Updated: April 10, 2024

Transmission Expansion Planning

1 Introduction

The transmission expansion planning (TEP) problem is a very complex problem, involving

- the interplay between resource needs and transmission needs;
- reliability, e.g., the performance of the system following "credible" outages of generation and/or transmission;
- resilience, e.g., the cost incurred for repair and restoration following extreme events;
- multiple decisions taken over an extended period of time;
- uncertainty, since the time is in the future;
- consideration of socio-political/economic/environmental influences from the general population as well as from local, state, regional, and national regulatory, legislative, and executive branches of government.

Figure 1 [1] illustrates conceptualization of 3 steps within the planning process (on left), related uncertainties (on right). Observe:

- The blue boxes which include load forecasting and the generation expansion planning (GEP) problem at the top;
- The yellow box which is the development of system representations (power flow and stability data) to be used in subsequent analysis;
- The green box which is the reliability analysis (not to be confused with resource adequacy analysis).

Figure 2 [2] illustrates a slightly more detailed view, with two important differences:

1. It is deterministic i.e., it does not assess uncertainty (uncertainty is addressed by performing the process with different load forecasts and/or generation plans).
2. It has, at the end, a "solutions" step (Figure 1 shows just the steps; the "solutions" step at the end is implied).

Figure 2 illustrates the way transmission planning has been done for many years. Observe the three large brown arrows, showing process input from (a) generation planning group; (b) NERC reliability criteria; and (c) cost data for potential solutions.

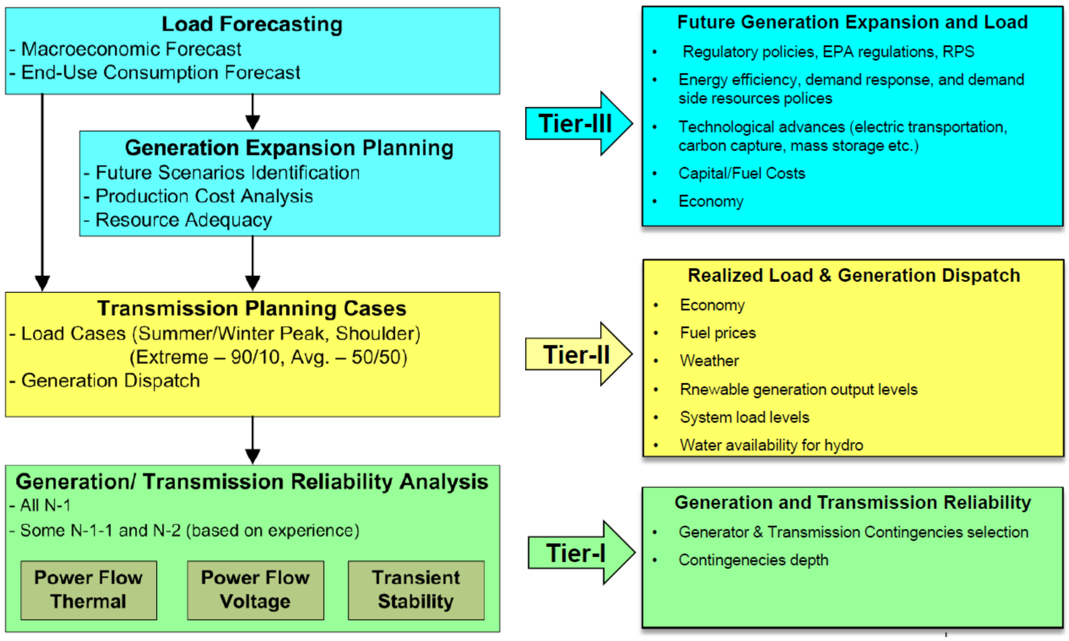


Figure 1: Transmission expansion planning, with uncertainty

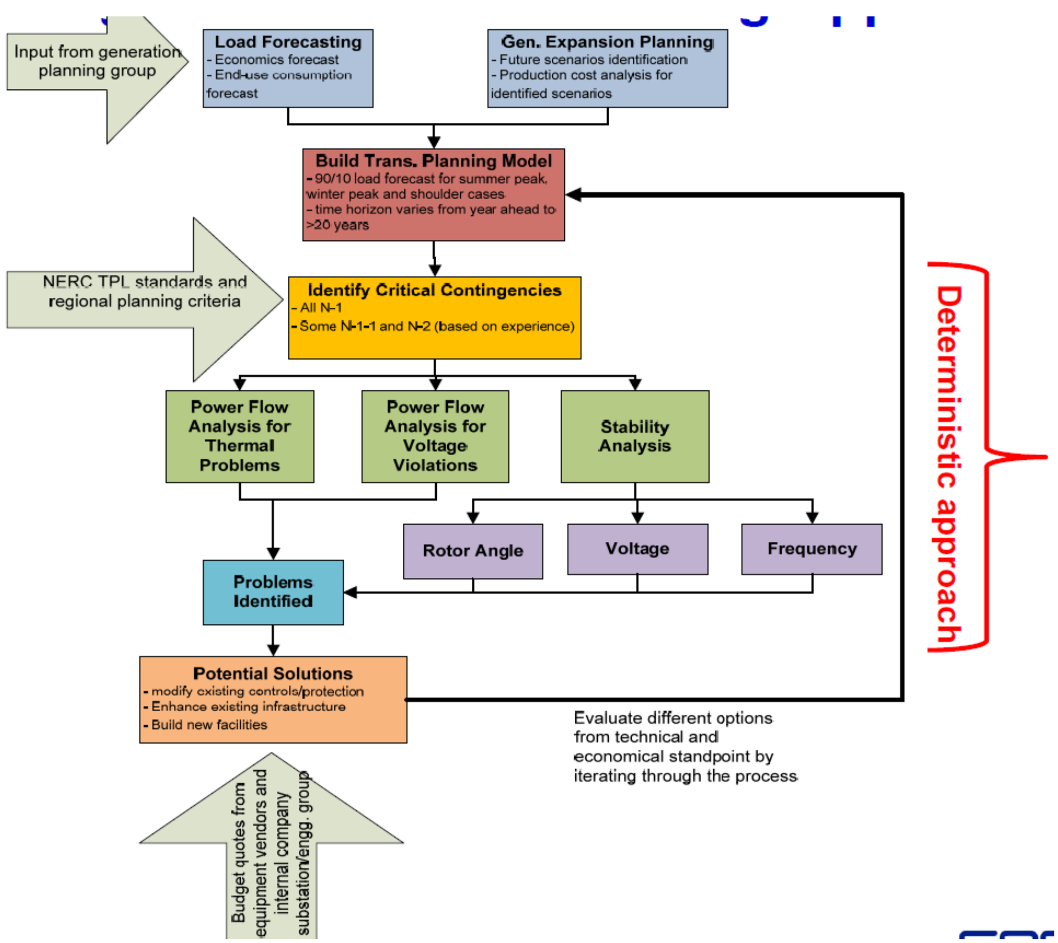


Figure 2: Deterministic transmission expansion planning

Figure 2 has been simplified in Figure 3 by

- Lumping the orange and green boxes together with the three blue boxes under "stability analysis, into "Assess steady-state and dynamic contingency performance."
- Making explicit the phrase at the bottom of Fig. 2 that says "Evaluate different options from technical and economical standpoint by iterating through the process," by adding the box "assess economic performance" in Fig. 3.
- Modifying the "Problems identified" to consider whether the solution can be improved, i.e., we assumed there is an objective function which provides a way to evaluate "potential solutions."

