Classical Model

1.0 Assumptions

In these notes, we cover Sections 2.5, 2.5.1 (but not, yet, 2.5.2, 2.6), and 2.7.

Your text, section 2.5, considers a very simplified representation of a synchronous machine, given in Fig. 1.

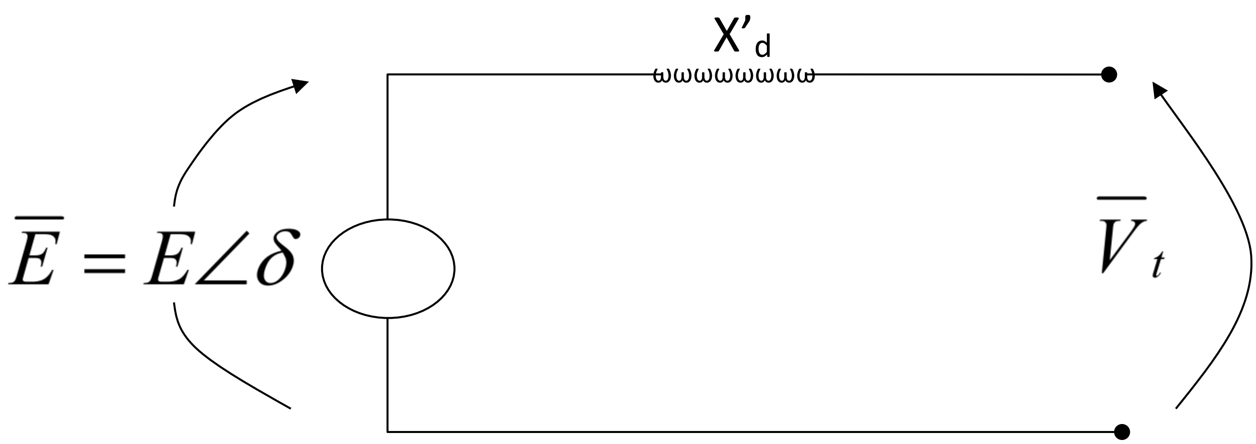


Fig. 1

In this model, $\overline{E}$ is the internal machine voltage and $\overline{V}_t$ is the machine terminal voltage. Both of these quantities are phasors. We could assign the terminal voltage phasor $\overline{V}_t$ as the reference, i.e., $\overline{V}_t = V_t\angle 0$, but in what follows, we will find it convenient to do otherwise.

This is the electrical basis of the so-called classical machine model. This model is based on the following assumptions:

1. $P_{au}=T_{au}$ (we neglect the error introduced by the power form of the swing equation, due to the fact that $\omega_m \neq \omega_R$).
2. P_m, the mechanical power into the machine, is constant. Therefore we consider that the governor is blocked, and that the effect on mechanical power for the non-regulated machine that we saw last time is too small for the kind of frequency changes of interest to us.
3. There is no damping internal to the machine representation.
4. Regarding voltages, consistent with Fig. 6b in the notes called "SwingEquation," we have that $\overline{E}$ leads $\overline{V}_t$ by the same angle that $\boldsymbol{\varphi}_f$ leads $\boldsymbol{\varphi}_r$, which is (in electrical degrees), the torque angle δ.
5. The internal voltage magnitude, $\left|\overline{E}\right|$, is constant, therefore we neglect the action of the excitation system.

6. The difference between internal voltage and terminal voltage is represented by a drop across a reactance given by X'_d, the reactance of the *direct axis* under transient conditions.

We will consider the case when the synchronous generator is connected to an *infinite bus*. An infinite bus is a bus in which the voltage magnitude, voltage angle, and bus frequency are constant, and it is unaffected by the amount of current drawn from it (or delivered to it).

Infinite buses do not really exist in a power system, but buses having machines with very large inertias connected to them tend to behave similar to an infinite bus under some kind of disturbances. For example, if we were studying the stability behavior of a 10 MVA machine, and there was another 1000 MVA machine close by, it would not be unreasonable to model the 1000 MVA machine as an infinite bus.

A one-line diagram of a synchronous generator connected to an infinite bus through a circuit of impedance Z_{TL} is provided in Fig. 2. The impedance Z_{TL} represents the sum of the step-up transformer impedance and a line impedance.

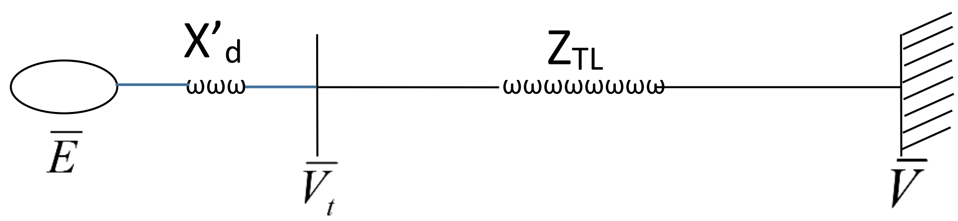


Fig. 2

We will assume that the synchronous machine of Fig. 2 has some station load which we can model as a constant impedance Z_S. The circuit diagram corresponding to Fig. 2 becomes, then, as in Fig. 3.

