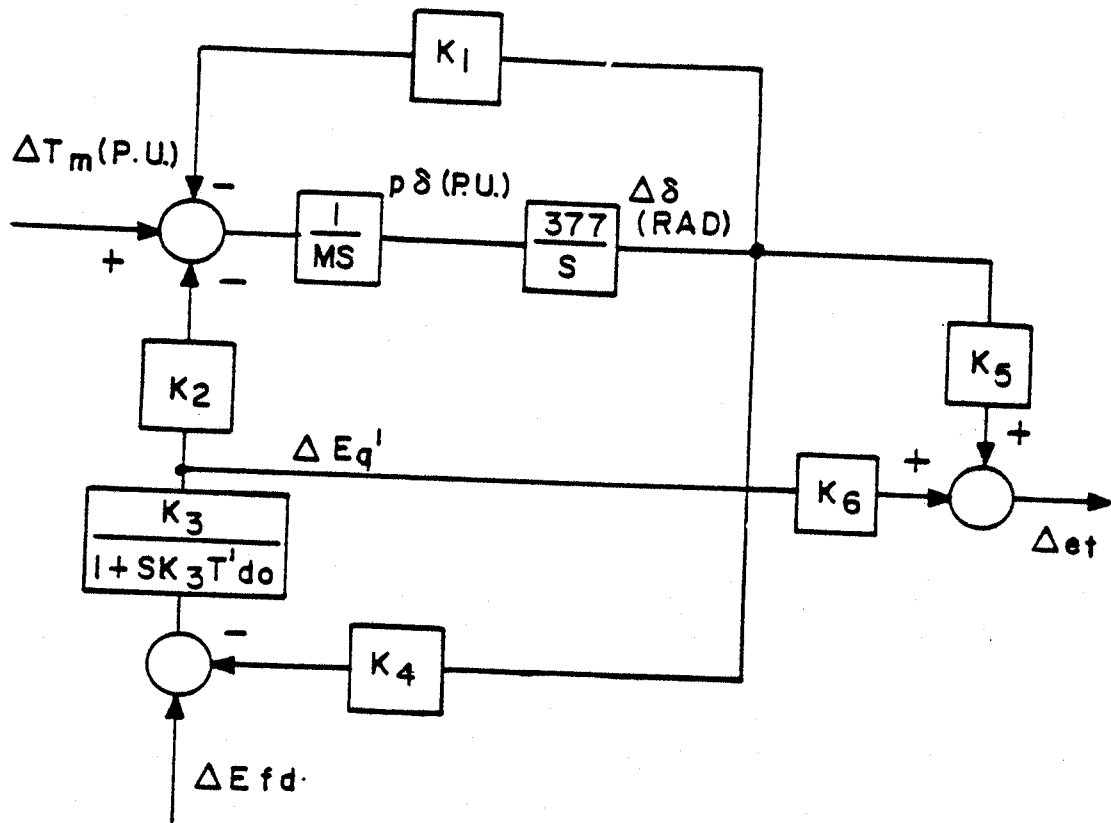


Fig. 1. Block diagram showing linearized small perturbation relations of a single generator supplying an infinite bus through external impedance.

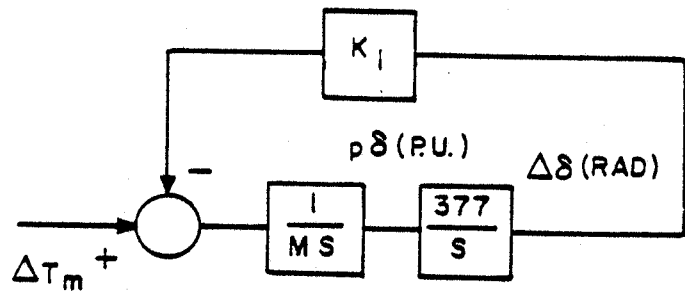


Fig. 2. Block diagram showing linearized torque-angle relationships for the condition of constant flux linkages in the d-axis.

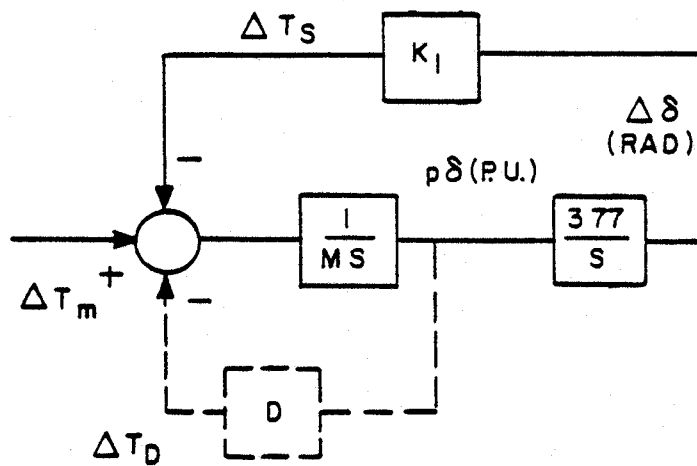


Fig. 3. Block diagram of torque-angle loop defining synchronizing torques ΔT_s and damping torques ΔT_D .