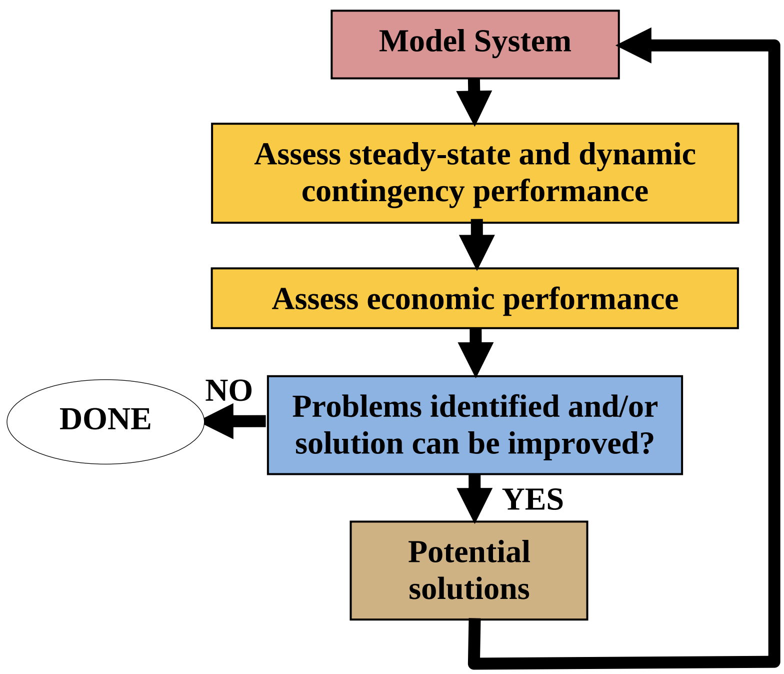


Figure 3: Simplification of transmission expansion planning

Figure 3 is not qualitatively different than Figure 2 but rather just a simplification and refinement of Figure 2. As such, the transmission planning process can be understood as an optimization process, i.e., a process whereby we attempt to identify solutions that provide feasibility (no problems identified) and optimality (solution cannot be improved). In these notes, we formalize this optimization problem.

Today, regional transmission organizations (RTOs), like MISO, have explicit responsibility to coordinate the planning process among its stakeholders. MISO has principles that guide their planning processes, as illustrated in Figure 4 [3]. It seeks to perform "value-based planning" as indicated in Figure 5 [4]. It is "value-based" because it utilizes production cost simulation (Promod or Plexos) to associated economic value to its plans.

Figure 4: MISO Planning Principles

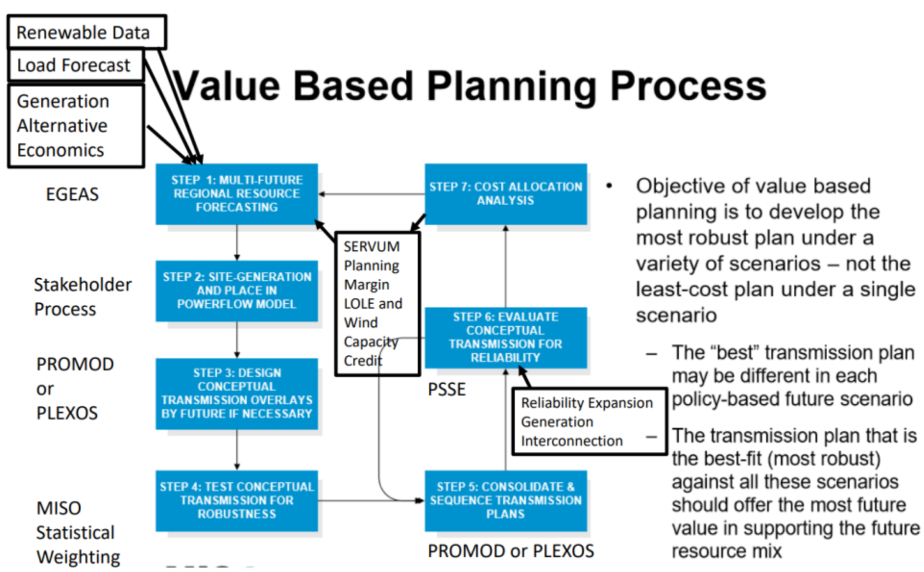


Figure 5: The MISO Value-Based Planning Process

It is important to realize that the transmission expansion planning cannot be reduced to a single optimization. It is an extended process which involves a great deal of human interaction, in terms of understanding ratings, capabilities, and locations of proposed generation, the potential for purchasing energy outside of the region, conducting analyses and coordinating such studies with stakeholders, deciding cost allocation, gaining regulatory approval, obtaining permits and siting (obtaining right-of-way), and finally building the circuit(s). Figure 6 (adapted from [5]) illustrates the complexities of transmission planning. This figure, together with the next five explanatory bullets, are taken from [6].

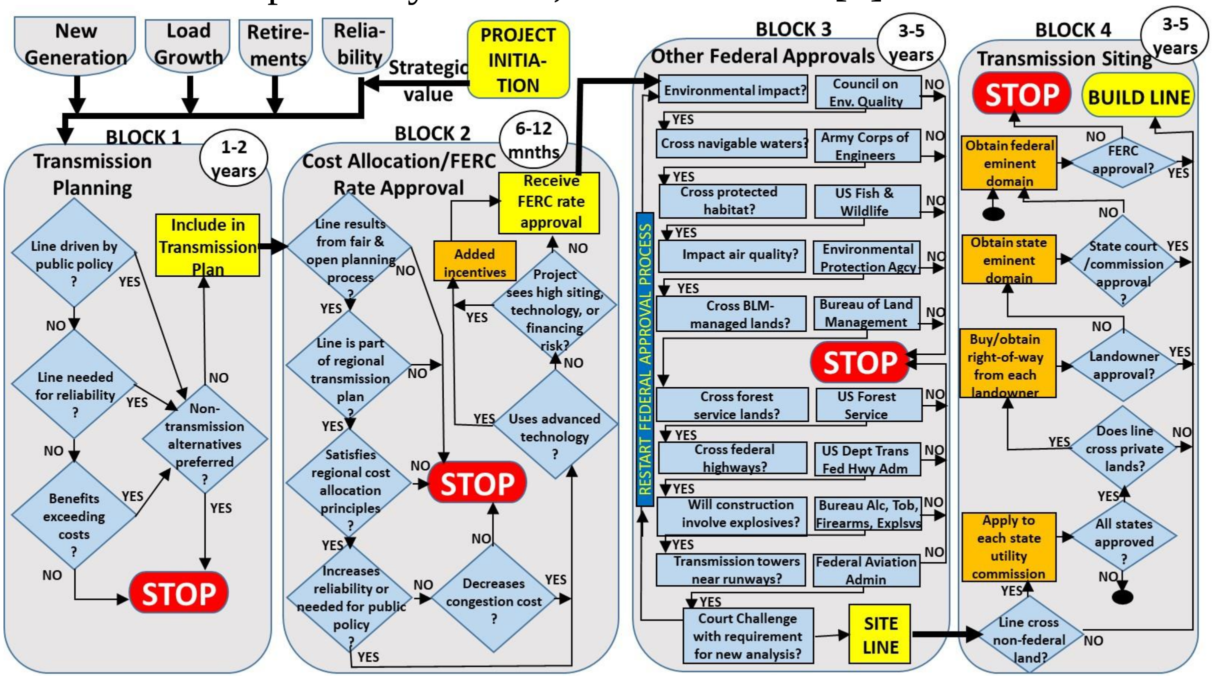


Figure 6: Transmission planning, cost allocation, approval, siting process in the US (adapted from [5])