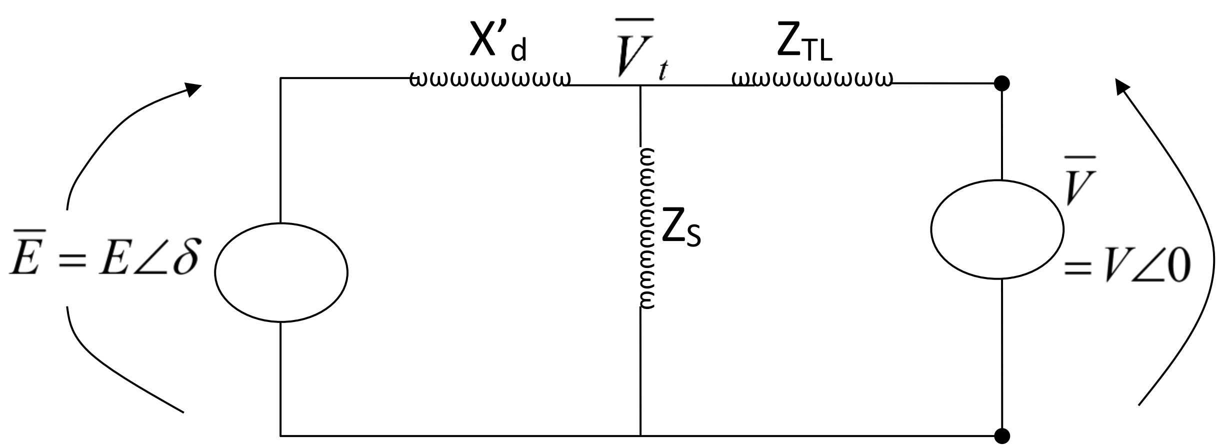


Fig. 3

Note that the voltage $\overline{V}$ of Fig. 3 is assumed to be the reference.

We can convert the circuit of Fig. 3 into a two-node network which will be more convenient to analyze. This can be done (for this simple case but not in general) using a Wye to Delta (Y-Δ) transformation.

Consider the networks of Fig. 4, where the one on the left has the same topology as the network of Fig. 3.

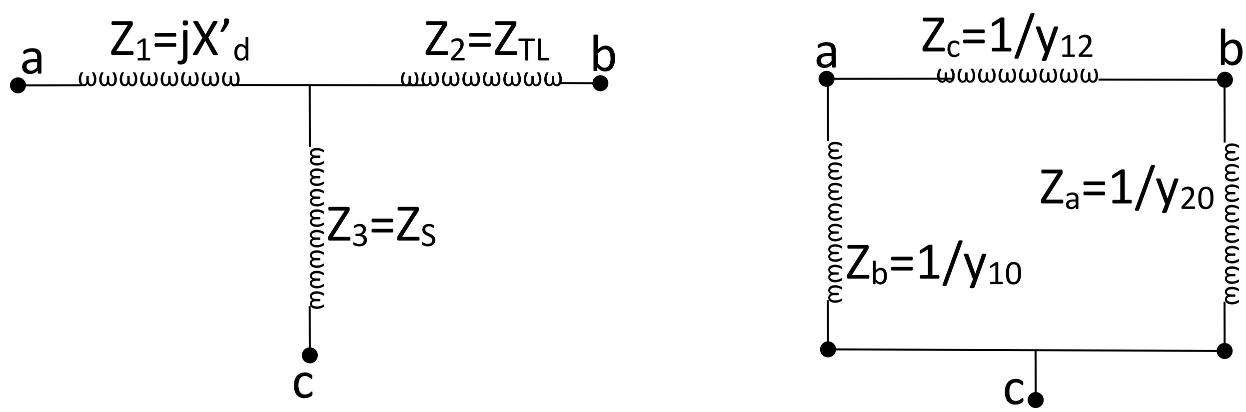


Fig. 4

If the Fig. 4 networks are equivalent, then the impedance seen between any two terminals, with the third terminal open, must be the same. This fact provides 3 equations:

$$a \rightarrow b: Z_1 + Z_2 = Z_c // (Z_a + Z_b)$$

The symbol "//" means "in parallel with".

$$b \rightarrow c: Z_2 + Z_3 = Z_a // (Z_b + Z_c)$$

$$a \rightarrow c: Z_1 + Z_3 = Z_b // (Z_a + Z_c)$$

Solving these equations for Z_a, Z_b, and Z_c results in

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

$$Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}$$

The values of our network, as indicated in Fig. 4, are $Z_1=jX'_d$, $Z_2=Z_{TL}$, and $Z_3=Z_S$. Substituting these into the above equations, and taking the inverse to obtain admittances, results in

$$Z_a = \frac{jX'_d Z_{TL} + Z_{TL} Z_S + jX'_d Z_S}{jX'_d} \Rightarrow y_{20} = \frac{jX'_d}{jX'_d Z_{TL} + Z_{TL} Z_S + jX'_d Z_S}$$

$$Z_b = \frac{jX'_d Z_{TL} + Z_{TL} Z_S + jX'_d Z_S}{Z_{TL}} \Rightarrow y_{10} = \frac{Z_{TL}}{jX'_d Z_{TL} + Z_{TL} Z_S + jX'_d Z_S}$$

$$Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_S} \Rightarrow y_{12} = \frac{Z_S}{jX'_d Z_{TL} + Z_{TL} Z_S + jX'_d Z_S}$$

With the admittance expressions, we may construct the Y-bus using the usual rules for Y-bus construction:

1. The matrix is symmetric, i.e., $Y_{ij}=Y_{ji}$.

2. A diagonal element Y_{ii} is obtained as the sum of admittances for all branches connected to bus *i*, including the shunt branch, i.e., where we emphasize that y_{ik} is non-zero only when there exists a physical connection between buses *i* and *k*.