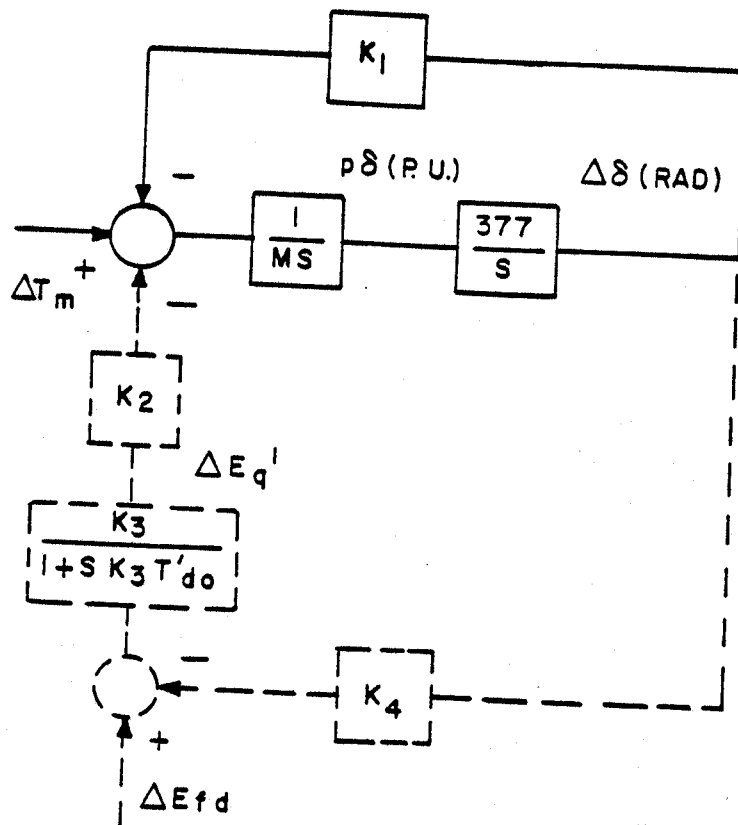


Fig. 4. Block diagram of torque-angle loop including direct axis field effects.

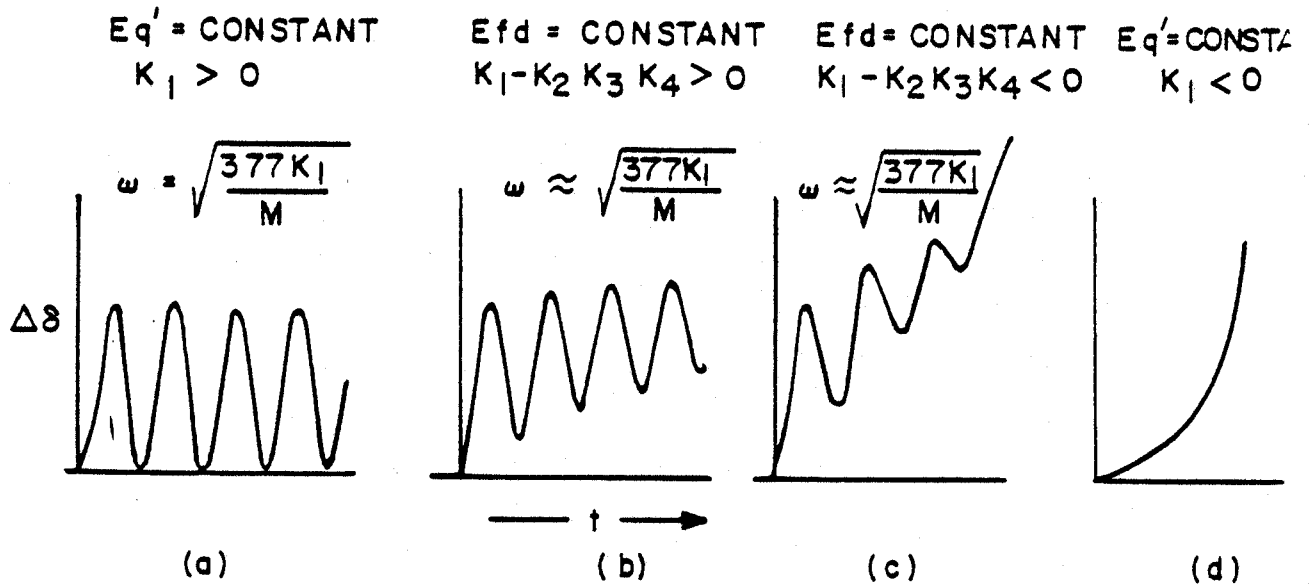


Fig. 5. Typical responses of rotor angle following small step increases in mechanical torque.