The central takeaway from Figure 6 is that the amount of time required to plan and build transmission is long, ranging from 7 to 13 years, and the overall process is exceedingly complex. Other important aspects of this process, as highlighted by Figure 6, are as follows:

- <u>Project initiation</u>: To initiate development of an transmission project, there ultimately must be an entity or coalition that identifies that the transmission project may be of strategic value. This step is critical because nothing moves forward without it; this step is difficult because it requires experience and understanding on how to evaluate the benefits of transmission together with the ability to bring together organizations interested in obtaining those benefits and able to provide funding towards pursuing them. The identified strategic value motivates a business plan to financially justify and guide the project.

- Transmission planning (Block 1): This process, typically requiring 1-2 years, needs the attention from experienced planners to design the transmission project and its technical features, consider alternatives, assess risks, ensure that the plan meets reliability requirements, and quantify costs and benefits and return on investment.
- Cost allocation/FERC rate approval (Block 2): FERC requires that the project be part of a fair and open planning process, that it be assessed within the planning processes of affected RTOs, and that it satisfy the RTO's cost allocation principles. FERC also has authority to adjust cost recovery based on "added incentives" [7][1]. This step typically requires 6-12 months.

[1] In 2006, FERC built into its processes (based on a section 219 Congress added to the Federal Power Act) the ability to add incentives for transmission projects proposed by a member of an RTO that ensure reliability or reduce cost of delivered power by reducing congestion, particularly for projects that present special risks or challenges. As described in [7], such incentives focus on risk and include higher return on equity; recovery of incurred costs if a project is abandoned for reasons outside the applicant's control; inclusion in rate base of 100% costs for construction work in progress; use of hypothetical capital structures; accelerated depreciation for rate recovery; and recovery of pre-commercial operations costs as an expense or through a regulatory asset. FERC recently issued a Notice of Proposed Rulemaking to extend and refine their approach for evaluating incentive requests.

- Other Federal approvals (Block 3): There are a variety of Federal permits that may need to be obtained depending on the nature of the project. Any of the various Federal agencies granting these permits can effectively stop the project. This step may require 3-5 years. Effort has been made to address the required Block 3 time by granting the US Department of Energy "lead agency" status [8], thereby coordinating and streamlining the process.
- Transmission siting (Block 4): The most significant uncertainties occur during efforts to obtain transmission siting. Block 4 uncertainties occur largely because of division of power between state and federal agencies. Unlike natural gas transmission, states are primary decision-makers for siting interstate electric transmission; there are strong arguments being made today that to obtain the very significant benefits of interregional transmission, FERC will need more siting authority [9], while state authorization and review processes need to be simplified [10].

It is not possible to account for this very complex process within a single optimization formulation. However, optimization may facilitate our understanding of the range of possible solutions, a step which is perhaps most useful at the beginning of the overall process, in order to identify what is and what is not a potential solution.

Finally, we refer to a planning group of the Western Electric Coordinating Council (WECC), called TEPCC. TEPPC stands for "Transmission Expansion Planning Policy Committee" and has four main functions [11]: 1) oversee and maintain public databases for transmission planning; 2) develop, implement, and coordinate planning processes and policy; 3) conduct transmission planning studies; and 4) prepare Interconnection-wide transmission plans. The following statement comes from a TEPPC document [12] and is revealing

"Electric power networks are a unique part of our national infrastructure. With current technology, long-distance high-voltage lines are not buried, so they become a visible part of the landscape through which they pass. Transmission facilities also have very long lives, so decisions made today have long-lasting effects. Therefore, the objective of long-term transmission planning is to make the best network design decisions today after considering possible future needs and expansion options. Few, if any, 10-year or 20-year transmission plans will come to fruition as originally conceived. However, by planning for possible future needs, flexibility is built into the network's design that allows options to be exercised and adaptation to occur as future conditions are revealed.

TEPPC's activities are an integral part of the Western Interconnection's overall approach to Interconnection-wide planning of the transmission system, which has two major aspects for consideration:

1) System reliability—characterized as "keeping the lights on" while responding in a predictable fashion to both planned and unplanned outages to generation and transmission system elements.
2) System utilization,—a measure of the economic performance of the transmission system. System production cost studies and associated capital cost estimates for those studies provide answers to the question, "While operating within the bounds of reliable operation, how well does the transmission system perform to deliver electricity services to consumers at a reasonable cost?"

2 TEP formulation

The formulation given in this section is adapted from that given in Section 6.3 of [13], an approach which was originally developed in [14]. The model is referred to as a *disjunctive*[2] model. It has been used in a number of TEP-related efforts, including [15].

[2] The word "disjunctive" means "lacking connection" or "marked by breaks" which fairly characterizes a network where one is considering adding new circuits (i.e., new connections between nodes).

References [16, 17] provide good background on the mathematical programming approaches used in solving the TEP problem.

Our initial model is based on the following assumptions:

- The planning horizon is over N_T periods with the variable t representing a single period so that $t=1,\ldots, N_T$. A period could be a single year, but it may be more appropriate to cover the range of loading conditions that it be quarters (i.e., fall, winter, spring, summer) or months. In the rest of this document, we assume that the period will be a year.
- Peak loading conditions are modeled for each period, and it is assumed that these conditions are constant throughout the period.
- All costs of planning and building a new transmission circuit are incurred in the period that the new circuit goes into service.

2.1 Objective function

Let the power production level of each generator j in year t be $P_{Gj}(t)$. (We assume only one unit is modeled at each bus and that buses having no generation will have $P_{Gj}(t)=0$. Therefore, the "j" index is the bus number. We assume that we have N buses.) One approach is to fix the production levels *a-priori*, i.e., to identify for each year (before determining transmission investments) the minimum-cost dispatch necessary to satisfy the load without violating reliability constraints. We would do this by solving a security-constrained unit commitment (SCUC) for each year. If a SCUC solution is not found for any year, then there would be some transmission necessary to achieve feasibility. However, assuming the SCUC finds a solution for each year, we know there is a feasible solution. Then, if for all years there is no transmission constraint that is binding (no congestion), then the answer is to invest nothing since we already fixed the production levels in each year, corresponding dispatches are feasible, and since there is no binding transmission, adding more transmission will not affect the solution, and so the answer is known: invest nothing. If there is binding transmission, then it is of interest to see if there is a level of investment for that binding transmission which, if made, will allow production cost savings equal to or greater than the cost of the transmission investment.

Alternatively, and preferably, generation levels $P_{Gj}(t)$ may be treated as decision variables and determined as part of the solution to a single transmission expansion optimization problem. In this case, the resulting solution will provide an optimal transmission plan *and* an optimal dispatch for the given yearly loading conditions. Transmission will be built if its investment cost is outweighed by the cumulative (over the simulation interval) savings in production cost which it enables, as illustrated in Figure 7. The savings in production cost will occur mainly because of reduced congestion (allowing less expensive generation to produce more), but there can also be influence from the impact of the transmission on losses (which may go up or down).