3. The off-diagonal elements are the negative of the admittances connecting buses *i* and *j*, i.e., $Y_{ij}=-y_{ji}$.

These observations enable us to formulate the admittance matrix very quickly from the network based on visual inspection. We write down the Y-bus for the network on the right-hand-side of Fig. 4, repeated here for convenience in Fig. 5. Remember that node c is the ground node.

$$\underline{Y} = \begin{bmatrix} y_{12}+y_{10} & -y_{12} \\ -y_{12} & y_{12}+y_{20} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

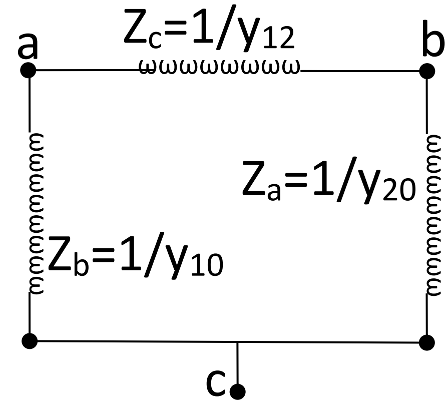


Fig. 5

In the previous equation, we introduced uppercase nomenclature, Y_{11}, Y_{12}, Y_{21}, and Y_{22} for Y-bus elements, which differ from the lower case nomenclature used for admittances y_{10}, y_{12}, and y_{20}. Both uppercase and lowercase values are complex.

Let's now insert the circuit obtained from the Y-Δ transformation, Fig. 5, into the system of Fig. 3, as shown in Fig. 6. The current injections have also been denoted in Fig. 6.

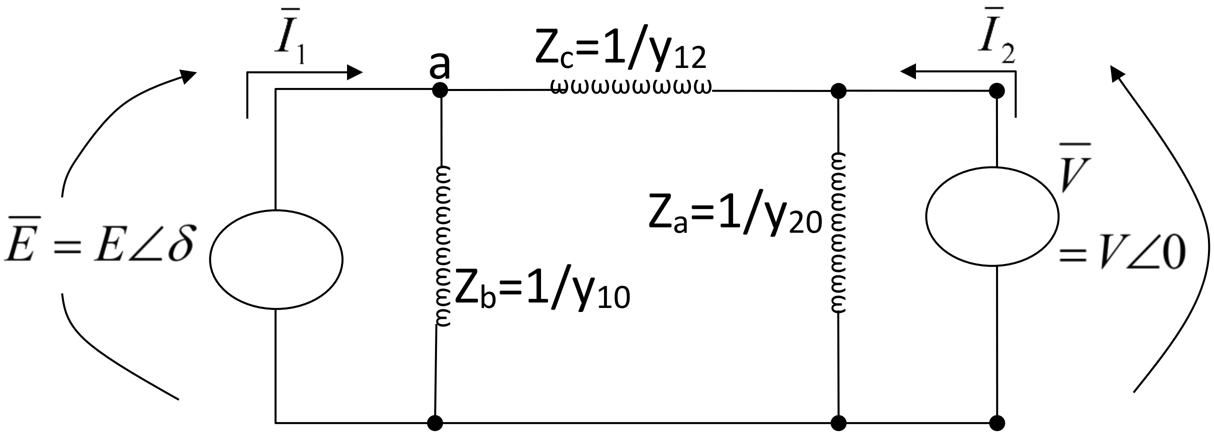


Fig. 6

The Y-bus relates the nodal current injections to the nodal voltages, according to eq. 2.40 in your text, repeated here:

$$\begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \overline{E} \\ \overline{V} \end{bmatrix} \tag{1}$$

Now recall that in per-unit, the power injected into the network from the source on the left of Fig. 6 is expressed as

$$S_1 = P_1 + jQ_1 = \overline{E}\bar{I}_1^* \tag{2}$$

However, by (1), we can express the current injected as

$$\bar{I}_1 = Y_{11}\overline{E} + Y_{12}\overline{V} \tag{3}$$

Substitution of (3) into (2) results in

$$
\begin{aligned}
S_1 = P_1 + jQ_1 = \overline{E}\left(Y_{11}\overline{E} + Y_{12}\overline{V}\right)^* \\
= \overline{E}Y_{11}^*\overline{E}^* + \overline{E}Y_{12}^*\overline{V}^* \\
= \left|E\right|^2 Y_{11}^* + \overline{E}\,\overline{V}^* Y_{12}^*
\end{aligned}
\tag{4}
$$

Now let

$$
\begin{aligned}
Y_{11} &= G_{11} + jB_{11} \\
Y_{12} &= \left|Y_{12}\right| \angle \theta_{12}
\end{aligned}
\tag{5}
$$

Substituting (5) into (4) results in

$$
S_1 = P_1 + jQ_1 = \left|E\right|^2 \left(G_{11} - jB_{11}\right) + \overline{E}\,\overline{V}^* \left|Y_{12}\right| \angle -\theta_{12} \tag{6}
$$

Replacing the voltages in the second term with their polar representation, we get

$$
\begin{aligned}
S_1 &= P_1 + jQ_1 \\
&= \left|E\right|^2 \left(G_{11} - jB_{11}\right) + \left|E\right|\angle\delta \left|V\right|\angle 0 \left|Y_{12}\right|\angle -\theta_{12} \\
&= \left|E\right|^2 G_{11} - j\left|E\right|^2 B_{11} + \left|E\right|\left|V\right|\left|Y_{12}\right|\angle\left(\delta - \theta_{12}\right)
\end{aligned}
\tag{7}
$$

P_1 is the real part of the expression in (7), and Q_1 is the imaginary part. The swing equation needs only the real part, and so concentrating on that, we get:

$$P_1 = |E|^2 G_{11} + |E||V||Y_{12}|\cos(\delta - \theta_{12}) \tag{8}$$

Now recall that cos(x)=sin(x+π/2). Using this in (8):

$$P_1 = |E|^2 G_{11} + |E||V||Y_{12}|\sin(\delta - \theta_{12} + \pi/2) \tag{9}$$

Define $\gamma=\theta_{12}-\pi/2$, and substituting into (9), we obtain:

$$P_1 = |E|^2 G_{11} + |E||V||Y_{12}|\sin(\delta - \gamma) \tag{10}$$

Defining $P_C=|E|^2G_{11}$ and $P_M=|E||V||Y_{12}|$, (10) becomes

$$P_1 = P_C + P_M \sin(\delta - \gamma) \tag{11}$$

This function is plotted in your text as Fig. 2.9, and it appears as in Fig. 7 below.

