## Homework \#1, EE 553, Fall 2012, Dr. McCalley, Due Wednesday August 29

1. Problem 4.1a in the book.
2. Before working this problem, you may review page 102 of your $\mathrm{W} \& \mathrm{~W}$ text (note that in class, we use V to denote voltages whereas your W\&W text uses E to denote voltages).
Observe that Jacobian derivative expressions given by equations T 7.47 and T 7.53 in your notes are almost the same. Also observe that Jacobian derivative equations T7.49 and T7.51 in your notes are almost the same. These two sets of equations are given below.

$$
\begin{gather*}
J_{j k}^{P \theta}=\frac{\partial P_{j}(\underline{x})}{\partial \theta_{k}}=\left|V_{j}\right|\left|V_{k}\right|\left(G_{j k} \sin \left(\theta_{j}-\theta_{k}\right)-B_{j k} \cos \left(\theta_{j}-\theta_{k}\right)\right)  \tag{T7.47}\\
J_{j k}^{Q V}=\frac{\partial Q_{j}(\underline{x})}{\partial\left|V_{k}\right|}=\left|V_{j}\right|\left(G_{j k} \sin \left(\theta_{j}-\theta_{k}\right)-B_{j k} \cos \left(\theta_{j}-\theta_{k}\right)\right)  \tag{T7.53}\\
J_{j k}{ }^{Q \theta}=\frac{\partial Q_{j}(\underline{x})}{\partial \theta_{k}}=-\left|V_{j}\right|\left|V_{k}\right|\left(G_{j k} \cos \left(\theta_{j}-\theta_{k}\right)+B_{j k} \sin \left(\theta_{j}-\theta_{k}\right)\right)  \tag{T7.49}\\
J_{j k}^{P V}=\frac{\partial P_{j}(\underline{x})}{\partial\left|V_{k}\right|}=\left|V_{j}\right|\left(G_{j k} \cos \left(\theta_{j}-\theta_{k}\right)+B_{j k} \sin \left(\theta_{j}-\theta_{k}\right)\right) \tag{T7.51}
\end{gather*}
$$

a. What is the difference between the right-hand-side expression of T7.47 and that of T7.53? What is the difference between the right-hand-side expression of T 7.49 and that of T 7.51 ?
b. Modify (T7.53) to revise the Jacobian element to $J_{j k}^{\prime}{ }^{Q V}=\left|V_{k}\right| \frac{\partial Q_{j}(\underline{x})}{\partial\left|V_{k}\right|}$
c. $\quad$ Modify (T7.51) to revise the Jacobian element to $J_{j k}^{\prime}{ }^{P V}=-\left|V_{k}\right| \frac{\partial P_{j}(\underline{x})}{\partial\left|V_{k}\right|}$
d. Express the relations in (T7.42e) below to account for the two modifications made above. This requires that you express the new Jacobian submatricies $J^{\mathrm{PV}, \text { new }}$ and $\mathrm{J}^{\mathrm{QV}, \text { new }}$ in terms of its revised elements (from parts (a) and (b) above). It also requires that you modify the elements of the voltage-related part of solution vector $\Delta|\underline{V}|^{\text {new }}$.

$$
\begin{align*}
& \underline{J}^{P \theta} \Delta \underline{\theta}+\underline{J}^{P V, \text { new }} \Delta|\underline{V}|^{\text {new }}=-\Delta \underline{P} \\
& \underline{J}^{Q \theta} \Delta \underline{\theta}+\underline{J}^{Q V, \text { new }} \Delta|\underline{V}|^{\text {new }}=-\Delta \underline{Q} \tag{T7.42e}
\end{align*}
$$

e. What is the advantage of this new formulation? Hint: Express and compare $J_{j k}^{P \theta}$ to $J_{j k}{ }^{Q V, \text { new }}$ and express and compare $J_{j k}{ }^{Q \theta}$ to $J_{j k}^{P V, n e w}$.
3. A transformer with an off-nominal tap ratio $t$ connects two buses as shown in Fig. 1. The transformer admittance is $y$. An equivalent representation of the Fig. 1 transformer, used for power flow calculations, is shown in Fig. 2. For each model, give the elements of the admittance matrix $\underline{Y}$, where $\underline{I}=\underline{Y V}, \underline{I}=\left[\begin{array}{ll}\underline{I}_{1} & \underline{I}_{2}\end{array}\right]^{T}$, and $\underline{V}=\left[\begin{array}{ll}\underline{V}_{1} & \underline{V}_{2}\end{array}\right]^{T}$. From your results, express the admittances of Fig. 2 in terms of tap $t$ and transformer admittance $y$. Doing so will allow us to model the off-nominal turns transformer of Fig. 1 in the standard $\pi$-equivalent model of Fig. 2 so that we can represent the tap changer in our power flow algorithm.


Fig. 1


Fig. 2
4. Consider that the transformer of Fig. 1 is a tap-changing-under-load (TCUL) transformer, regulating bus 1 , and that buses 1 and 2 are interconnected to a larger system. Bus 2 is a type PQ bus. Which parameters associated with the bus 1 to bus 2 subsystem would be included in the state vector $\Delta \underline{x}$ used to solve the equation $\underline{J} \Delta \underline{x}=-f(\underline{x})$ for one iteration of the Newton-Raphson power follow solution? That is, what are the unknowns for this subsystem? Note that the matrices $\underline{J}$ and $f(\underline{x})$ are the Jacobian and the mismatch vector, respectively.
5. Consider the two-bus system of Fig. 3, where the tap $t$ is used to regulate the voltage at bus 2 to be equal to 1.0 pu .


Fig. 3
To solve this system by Newton-Raphson, we need to develop the equation $\underline{J} \Delta \underline{x}=-f(\underline{x})$. For this system, specify $\underline{J}, \Delta \underline{x}$, and $f(\underline{x})$ in terms of the unknowns in the problem and the admittance $y=|y|\llcorner\gamma$.