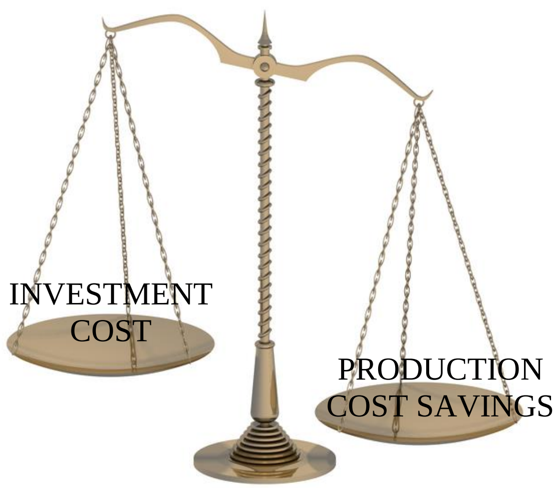


Figure 7: Investment cost vs. production cost savings

The difference in these two approaches is that the latter approach considers the continuous interdependency between the transmission plan and the optimal dispatch, i.e., the transmission plan affects the optimal dispatch, and the optimal dispatch affects the transmission plan.

Therefore, our objective function is a combination of two costs, the aggregate production costs in future periods and the aggregate transmission investment costs in future periods.

One can see that this problem is inherently a mixed integer program (MIP) because it involves

(a) the minimization of production costs (a function of the continuous variable P_{Gj} at each plant) and

(b) the minimization of investment costs, where an investment is to either build a new circuit (1) or not (0).

We discuss each of the two costs below.

2.1.1 Aggregate production costs in future periods

We already defined the generation level of unit j at time t as $P_{Gj}(t)$. We assume here that $P_{Gj}(t)$ is in per-unit (pu). (Per-unitization is generally preferred when modeling transmission because it avoids voltage transformation across transformers having turns ratios equal to nominal voltage ratios. It is required when modeling the DC-flow approximation because the DC-flow linearization depends on the assumption that all voltage magnitudes are 1.0, an assumption which only holds in per-unit.)

We also define the average cost of producing 1 per-unit power at node j during period t as $C_j(t)$. It has units of \$/pu-year. It is obtained as the slope of a line from the origin to the peak point on the unit's cost-rate curve, multiplied by the number of hours in the period. This is illustrated in Figure 8 below. We use average cost instead of marginal cost here because we desire to reflect total costs over the time period, not the cost of the next MW produced.

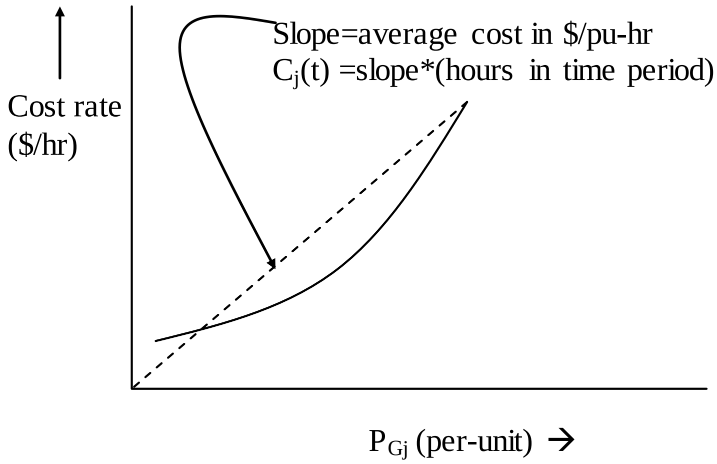


Figure 8: Illustration of generation cost coefficients

We also need here the discount factor for period *t*, given by

$$\zeta^t = \frac{1}{(1+i)^t}$$

where *i* is the discount rate. We assume the investments made in year 1 are already present value, and so it is not until year 2 that we need to discount to present worth; therefore we utilize ζ^{t-1} as the discount factor.

With these definitions, we can express the aggregate production costs in the planning horizon, C_E (where E is for energy) as:

$$C_E = \sum_{t=1}^{N_t} \sum_{j=1}^{N} \zeta^{t-1} C_j(t) P_{Gj}(t) \qquad (1)$$

We note that the decision variables in eq. (1) are continuous.

2.1.2 Aggregate facility investments costs in future periods

We make the following definitions:

- $K_{ij}(t)$ is the investment cost of branch *ij* in period *t*.
- A_n is the set of *candidate* branches (*n* is for "new")
- $z_{ij}(t)$ is an integer 0 or 1. It is 1 if branch $ij \in A_n$ is put in service ***during*** period *t*, and 0 otherwise.

- $S_{ij}(t)$ is an integer 0 or 1. It is 1 if circuit $ij \in A_n$ is put in service ***before or during*** period t, and 0 otherwise. Therefore

$$S_{ij}(t) = \sum_{n=1}^{t} z_{ij}(n) \tag{2}$$

We will not use $S_{ij}(t)$ in expressing the objective function but will use it in expressing the constraints. It is convenient to define it now since it depends on $z_{ij}(t)$.

With these definitions, we express the aggregate investment costs in the planning horizon, C_I, as:

$$C_I = \sum_{t=1}^{N_T} \sum_{ij \in A_n} \varsigma^{t-1} K_{ij}(t) z_{ij}(t) \tag{3}$$

The objective function of our optimization problem can therefore be formulated as the sum of the aggregate production costs and the aggregate facility investment costs, according to:

$$C = C_E + C_I =$$

$$\sum_{t=1}^{N_T} \sum_{j=1}^{N} \varsigma^{t-1} C_j(t) P_{Gj}(t) + \sum_{t=1}^{N_T} \sum_{ij \in A_n} \varsigma^{t-1} K_{ij}(t) z_{ij}(t) \tag{4}$$

2.2 Equality constraints – first attempt

In this section, we attempt to formulate the equality constraints.

The equality constraints that we need are those which will force the solution to satisfy electrical laws associated with how the power flows in the network. This, you will recall, is accomplished by enforcing the DC power flow equations.

$$\underline{P} = \underline{B}' \underline{\theta} \tag{5}$$

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \tag{6}$$

Equation (5) is the only one we really need to enforce the DC power flow equations, but (6) is needed to enforce branch flow constraints. The nomenclature is defined below:

- $\underline{P}$ is the $N\times 1$ column vector of nodal injections P_j, $j=1,\ldots,N$, where

$$P_j=P_{Gj}-P_{Dj} \qquad (7)$$

and P_{Gj} and P_{Dj} are generation and load, respectively, at bus j.
- $\underline{B}'$ is the so-called "B-prime" matrix which is the negative of the imaginary part of the network's admittance matrix $\underline{Y}$, i.e.,

$$\underline{B}' = -\operatorname{Im}\{\underline{Y}\} \qquad (8)$$

The B-prime matrix here must be $N\times N$, i.e., it must have dimension equal to the number of buses in the network.
- $\underline{\theta}$ is the $N\times 1$ column vector of bus angles, in radians.
- $\underline{P}_B$ is the $M\times 1$ column vector of branch flows; branches are ordered arbitrarily, but whatever order chosen must also be used in constructing $\underline{D}$ and $\underline{A}$.
- $\underline{D}$ is an $M\times M$ matrix having non-diagonal elements of zeros; the diagonal element in row k, column k contains the negative of the susceptance of the k^{th} branch.
- $\underline{A}$ is the $M\times N$ *node-arc incidence matrix*. It is also called the adjacency matrix, or the connection matrix. We saw an example of the node-arc incidence matrix in our GEP notes, as shown in Figure 9.