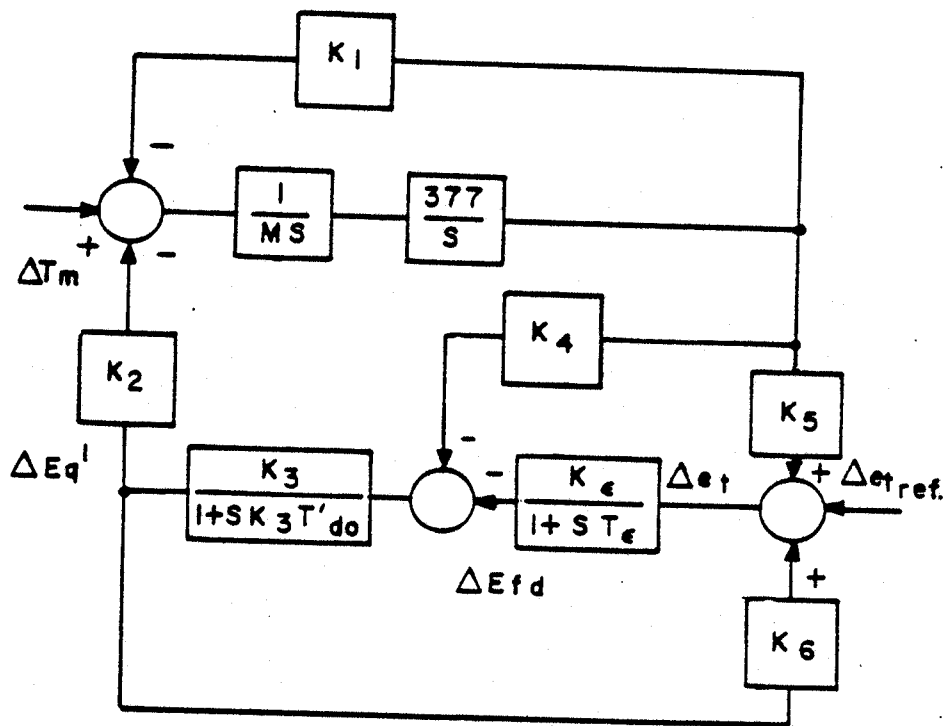


Fig. 6. Block diagram of single machine supplying an infinite bus through external impedance including effects of voltage regulator-excitation system.

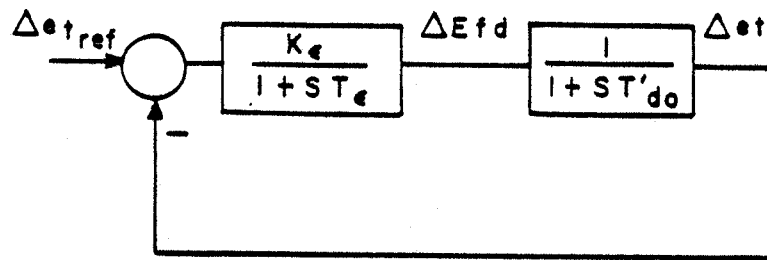


Fig. 7. Block diagram of voltage regulating loop with machine on open circuit.

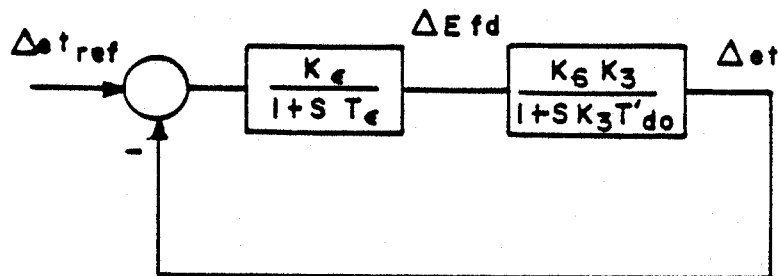


Fig. 8. Block diagram of voltage regulating loop with machine connected to system assuming no changes in rotor angle.

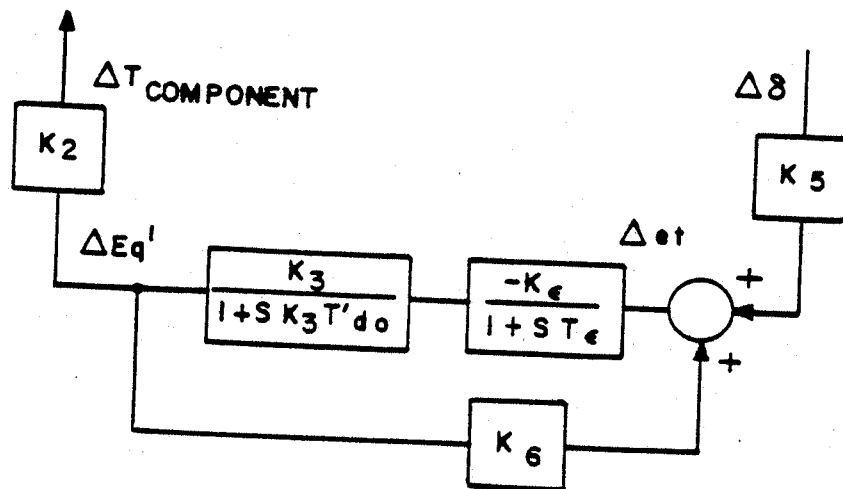


Fig. 9. Block diagram describing the component of torque produced by voltage regulator action in response to voltage deviations produced by rotor angle deviations.