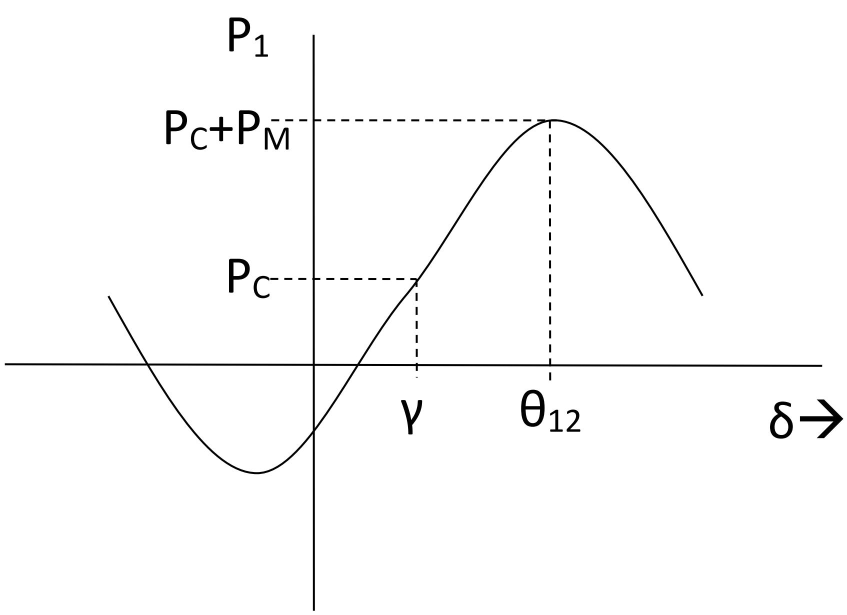


Fig. 7

Two things to note about Fig. 7:

- The generator must supply both the transferred power, $P_M\sin(\delta-\gamma)$, and the power in the station load and dissipated in the network, P_C. (Observe that P_C depends on G_{11}, where $Y_{11}=G_{11}+jB_{11}$ is the Y-bus element in row 1, column 1 of the Y-bus, given by $Y_{11}=y_{10}+y_{12}$; here, y_{10} represents the station load and y_{12} represents the network).

- Based on (10), the maximum power generation occurs at $\delta-\gamma=\pi/2$. But we defined (previous page) $\gamma=\theta_{12}-\pi/2$; substitution yields $\delta-\theta_{12}+\pi/2=\pi/2 \Rightarrow \delta=\theta_{12}$, as indicated in Fig. 7.

Example 2.3 (pre-fault condition):

This example is worked in the text but I want to clarify parts of it. The system appears in Fig. 2.10 of your text and Fig. 8 below. There is no station load, i.e., in Fig. 3, $Z_S=0$.

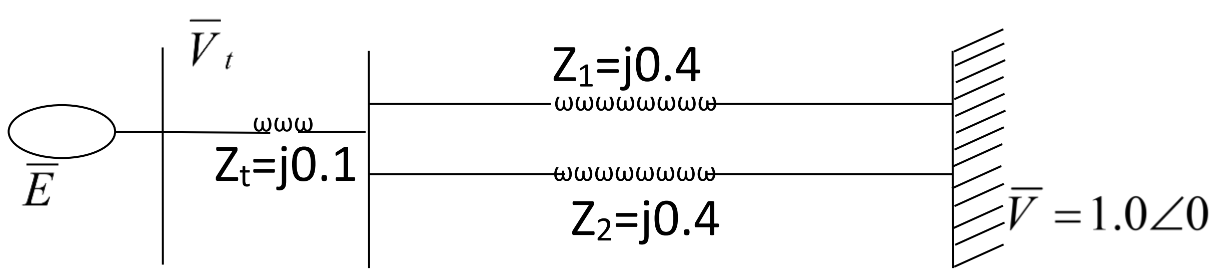


Fig. 8

The problem states that H=5 seconds, X'_d=0.20 pu, the machine is delivering 0.8 pu power at a terminal voltage of $|V_t|$=1.05 pu, all data is given on the machine power base, and **all resistances are neglected**. It is desired to obtain the equation of motion for the machine rotor. **No disturbance is specified, and so we want to determine the swing equation for the given topology**. Doing so is useful for obtaining the "pre-fault" condition and for studying the machine for a "small" network disturbance.

Let's understand better what this problem is asking us to do. We are trying to express the swing equation below in a way that it can actually be solved.

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t) = P_{mu} - P_{eu} \tag{12}$$

We know the power delivered is 0.8 pu. Since we assume no losses in the generator, this means $P_{mu}=0.8$ pu, and based on classical model assumptions, we assume this mechanical power into the machine remains constant.

What we do not know at this point, and what we need to determine, is an expression for the electrical power out of the machine during a small disturbance condition. And this expression needs to be written in terms of δ as the unknown if we are to have any hope of solving (12). So our initial goal is to express P_{eu} as a function of δ.

To solve this problem, one must first understand that Fig. 8 omits the transient reactance, X'_d. We show how this is included in Fig. 9.