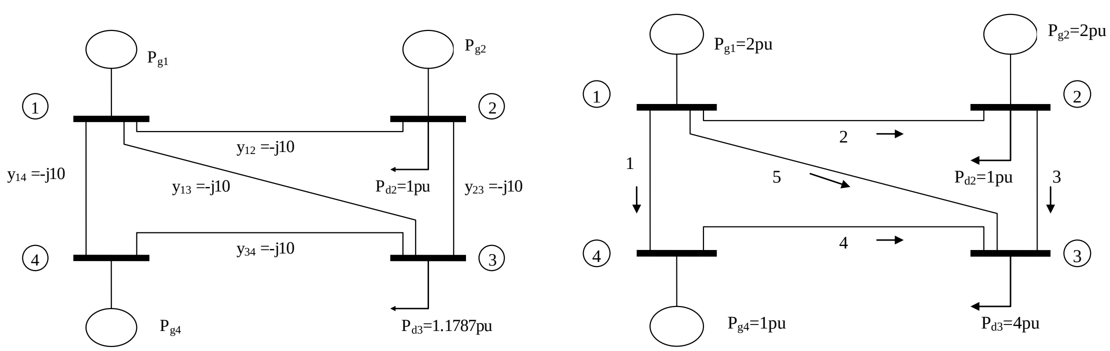


Figure 9: admittances (left) and branch numbers (right)

$$\underline{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

We will also obtain $\underline{Y}$, $\underline{B'}$, and $\underline{D}$ for this system, just to illustrate.

$$\underline{Y} = j\begin{bmatrix} -30 & 10 & 10 & 10 \\ 10 & -20 & 10 & 0 \\ 10 & 10 & -30 & 10 \\ 10 & 0 & 10 & -20 \end{bmatrix} \Rightarrow \quad \underline{B}' = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix}$$

A useful relationship between $\underline{D}$ and $\underline{B'}$ is:

$$\underline{A}^T \underline{D}\underline{A} = \underline{B}' \qquad (9)$$

To illustrate using the matrices for the sample system of Figure 9:

$$\underline{B}' = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix} \quad \underline{D} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \quad \underline{A} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$\Rightarrow$

$$\underline{A}^T \underline{D}\underline{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 10 & 0 & 0 & 10 \\ 0 & -10 & 10 & 0 & 0 \\ 0 & 0 & -10 & 0 & -10 \\ -10 & 0 & 0 & 10 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix}$$

And if (9) is true, then we can also derive:

$$\underline{A}^T \underline{D}\underline{A} = \underline{B}' \Rightarrow \underline{A}^T \underline{D}\underline{A}\underline{\theta} = \underline{B}'\underline{\theta},$$

and since, by (6), $\underline{D}\underline{A}\underline{\theta} = \underline{P}_B$

$$\Rightarrow \underline{A}^T \underline{P}_B = \underline{P} \qquad (10)$$

In formulating the constraints, a key requirement we will try to satisfy is to retain linearity in the decision variables, because linear problems (linear programs or LPs) are much easier to solve than nonlinear ones (NLPs). In considering this, there are two complications, which we discuss in the following subsections.

2.2.1 Changing loading conditions

The loading conditions will change from time period to time period. Therefore, one set of equality constraints will not be satisfactory, we must write a distinct set of equality constraints for every time period in the optimization. Although this will increase our problem size, it does not present any fundamental problem. That is, as long as our problem in one time interval is linear in the decision variables, the multi-time interval problem will also be linear in the decision variables.

2.2.2 Changing topology

The elements of $\underline{B}'$, $\underline{D}$, and $\underline{A}$ depend on the topology of the network. In fact, the dimension of $\underline{D}$ and $\underline{A}$ depend on the topology of the network. And if we allow the expansion plans to include construction of new substations (nodes), the dimension of $\underline{B}'$ also depends on the topology of the network.

Yet the problem we are trying to solve is exactly "what should be the future topology of the network"! Therefore it seems difficult to formulate any of these matrices until we have the solution, a condition which seems to eliminate our use of these matrices in the solution procedure.

So how to enforce the network flow equations?

One approach is as follows:

(a) Construct the matrices so that all existing transmission is modeled (of course) *AS WELL AS* all possible expansion plans; we will make individual expansion-related elements of the matrices to be a function of a binary variable.

(b) Solve the resulting optimization problem.

Let's refer to this as the "expanded matrix" approach. As an example, in the network of Figure 10, we may like to consider an expansion plan that includes a new branch between nodes 2 and 3, shown as a dashed line; we will assume that the new branch is identical to the existing branch between nodes 2 and 3, i.e., it has an admittance of -j3.33 and a capacity of 250 MW.

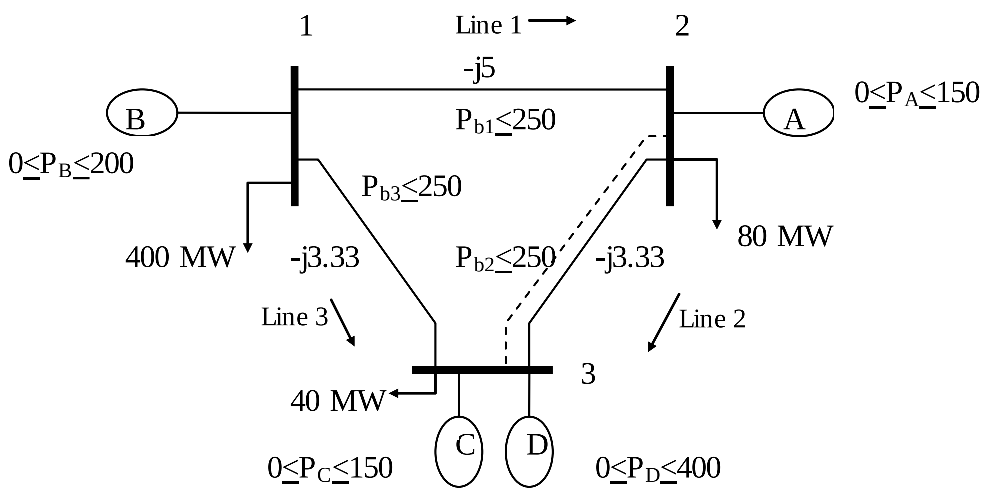


Figure 10: Example system for TEP problem

Continuing our Figure 10 example, let's define Z as a binary variable that is 1 if we accept the new line and 0 otherwise. If we assume that the new line will have the same admittance as the existing line between nodes 2 and 3, then the various matrices are:

$$\underline{B}' = \begin{bmatrix} 8.33 & -5 & -3.33 \\ -5 & 8.33 + Z(3.33) & -3.33 - Z(3.33) \\ -3.33 & -3.33 - Z(3.33) & 6.66 + Z(3.33) \end{bmatrix}$$

$$\underline{D} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3.33 + Z(3.33) & 0 \\ 0 & 0 & 3.33 \end{bmatrix}, \qquad \underline{A} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

The resulting equality constraints are as follows:

$$\underline{P} = \underline{B}'\underline{\theta} = \begin{bmatrix} 8.33 & -5 & -3.33 \\ -5 & 8.33 + Z(3.33) & -3.33 - Z(3.33) \\ -3.33 & -3.33 - Z(3.33) & 6.66 + Z(3.33) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 3.33 + Z(3.33) & 0 \\ 0 & 0 & 3.33 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

The problem with this approach can be observed by noting that the two above equations, one for nodal injections $\underline{P}$ and the other for branch flows $\underline{P}_B$, contain nonlinear terms, i.e., they have products of Z and θ_j, $j=1,2,3$. Therefore, this is a *nonlinear integer programming problem*, since it has product terms, and as a result, we become unhappy, because this problem is difficult to solve.