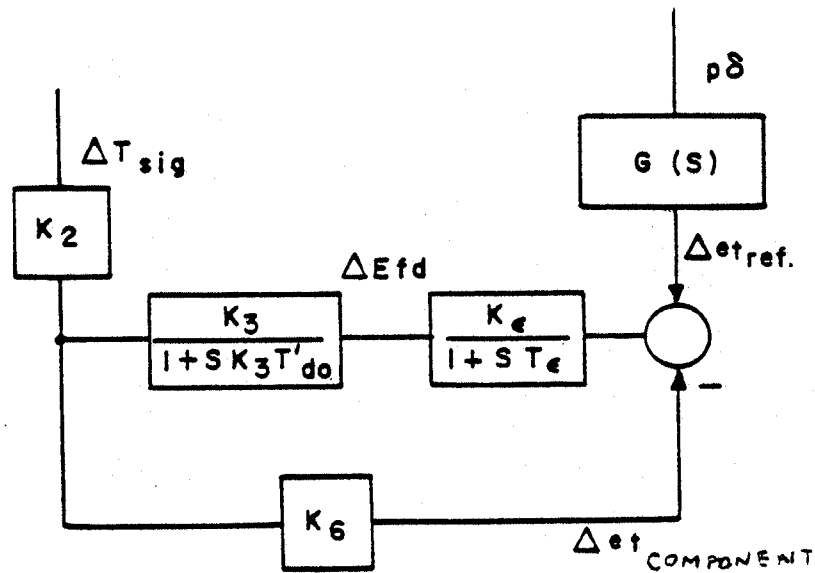
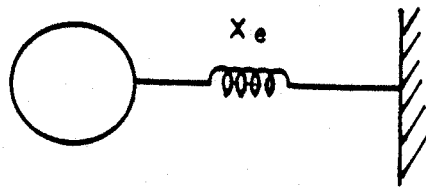
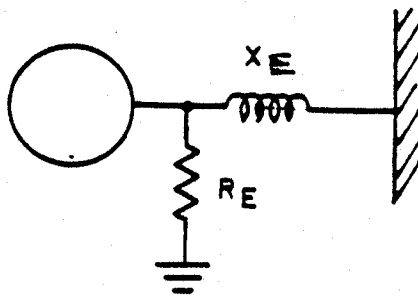


Fig. 10. Block diagram describing the component of torque produced by voltage regulator action in response to a speed derived signal.



(a)



(b)

Fig. 11. External impedance system configurations for single machine against infinite bus.