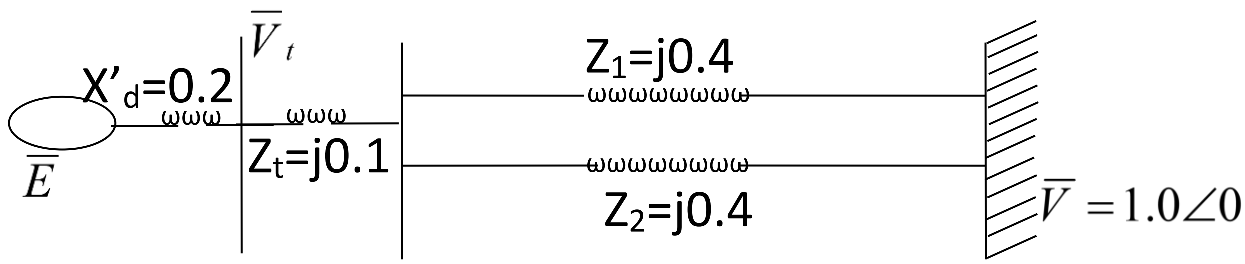


Fig. 9

We can get an equivalent impedance between the internal voltage and the infinite bus according to

$$Z_{12} = jX'_d + Z_t + \left(Z_1 // Z_2\right) \tag{13}$$

where the notation "//" means "paralleled with." Substituting the numerical values results in Z_{12}=j0.5. This means that y_{12}=-j2.0. We can then construct the Y-bus:

$$\underline{Y} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j2.0 & j2.0 \\ j2.0 & -j2.0 \end{bmatrix} \tag{14}$$

We found in (10) and (11) that

$$P_1 = |E|^2 G_{11} + |E||V||Y_{12}|\sin(\delta - \gamma) \tag{10}$$

$$P_1 = P_C + P_M \sin(\delta - \gamma) \tag{11}$$

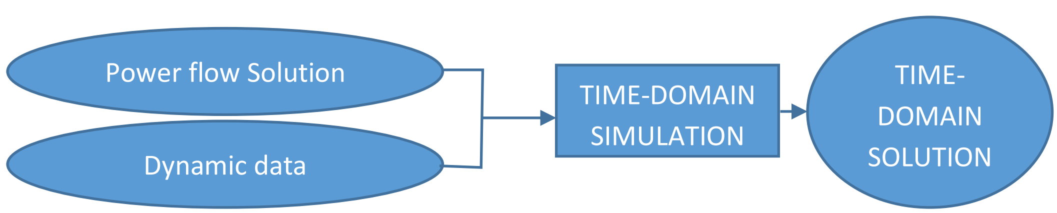
where $P_C=|E|^2G_{11}$ and $P_M=|E||V||Y_{12}|$. Noting that our system is lossless, $G_{11}=0$ implies $P_C=0$, and $\theta_{12}=\angle Y_{12}=\angle j2.0=\pi/2$. Therefore, $\gamma=\theta_{12}-\pi/2=0$. So (11) becomes:

$$P_1 = P_M \sin\delta \tag{15a}$$

where $P_M=|E||V||Y_{12}|$. We do know that $|V|=1.0$ and $|Y_{12}|=2.0$. Therefore

$$P_1 = 2|E|\sin\delta \tag{16a}$$

One problem we are faced with at this point is that we do not know $|E|$. So what do we do? The below figure shows standard setup to solve time-domain simulations.



This (not knowing internal machine conditions) is a quite typical situation in stability analysis, even for large-scale multi-machine models. The situation is this: We know terminal conditions of each machine at t=0 because a time-domain simulation is always initiated from a steady-state solution. We usually call this steady-state solution the power flow. Remember, a power flow solution tells us

- the complex power out of the machine terminals, and
- the voltage and angle at the machine terminals.

To perform time-domain simulation of the system, beginning from the initial conditions at the machine terminals provided by a power flow program, we will always need to "back-calculate" from the terminal conditions in order to determine conditions necessary to initialize the machine model internal dynamics (which must be represented for time-domain simulation but not for power flow).

Thus, an important step for time-domain simulation is always to perform this "back calculation." For machine models we will study later in the course, this effort has some complexities. For the classical machine model, the effort is straightforward. Let’s do it.

The strategy for performing the “back calculation” is fundamentally the same, however, independent of the machine model used. The power flow solution tells us the power flow and voltage magnitude and angle at the terminals. We use this information to get the current out of the machine. Then we “back calculate” using the current and machine internal impedances to obtain machine internal voltages.

In this case, however, the terminal conditions we know are power out and voltage magnitude. But we do not know the machine terminal voltage angle which we do need. The reason we do not know this is because we have not done a power flow calculation. So we need to do that. In this case, it is simple to obtain by inspecting the circuit, as given in Fig. 10, which is a circuit diagram for the one-line of Fig. 9.