So… we consider a different approach.

2.3 Equality constraints – second attempt; concept

The nonlinearity in the expressions of the previous section result in a mixed integer nonlinear programming (MINLP) problem. The approach to handling this is to utilize what is known in the literature as the disjunctive method. Before presenting the disjunctive method, we need to make one clarification.

To represent the power system flow relations, we can implement equations (5) and (6) above, repeated below for convenience.

$$\underline{P} = \underline{B}'\,\underline{\theta} \qquad (5)$$

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \qquad (6)$$

We show in the appendix that eqs. (5) and (6) are equivalent to eqs. (6) and (10), given here:

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \qquad (6)$$

$$\underline{A}^T \underline{P}_B = \underline{P} \qquad (10)$$

Therefore we may implement either set to characterize the network. We choose to implement (6) and (10); the reason for this choice is that doing so isolates our nonlinear problem to (10).

When written as scalar relations, eq. (6) becomes:

$$P_{ij} = B_{ij}\left(\theta_i - \theta_j\right) \qquad (6a)$$

where P_{ij} is the flow on the branch connecting nodes i and j (P_{ij} is an element of vector $\underline{P}_B$), B_{ij} is the element in row i column j of $\underline{D}\times\underline{A}$, and θ_i, θ_j are the voltage phasor angles at nodes i and j, respectively.

When written as a scalar, eq. (10) becomes:

$$\sum_j k_{ij} P_{ij} = P_{Gi} - P_{Di} \qquad (10a)$$

where k_{ij} is +1 if the branch flow P_{ij} is defined positive from node i to node j and -1 if the branch flow P_{ij} is defined positive from node j to node i, P_{Gi} is the positive (generation) injection at node i, and P_{Di} is the negative (load) injection at node i.

If a branch from node i to node j is an existing branch and is not considered for expansion, then equations (6a) and (10a) are written for that branch. However, if we want to consider expanding a branch from node i to node j, then we have to introduce our integer decision variable z_{ij}, which results in (6a) and (10a) becoming

$$P_{ij} = (B_{ij} + z_{ij} B_{ij,exp})\left(\theta_i - \theta_j\right) \qquad (6b)$$

$$\sum_j k_{ij} P_{ij} = P_{Gi} - P_{Di} \qquad (10a)$$

We observe that (6b) contains the product term between the integer decision variable z_{ij} and the angles angles θ_i and θ_j, respectively. We also observe (10a) does not change (P_{ij} is computed from (6b)).

Equation (10b) presents no nonlinearity (k_{ij} is a constant). So we do not need to consider equation (10b) further.

In considering (6b), we will also consider the branch flow constraint. Thus, (6b) becomes

$$P_{ij} = (B_{ij} + z_{ij} B_{ij,exp})\left(\theta_i - \theta_j\right)$$

$$-P_{ijmax} - z_{ij} \Delta P_{ijmax} \leq P_{ij} \leq P_{ijmax} + z_{ij} \Delta P_{ijmax} \qquad (6c)$$

If we are considering building a branch between two nodes that were previously unconnected, then (10b) remains unchanged, but (6c) becomes

$$P_{ij} = z_{ij} B_{ij,exp}\left(\theta_i - \theta_j\right)$$

$$-z_{ij} \Delta P_{ijmax} \leq P_{ij} \leq z_{ij} \Delta P_{ijmax} \qquad (6d)$$

or it may be written as

$$P_{ij} - z_{ij} B_{ij,exp}\left(\theta_i - \theta_j\right) = 0$$

$$-z_{ij} \Delta P_{ijmax} \leq P_{ij} \leq z_{ij} \Delta P_{ijmax} \qquad (6e)$$

We may write (6e) in a different form to avoid the nonlinearity. The form is given below as eq. (6f):

$$\begin{aligned} &-M\left(1 - z_{ij}\right) \leq P_{ij} - z_{ij} B_{ij,exp}\left(\theta_i - \theta_j\right) \leq M\left(1 - z_{ij}\right) \\ &-z_{ij} \Delta P_{ijmax} \leq P_{ij} \leq z_{ij} \Delta P_{ijmax} \end{aligned} \qquad (6f)$$

It is observed that eq. (6f) contains no nonlinearity. Furthermore, it is easy to see that eq. (6f) is equivalent to (6e) by comparing the equations for the case of z_{ij}=1 and for the case of z_{ij}=0. This is done in Fig. 11 below, where M is chosen to be a large number.

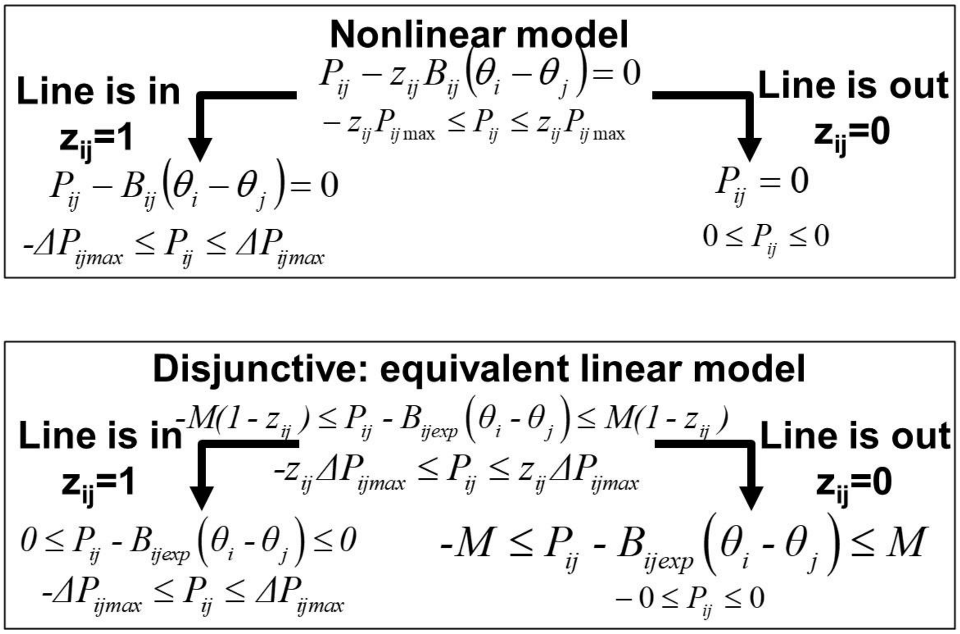


Fig. 11

When the branch is in, z_{ij}=1, the left-hand-side of the Fig. 11 disjunctive model is constrained above and below by 0, therefore what is in the middle must equal zero, which imposes the DC power flow equation together with the branch constraints.