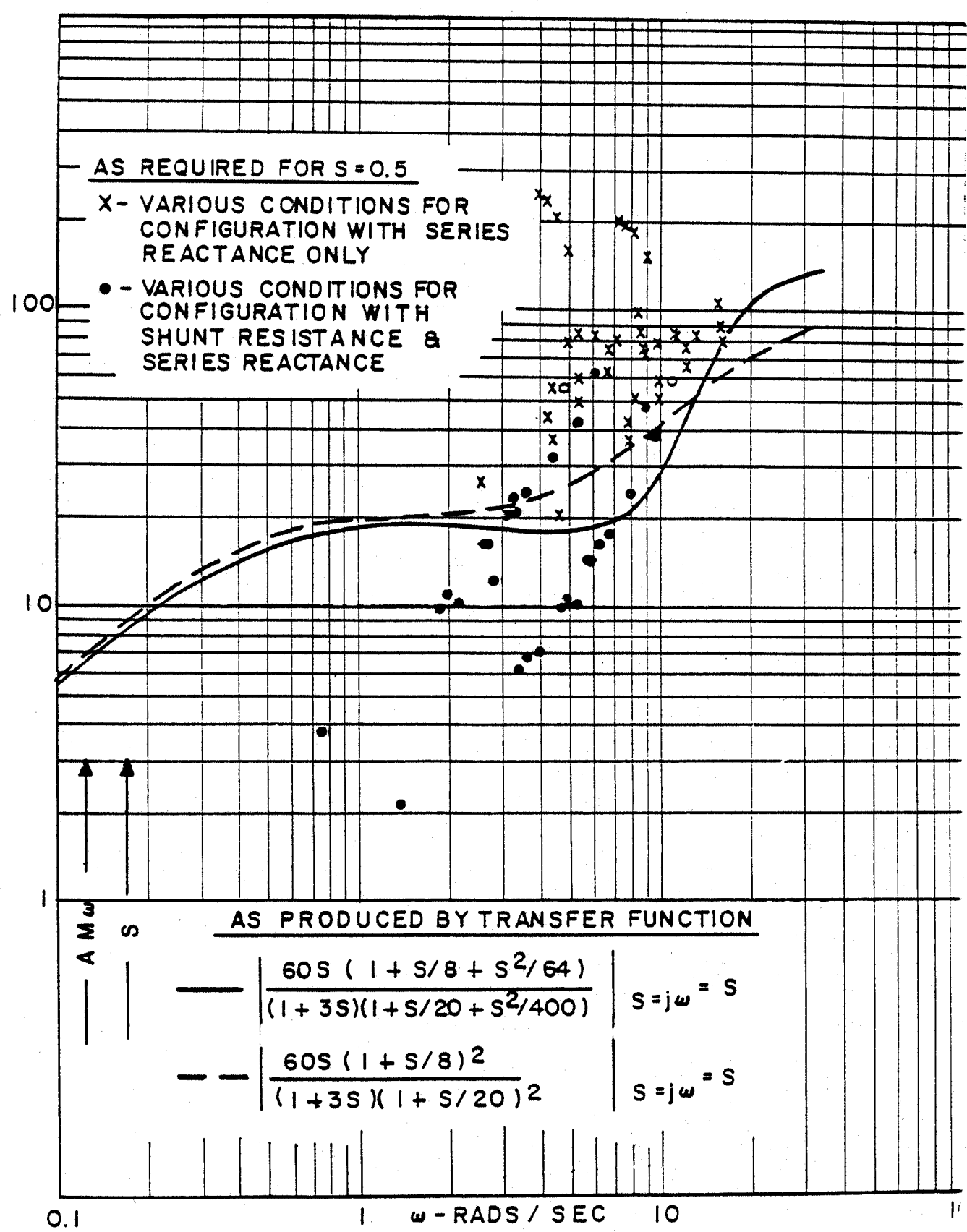


Fig. 12. Plot of magnitude of stabilizing function vs. frequency of oscillation. Scatter points are values of $AM\omega$ for a variety of steam machine-system parameters and loading conditions as listed in tables I and II.