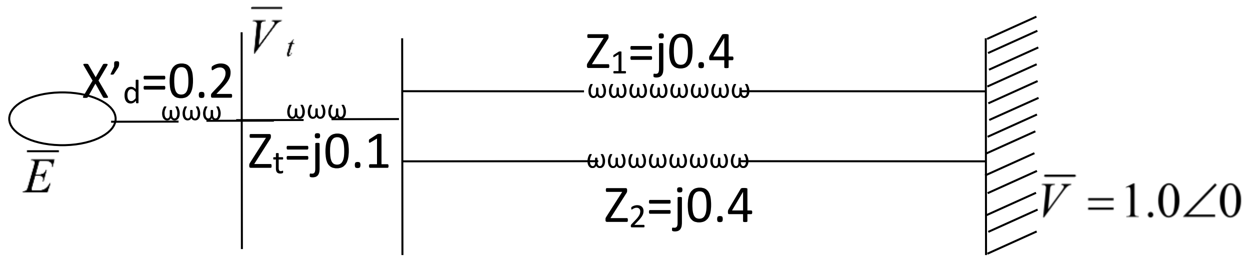


Fig. 9

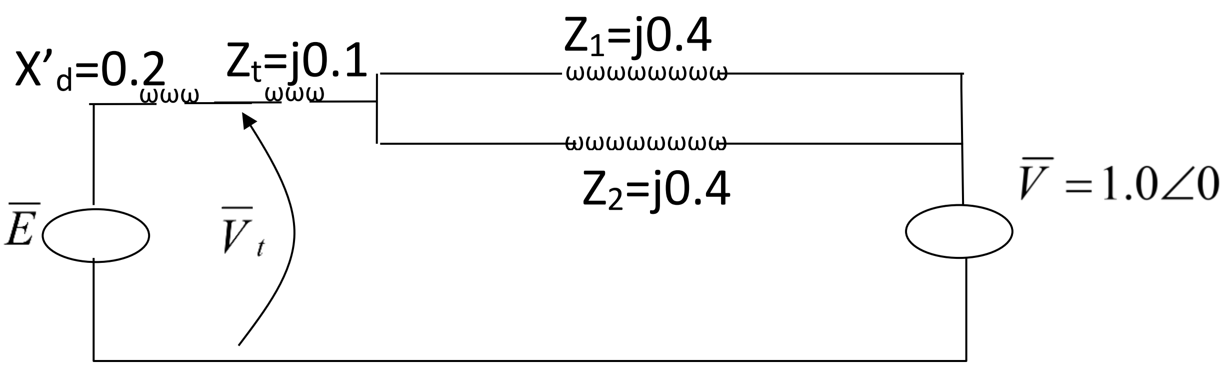


Fig. 10

We want to find $\overline{E}$ and we could do so if we knew the current. We can find the current if we can identify magnitude and angle for two different voltages. We know magnitude and angle at the infinite bus, and we know terminal voltage magnitude, so we need to obtain terminal voltage angle.

This we can do by recognizing that our equation (15a) also applies to the portion of the circuit between terminal voltage and infinite bus (there is no reason why it should not since our circuit is identical to the circuit used to derive (15a) - a voltage at either end of a lossless line).

$$P_1 = P_M \sin\theta_t \tag{15b}$$

where we notice that we replaced the angle with θ_t (the angle of V_t) since it is the angle across the impedances over which we compute the power flow. P_M is given by:

$$P_M=|V_t||V|/(X_t+X_1//X_2)=(1.05)(1)/(0.1+0.2)=3.5 \tag{16b}$$

We know the electrical power out is 0.8 pu, therefore (15b) becomes:

$$0.8 = 3.5\sin\theta_t \tag{17}$$

From (17), we solve to obtain

$$\sin\theta_t = 0.8/3.5 = 0.2286 \tag{18}$$

And so θ_t=0.2306 radians or 13.2147°.

Now we can obtain the current, which will be

$$I = \frac{\overline{V}_t - \overline{V}}{j(X_t + X_1 // X_2)} = \frac{1.05\angle 13.2147 - 1\angle 0}{j0.3}$$
$$= 0.8 - j0.074 = 0.8034\angle -5.285^\circ \tag{19}$$

Now that we have the current, we can compute the internal voltage according to

$$\overline{E} = \overline{V}_t + jX'_d\,\overline{I} = 1.05\angle 13.2147 + j0.2(0.8034\angle -5.285^\circ)$$
$$= 1.037 + j0.4 = 1.1115\angle 21.09^\circ \tag{20}$$

Now we may obtain the equation (16a):

$$P_1 = 2\,|\,E\,|\sin\delta \tag{16a}$$

as

$$P_1 = 2 * 1.1115\sin\delta = 2.223\sin\delta \tag{16c}$$

Now inserting into (12)

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t) = P_{mu} - P_{eu} \tag{12}$$

and recognizing that P_{eu} is the same as P_1, we have

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t) = P_{mu} - 2.223\sin\delta \tag{21}$$

With H=5, ω_{Re}=377, and P_{mu}=0.8, we have:

$$\frac{10}{377}\ddot{\delta}(t) = 0.8 - 2.223\sin\delta \tag{22}$$

And pushing the constant 10/377 to the right-hand-side,

$$\ddot{\delta}(t) = 30.16 - 83.8071\sin\delta \tag{23}$$

<u>Example 2.4 (fault-on condition)</u>:

Here, we want to obtain the equation of motion as in Example 2.3, but we want to do so for a faulted condition, where the fault is a balanced three-phase fault having impedance to ground of j0.1 pu, occurring on one of the transmission lines very close to the sending end.

The situation is illustrated in Fig. 11.

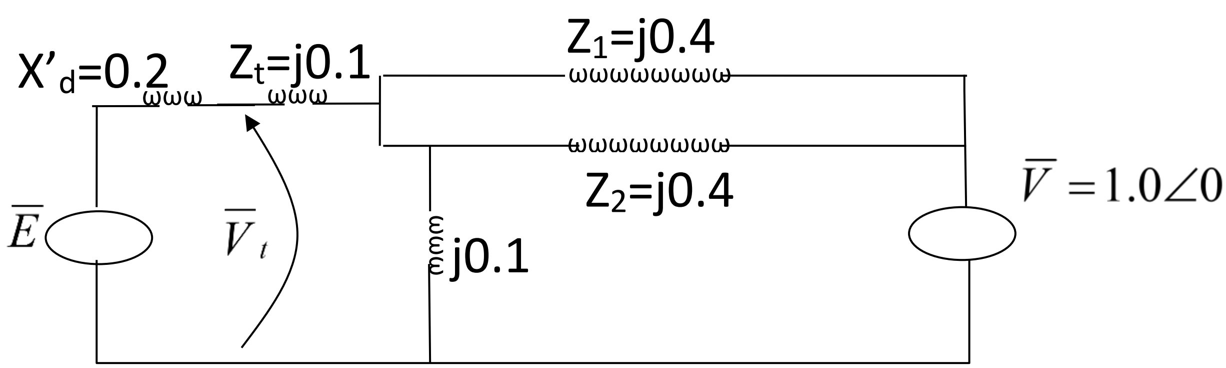


Fig. 11

This circuit may be redrawn in a simpler fashion as in Fig. 12.