But when the branch is out, z_{ij}=0, the right-hand-side of the Fig. 11 disjunctive model has its flow constrained (at the bottom of Fig. 11) above and below by 0; this means that P_{ij}=0. If P_{ij}=0, then the upper constrain on the right-hand-side of Fig. 11 becomes

$$-M \le -B_{ij} (\theta_i - \theta_j) \le M$$

and so, with M very large, the angular separation between nodes i and j, θ_i-θ_j, must be unconstrained, which indeed it should be if there is no branch connecting them.

<u>**Suggestion**</u>: The above is the disjunctive model for the case when we are adding a branch to two previously unconnected nodes. Develop the disjunctive model for the case when we are adding a branch to two previously connected nodes; show that it works. To do this, review the example of Section 2.2, and start with eq. (6c). And see below formulation.

<u>**Comment 1**</u>: In disjunctive part of below formulation, we replace z_{ij} with S_{ij}. This is necessary to reflect all expansions made, current yr & previous yr.

Model: For N_T years, N buses, and a candidate branch set A_n:
Minimize:

$$C = C_E + C_I =$$

$$\sum_{t=1}^{N_T}\sum_{j=1}^{N}\varsigma^{t-1}C_j(t)P_{Gj}(t) + \sum_{t=1}^{N_T}\sum_{ij\in A_n}\varsigma^{t-1}K_{ij}(t)z_{ij}(t)$$

Subject to:

- For existing branches for which expansion is not considered:

$$P_{ij}(t) = B_{ij}\left(\theta_i(t) - \theta_j(t)\right)$$

$$-P_{ijmax} \le P_{ij}(t) \le P_{ijmax}$$

- For candidate branches A_n (a new connection, or a parallel expansion of an existing connection):

$$-M\left(1 - S_{ij}(t)\right) \le P_{ij}(t) - \left(B_{ij} + S_{ij}(t)B_{ij,exp}\right)\left(\theta_i(t) - \theta_j(t)\right) \le M\left(1 - S_{ij}(t)\right)$$

$$-\left(P_{ijmax} + S_{ij}(t)\Delta P_{ijmax}\right) \le P_{ij}(t) \le \left(P_{ijmax} + S_{ij}(t)\Delta P_{ijmax}\right)$$

$$S_{ij}(t) = \sum_{n=1}^{t} z_{ij}(n) \qquad (*)$$

- For all nodes i:

$$\sum_j k_{ij}P_{ij}(t) = P_{Gi}(t) - P_{Di}(t)$$

$$P_{Gi}(t) \le P_{Gi,max}$$

Comment 2: In the above, each branch (existing and candidate) must be assigned a direction so that it has a "begin" node and an "end" node; this directionality is reflected in k_{ij} (which is either 1 if node i is begin node and -1 if node i is end node). "Candidate" nodes (new substations) are not modeled in the above, but can be[3].

Comment 3: If the planning horizon contains only 1 period ($N_T=0$), then $S_{ij}(t)=z_{ij}(t)$, and we may eliminate $S_{ij}(t)$ and eq. (*) above and replace every occurrence of $S_{ij}(t)$ in our formulation with $z_{ij}(t)$.

[3] Normally, only existing substations are included; when candidate substations need to be considered, it may be necessary to include them as "fictitiously existing" by connecting them to the existing network with at least one high-impedance line. This is a topic that needs to be developed further in these notes. But it is not conceptually difficult to do so. Indeed, recent work on designing an East Coast offshore transmission grid has used such a formulation.

2.4 An equivalent model

The model given on the previous page is more or less consistent with the model given in [15] and to most models given in the general literature. There is another model given by Wang in [13] that looks different, especially the disjunctive part of it, and the model used in [18] is similar. However, I have shown in the Appendix A that the disjunctive model given in [13] (and the similar one in [18]) is indeed equivalent to the model given here and in [15]. I have also provided in Appendix B the model given in [15]. In the last part of these notes, I will present the model given in [18], which uses the disjunctive representation of Wang [13].

3 Extended TEP formulation

Several extensions are of interest in developing a TEP formulation. These are:

- Investment cost variation with technology and design
- Variation in AC loadability with distance
- Transmission losses

We address these in the following three subsections. We make some notational changes: time will be denoted by *y* (for year); the transmission circuit will be denoted by *t*; the transmission technology will be denoted by *k*.

3.1 Investment cost variation with technology and design

There are two overriding issues related to the investment cost of any transmission line design, independent of whether it is AC or DC. Again, the technology is denoted by *k*. Then the two overriding investment cost issues are

- Investment cost of the lines: For both AC and DC, this cost is proportional to the distance of the line. We represent the distance of line *t* as l_{at} (**a**ctual route distance on branch *t*). However, this cost will also depend on the terrain over which the line must cross. The per-mile cost of the following three lines will be very different (assuming the same technology and capacity):

- In a highly urban area near Los Angeles
- Across the Midwestern plain
- Across the Rocky Mountains

To account for the impact on terrain, we will represent the investment cost of the line with a base cost c_{Lk} multiplied by the distance weighted by a factor m_t. Thus, this cost will be

$$c_{L,k} l_{at} m_t.$$

- Investment cost associated with the substations: The situation depends on whether the technology is AC or DC. We assume a base cost for an AC substation for technology k is given by $c_{S,k}$.
  - AC: The substation cost for an AC line will depend on how many substations are deployed; the number of substations deployed will depend on the line distance l_{at}. We will assume that substations for AC lines should be separated by less than l_0 miles. Then the number of substations necessary for that line will be $Int[(l_{at}+2l_0)/l_0]$, where the "Int" function rounds the argument to the next lower integer. Thus, for example, if $l_0=200$ miles, then the number of substations, per Table 1, result from use of this function.

Table 1: Illustration of function for number of AC substations

Distance, l_{at}	$(l_{at}+2l_0)/l_0$	Int$[(l_{at}+2l_0)/l_0]$
50	2.25	2
200	3	3
300	3.5	3
400	4	4
1000	7	7

Note that the distance between substations is the distance divided by the number of segments (which is the number of substations minus 1), i.e.,

$$DistanceBetweenSubs=Distance /\{ Int[(l_{at}+2l_0)/l_0] -1]$$

For example, the distance between substations for the 1000 mile-long-line is 1000/{7-1}=166miles. If we only used 6 substations, then the distance between substations for the 1000 mile-long-line would be 1000/{6-1}=200, in violation of our requirement that AC lines should be separated by less than l_0 =200 miles.