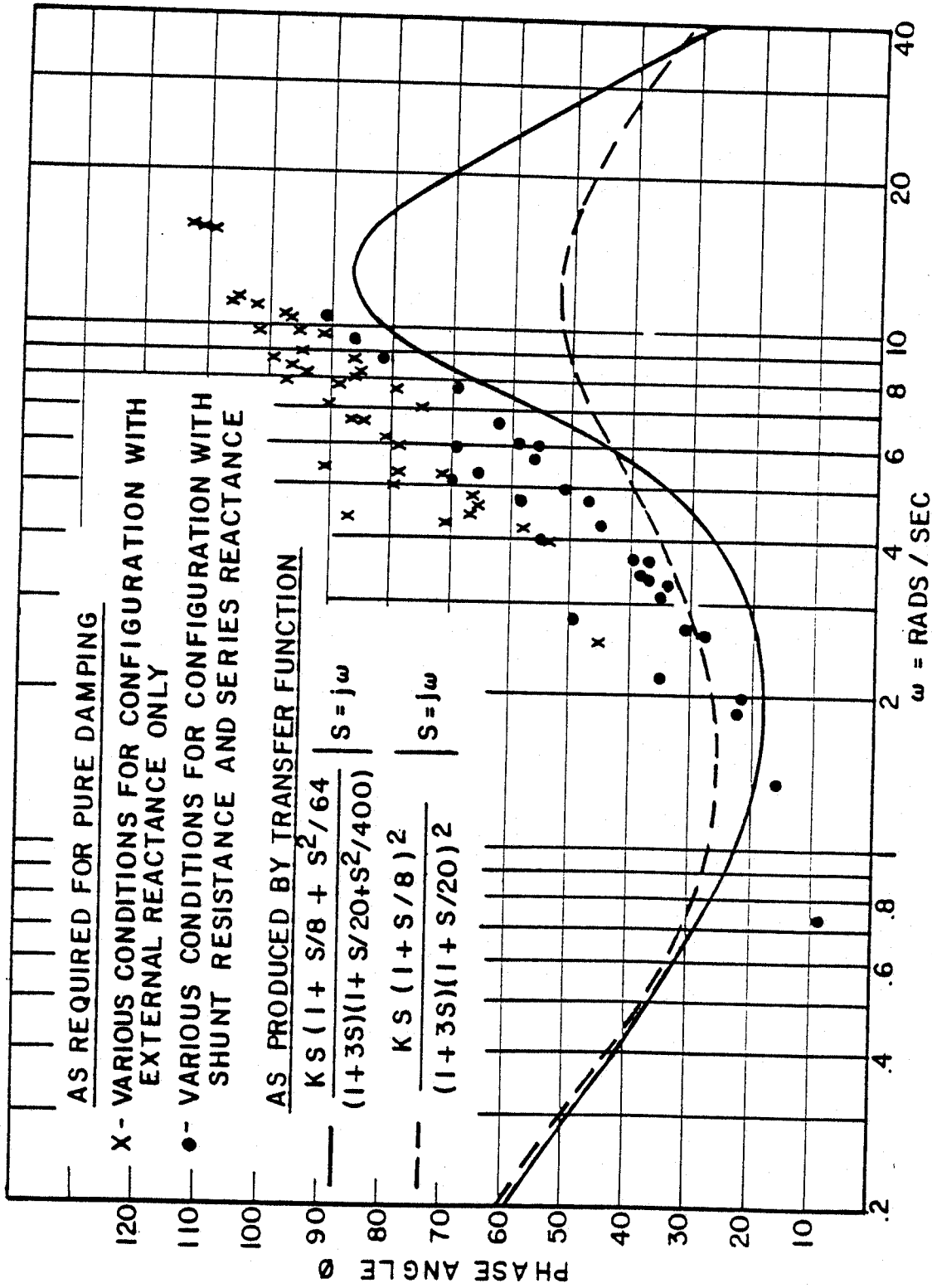
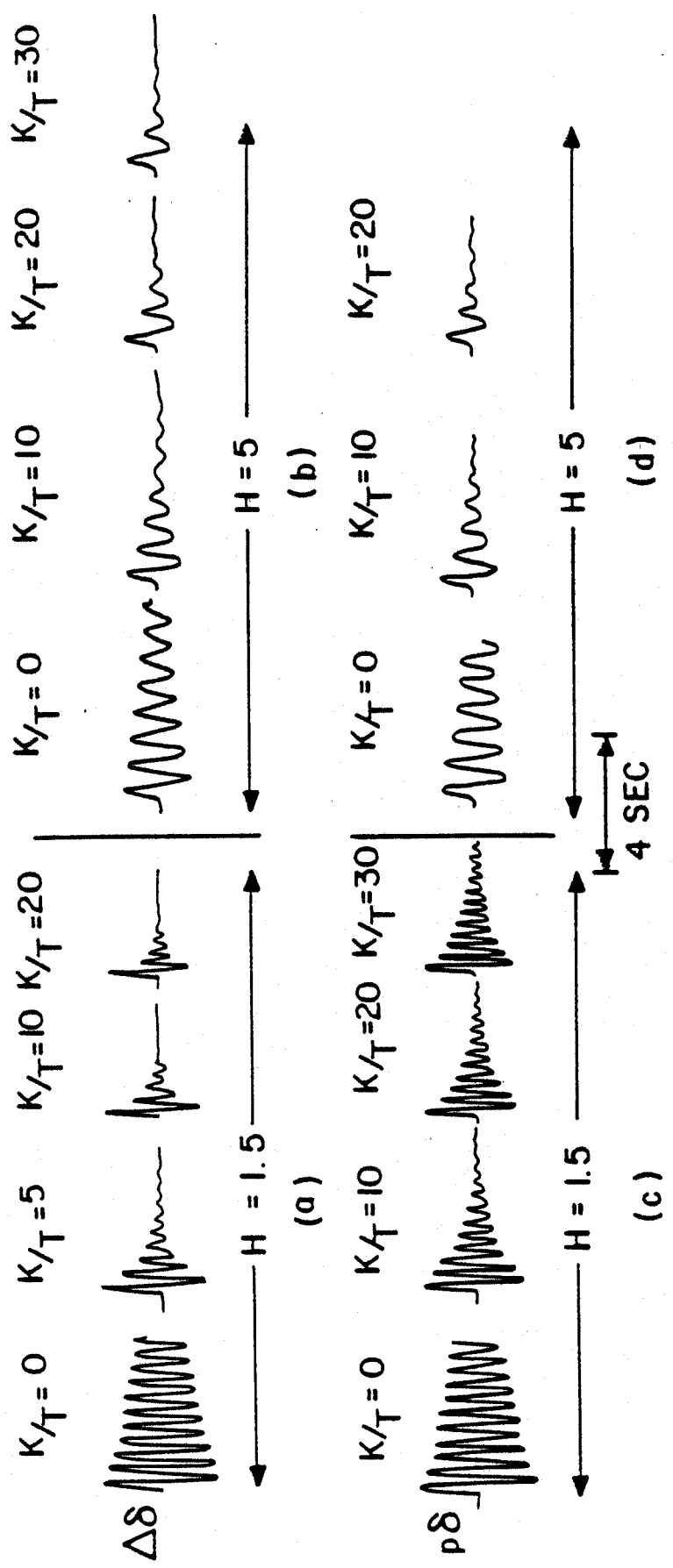


Fig. 13. Plot of phase angle ϕ vs. frequency of oscillation ω . Scatter plot points are phase angles required to cancel the lagging phase angles of the machine function, $\frac{\Delta E_t}{\Delta E_t \text{ ref}}$ for a variety of steam machine-system parameters and loading conditions as listed in tables I and II.