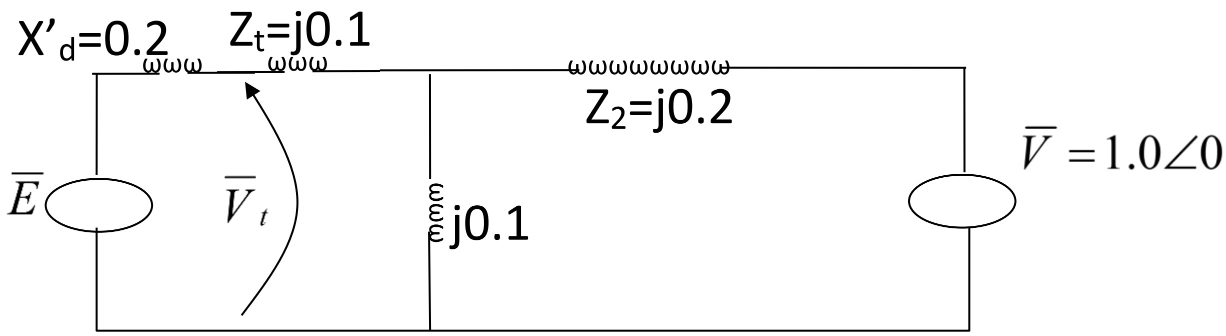


Fig. 12

We again use a Y-Δ transformation on this. The two equivalent networks are given in Fig. 13.

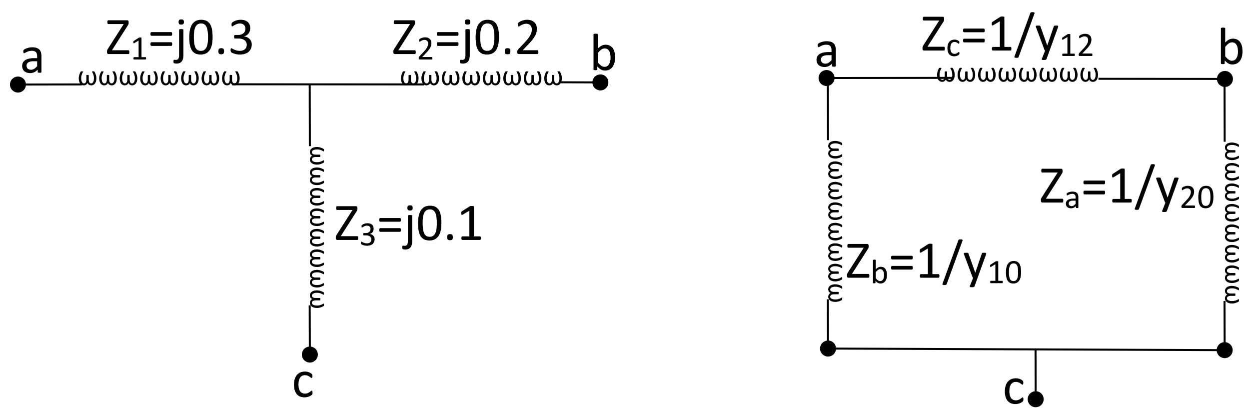


Fig. 13

From the relations at the top of page 6, we can write:

$$y_{20} = \frac{1}{Z_a} = \frac{Z_1}{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}$$

$$= \frac{j0.3}{j0.3 * j0.2 + j0.2 * j0.1 + j0.3 * j0.1} = \frac{j0.3}{-0.11} = -j2.7273$$

$$y_{10} = \frac{1}{Z_b} = \frac{Z_2}{-0.11} = \frac{j0.2}{-0.11} = -j1.8182$$

$$y_{12} = \frac{1}{Z_c} = \frac{Z_3}{-0.11} = \frac{j0.1}{-0.11} = -j0.9091$$

The equivalent circuit is shown in Fig. 14.

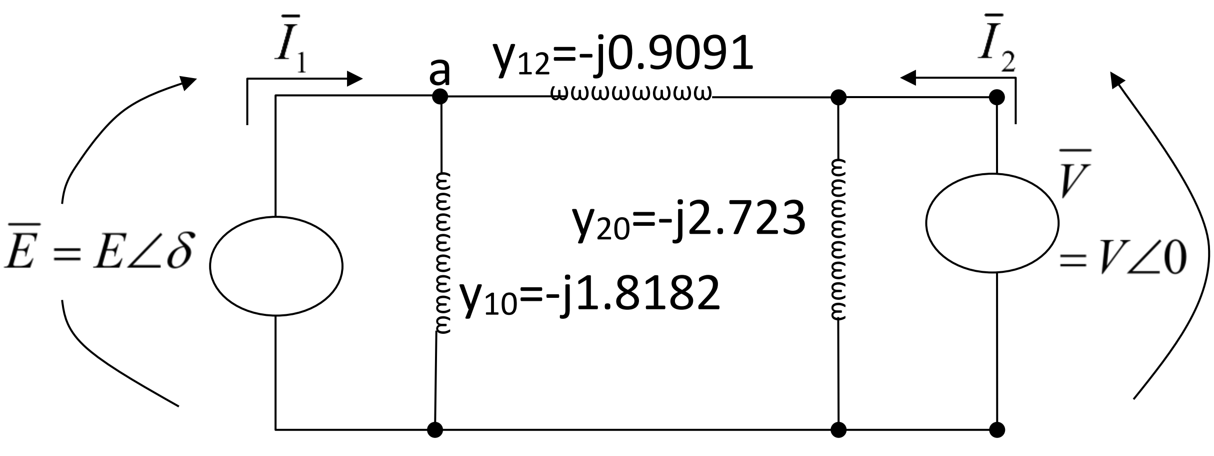


Fig. 14

Assuming the internal voltage magnitude remains constant at the value we found in the last example (an assumption inherent to the classical machine), which is