Stabilizing function

$$\frac{KS}{(1+TS)} \frac{(1+S/g+S^2/64)}{(1+S/20+S^2/400)}$$

for cases a and b

$$\frac{KS}{(1+TS)} \frac{(1+S/g)^2}{(1+S/20+S^2/400)^2}$$

for cases c and d

Fig. 14. Speed deviations of a steam unit following a small step decrease in mechanical torque.

Effect of stabilizing with speed derived signal. Conditions: Regulator exciter gain $K_e = 25$, $T_e = 0.05$ sec.

X_0	$P+jQ$	K_1	K_2
0.1	$1.0+j0.5$	1.01	$1.1j9$
K_3	K_4	K_5	K_6
2.78	1.47	-0.097	0.419

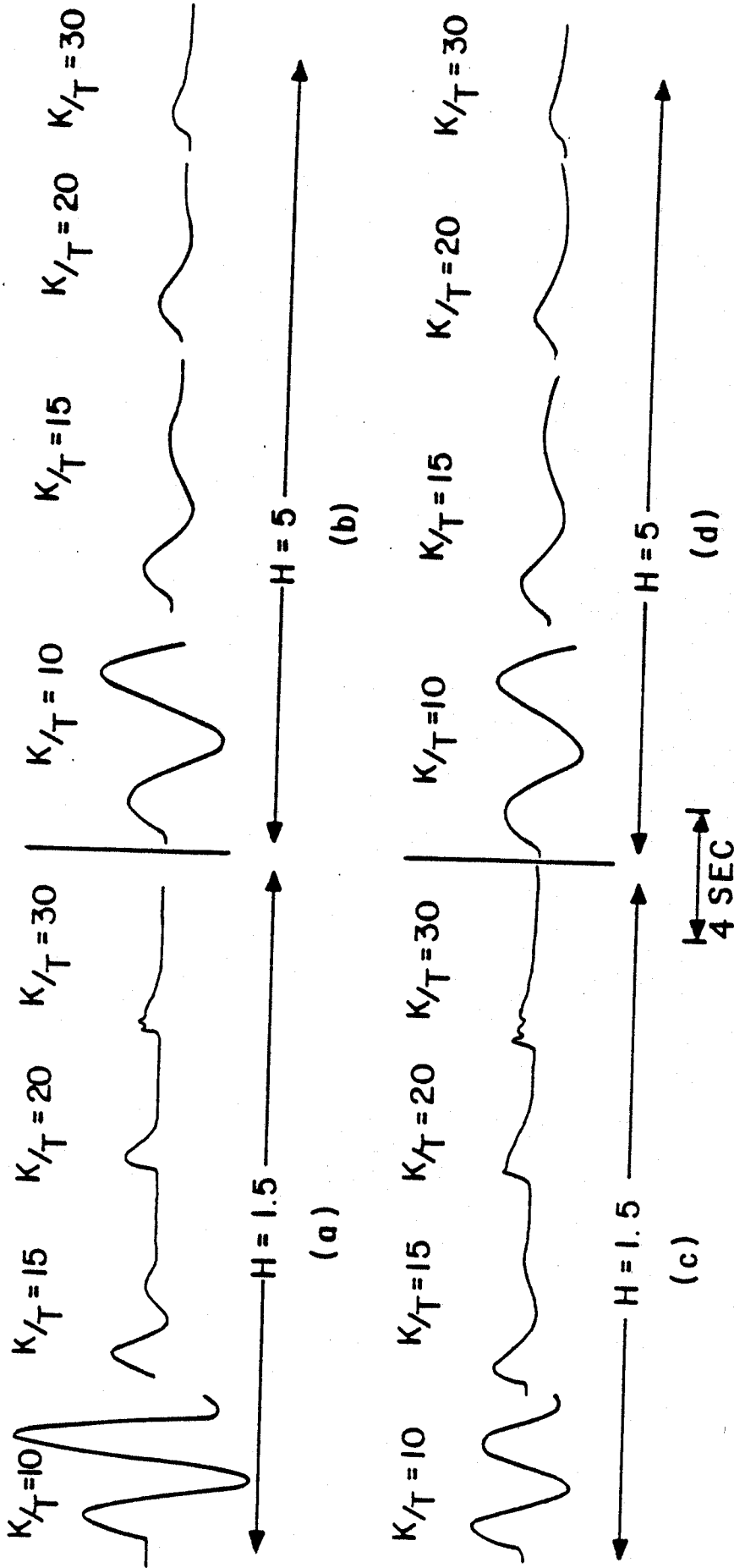


Fig. 15. Speed deviation of a steam unit following a small step decrease in mechanical torque.

Effect of stabilizing with speed derived signal. Conditions: Regulator exciter gain $K_c = 25$ $T_e = 0.05$ sec.