$|E|=1.1115$, and with $y_{12}=-j.9091 \Rightarrow Y_{12}=j.9091$, we have that

$$P_1 = P_M \sin\delta \tag{24}$$

where P_M is given by:

$$P_M=|E||V|Y_{12}|=(1.1115)(1)(.9091)=1.0105 \tag{25}$$

So (24) becomes

$$P_1 = 1.0105 \sin\delta \tag{26}$$

Plugging back into (12)

$$\frac{2H}{\omega_{\mathrm{Re}}}\ddot{\delta}(t) = P_{mu} - P_{eu} \tag{12}$$

(where again, $P_{eu}=P_1$), we have

$$\frac{10}{377}\ddot{\delta}(t) = 0.8 - 1.0105 \sin\delta \tag{27}$$

Or

$$\ddot{\delta}(t) = 30.16 - 38.0958 \sin\delta \tag{28}$$

Post-fault condition (also in Example 2.4):

Finally, it is of interest to consider what happens when the fault is cleared. In this case, the j0.1 impedance to ground is eliminated; so is the faulted line, as illustrated in Fig. 15.

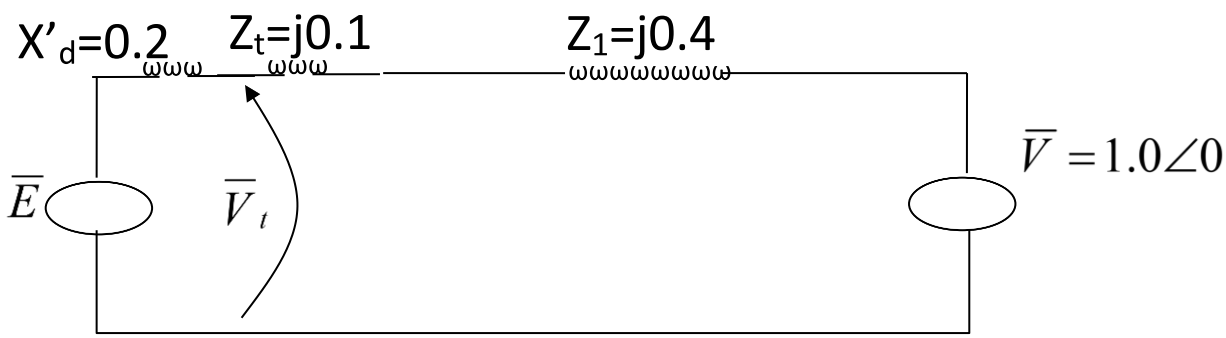


Fig. 15

Now the impedance between the voltages is j0.7, and the transfer admittance is Y_{12}=-j1/0.7=-j1.4286, therefore

$$P_M=|E||V|Y_{12}|=(1.1115)(1)(1.4286)=1.5879 \qquad (29)$$

The swing equation for the post-fault network is then

$$\frac{10}{377}\ddot{\delta}(t) = 0.8 - 1.5879\sin\delta \qquad (30)$$

or

$$\ddot{\delta}(t) = 30.16 - 59.8638\sin\delta \qquad (31)$$

Let's summarize the three swing equations for the three different time frames:

Pre-fault: $\ddot{\delta}(t) = 30.16 - 83.8071\sin\delta$

Fault-on: $\ddot{\delta}(t) = 30.16 - 38.0958\sin\delta$

Post-fault: $\ddot{\delta}(t) = 30.16 - 59.8638\sin\delta$

What do you notice?

Here, we observe that the coefficient of the sin term varies as follows:

- The largest one, 83.807, corresponds to the pre-fault condition when the transmission system is strongest;
- The smallest one, 38.095, corresponds to the fault-on condition when the transmission system is weakest.
- The middle one, 59.863, corresponds to the post-fault condition. Here the transmission system is stronger than the fault-on condition because the fault has been eliminated. But it is weaker than the pre-fault condition because a transmission line has been eliminated.

Although this example has been for a very simple system, the observations of the above three bullets apply for real systems as well.

One concept to clarify here: what does it mean to have a “strong” or “weak” transmission system? Note that the coefficient of the sin term in the above equations is

$$\frac{|E||V|}{X}\frac{\omega_{\mathrm{Re}}}{2H}$$

The second fraction, $\omega_{Re}/2H$, is present for all three conditions (pre-fault, fault-on, and post-fault) and so does not play a role in distinguishing between “strong” and “weak” transmission.

For the first fraction, $|E||V|/X$, the product $|E||V|$ is the same among the three conditions, but the parameters X is different. Here are the differences:

Pre-fault: X=0.3 (see eq. (16) above)

Fault-on: X=1/0.9091=1.1 (see eq. (25) above)

Post-fault: X=1/1.4286=0.7 (see. eq. (29) above)

So we see that

- a “strong” transmission system (pre-fault) means the transfer impedance between source and sink is low (0.3), and
- a “weak” transmission system (fault-on) means the transfer impedance between source and sink is high (1.1).