R_E	X_E	$P+jQ$	K_1
1.0	5.0	$1.2+j0.1$	-0.191
K_2	K_3	K_4	K_5
2.35	2.227	0.046	-0.136
			K_6
			1.062

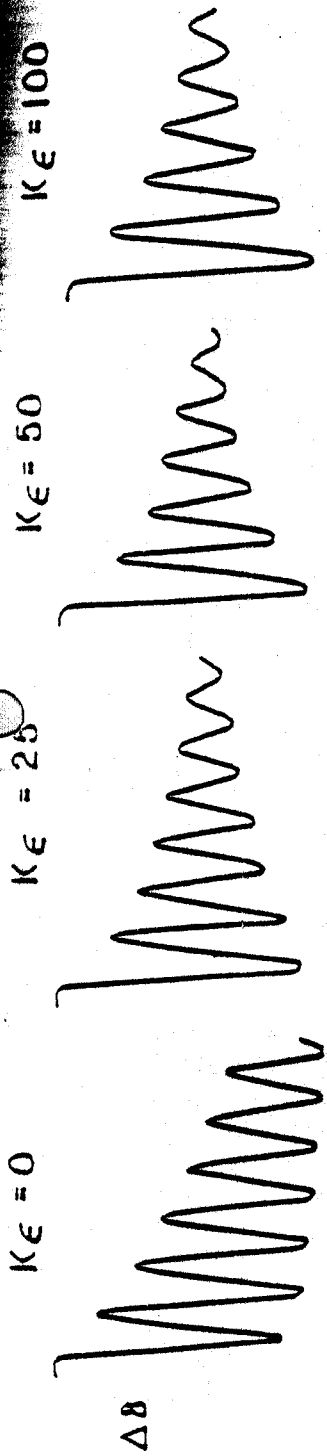
H	Stabilizing function
(a) 1.5	$\left\{ \frac{KS}{(1+TS)} \frac{(1+S/8+S^2/64)}{(1+S/20+S^2/400)} \right\}$
(b) 5.0	$\left\{ \frac{KS}{(1+TS)} \frac{(1+S/8+S^2/64)}{(1+S/20+S^2/400)} \right\}$

$T = 3$ sec.

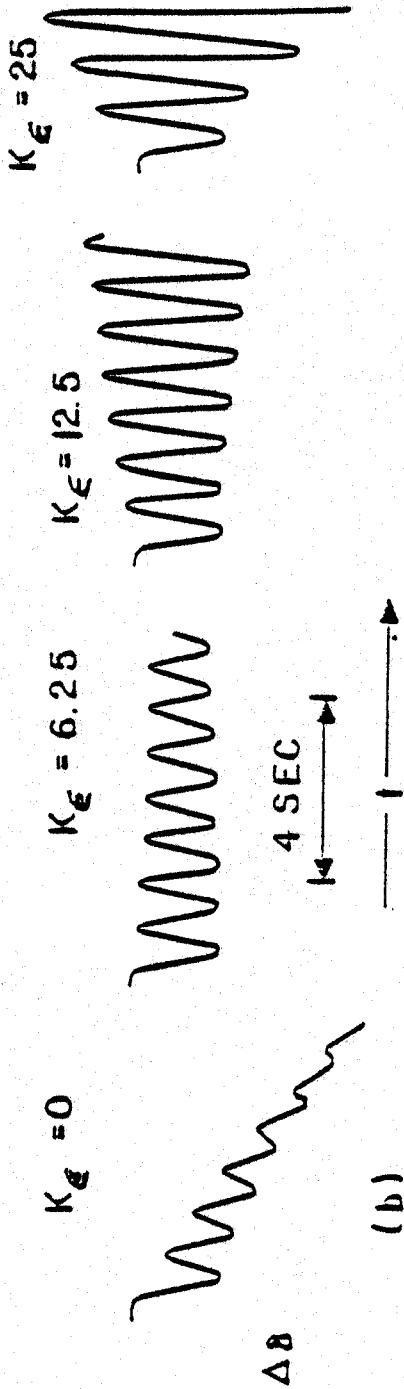
(c) 1.5	$\left\{ \frac{KS}{(1+TS)} \frac{(1+S/8)^2}{(1+S/20)^2} \right\}$
(d) 5.0	$\left\{ \frac{KS}{(1+TS)} \frac{(1+S/8)^2}{(1+S/20)^2} \right\}$

$T = 3$ sec.

Configuration of Fig. 11(b)



(a)



(b)

Fig. 16. Angle deviations of a steam unit following a small step decrease in mechanical torque.

Effect of voltage regulator-exciter gain K_E at two different loading conditions.

$\frac{Y}{a}$	$P+JQ$	$\frac{K_1}{1}$	$\frac{K_2}{1}$
0.4	$0.5+J0.5$	5	1.05
		0.954	1.07
K_3^{-1}	$\frac{K_4}{4}$	$\frac{K_5}{5}$	$\frac{K_6}{6}$
2.78	1.37	0.0544	0.439

$\frac{Y}{a}$	$P+JQ$	$\frac{K_1}{1}$	$\frac{K_2}{1}$
0.4	$1.0-J0.5$	5	1.10
		1.73	
K_3^{-1}	$\frac{K_4}{4}$	$\frac{K_5}{5}$	$\frac{K_6}{6}$
2.78	2.21	-0.755	0.080
		0.913	