The substation cost will be, therefore

$$c_{S,k}Int[(l_{at}+2l_0)/l_0].$$

Another issue which we will encounter in illustrative results provided at the end of this section is if an AC circuit interconnects two asynchronous grids, e.g., Eastern interconnection and WECC. In this case, we will have to build back-to-back (B2B) DC substations, because an AC interconnection between two grids will be unstable otherwise. We assume a "base" cost per DC substation per GW to be $c_{s,bb}$, so that the base cost per GW of the back-to-back installation would be $2c_{s,bb}$. We call this a base cost because we assume the actual cost increases linearly with line capacity, TC_{kt}. Thus, the back-to-back DC substation cost for an AC line spanning two asynchronous grids is

$$2c_{s,bb}TC_{kt}$$

The total cost of an AC line of technology k that spans two asynchronous grids, therefore, will be:

$$c_{L},_{k}l_{at}m_t+c_{S,k}Int[(l_{at}+2l_0)/l_0]+2c_{s,bb}TC_{kt}$$

The above assumes there is no existing B2B DC substations; if there is, and there is no need to increase existing B2B DC capacity, then the corresponding term is not needed. Code would need to recognize this situation. Alternatively, an existing B2B DC substation may require capacity increase; this would likely be a less expensive situation than building a brand new B2B DC substation, and code would also need to recognize this situation.

- DC: We assume that every DC line will have two primary substations, one at the sending end and one at the receiving end. We also assume the cost of these two substations will be proportional to the line's capacity TC_{kt}. Therefore, the total cost will be

$$2c_{S,k}TC_{kt}$$

We also include the possibility of having multi-terminal DC lines, with n_{it} additional terminals for line t. Therefore, the total cost of a DC circuit is given by

$$c_{L,k}l_{at}m_t+2c_{S,k}TC_{kt}+c_{S,k}n_{it}TC_{kt}$$

NOTE! This approach can be improved by distinguishing between VSC and LCC terminals in terms of converter station cost, the benefits of control capabilities, and converter stations needed for multi-terminal configurations (in LCC, only one line can be connected to each terminal, but DC breakers are not needed; VSC, on the other hand, allows multiple lines to be connected at each terminal but requires DC breakers).

A set of representative data for four different technologies are provided in [18]. These data should be compared to the data provided in Table 2 [19] (this data is old and should be updated).

Table 2: Basic data for transmission technologies

Technology	765kV	500kV	600kV	800kV
Typical Rating(GW)	SIL=2.25 @300mile	SIL=1 @300mile	3GW	6GW
Circuit Breaker(M$)	2.88	2.27	–	–
Transformer(M$)	9.02	6.8	–	–
Voltage Control(M$)	4.24	3.5	–	–
Converter(M$/MW)	–	–	0.155	0.17
Line Cost (M$/mile)	3.49	2.75	1.8	1.95
ROW (ft.)	200	200	250	270
($w_k l_{at}$) losses@SIL(10^{-5})	6.47 l_{at}	12.6 l_{at}	6.58 l_{at}	4.58 l_{at}
X for AC (Ω/mile)	0.5069	0.5925	–	–

From the data provided in Table 2, we may construct investment cost functions for four different technologies, as follows (in $M):

$$765\text{kV AC: } CT_{1t} = 3.49 l_{at} m_t + 16.14 \times Int[\frac{l_{at} + 2l_0}{l_0}] + 170 n_{at} TC_{1t} \quad (3)$$

$$500\text{kV AC: } CT_{2t} = 2.75 l_{at} m_t + 12.57 \times Int[\frac{l_{at} + 2l_0}{l_0}] + 155 n_{at} TC_{2t} \quad (4)$$

$$600\text{kV DC: } CT_{3t} = 1.8 l_{at} m_t + 2 \times 155 TC_{3t} + 155 n_{it} TC_{3t} \quad (5)$$

$$800\text{kV DC: } CT_{4t} = 1.95 l_{at} m_t + 2 \times 170 TC_{4t} + 170 n_{it} TC_{4t} \quad (6)$$

3.2 Variation in loadability with distance

AC Line loadability is estimated based on St. Clair Curves [20], as approximated by the function $f\left(l_{\mathrm{at}}\right) \approx 43.261 l_{\mathrm{at}}^{-0.6678}$. We select a typical rating for a single circuit of each technology, as listed in Table 2. For EHVAC options, we use Surge Impedance Loading (SIL) values. Equations (7)–(10) express the location-specified loadability data.

$$765\text{kV AC: } TC_{1t} = SIL_1 f(l_{at}) \quad (7)$$

$$500\text{kV AC: } TC_{2t} = SIL_2 f(l_{at}) \quad (8)$$

$$600\text{kV DC: } TC_{3t} = 3 \quad (9)$$

$$800\text{kV DC: } TC_{4t} = 6 \quad (10)$$

3.3 Transmission losses

To precisely reflect transmission losses, one may need to use a more accurate model of the power grid using so-called "AC" power flow equations, which is non-linear and thus is very challenging to solve for large systems. In order to improve model accuracy without introducing excessive computational load, i.e., in order to account for losses while maintaining linearity of the formulation, we need to approximate losses.

One way to do this is to estimate losses as a function of the loads and add the increment into the loads. However, this approach is essentially a “fixed losses” approach in that it does not account for variation in losses with transmission flows.

Another approach is to assume that losses in each line are linearly proportional to the flow. This approach reflects loss variation with flow, but over-estimates for low flows and under-estimates for high flows.

A third approach is to do both, which is the approach taken in [18]. This approach is fully explained in [21].

Loss approximation for linearized power flow analysis has been fairly well addressed in the literature, e.g., [22].

3.4 Optimization statement

The complete model follows:

$$Min \quad \sum_{y=1}^{Ny}\sum_{s=1}^{Ns}\sum_{g=1}^{Ng}\sum_{h=1}^{Nh}(1+r)^{-y}P_{ysgh}\Delta_s CG_{gh}+$$

$$\sum_{\substack{y=1\\ y\in N_{inv}}}^{Ny}\sum_{k=1}^{Nk}\sum_{t=1}^{Nt}\sum_{b=1}^{Nb}2^{b-1}(1+r)^{-y}v(y)CT_{kt}x_{yktb}+\sum_{y=1}^{Ny}\sum_{s=1}^{Ns}\sum_{t=1}^{2Nt}(1+r)^{-y}(1-\eta_0)\Delta_s E_r B_{yst} \qquad (26)$$

SUBJECT TO

$$\sum_{h=1}^{Nh}P_{ysgh}-D_{ysg}=\sum_{t=1}^{2Nt}A^T(g,t)B_{yst} \qquad \text{(a)}$$

$$\sum_{i=1}^{y}\sum_{b=1}^{Nb}x_{iktb}=\sum_{b=1}^{Nb}2^{b-1}S_{yktb} \qquad \text{(b)}$$

$$B_{yst}=B_{yst0}+\sum_{k=1}^{Nk}\sum_{b=1}^{Nb}B_{ystkb} \qquad \text{(c)}$$

$$\theta_{ysgi}-\theta_{ysgj}=X0_{ty}(B_{yst0}-B_{ys(t+Nt)0}) \qquad \text{(d)}$$

$$\theta_{ysgi}-\theta_{ysgj}=X_{tkb}(B_{ystkb}-B_{ys(t+Nt)kb})+(S_{yktb}-1)G+UB_{ysktb} \qquad \text{(e)}$$

$$0\le UB_{ysktb}\le 2(1-S_{yktb})G \qquad \text{(f)}$$

$$0\le B_{ysktb}\le 2^{b-1}S_{yktb}TC_{kt} \qquad \text{(g)}$$

$$0\le B_{yst0}\le TC0_{ty} \qquad \text{(h)}$$

$$0 \le P_{ysgh} \le CF_{gh} PC_{ygh} \tag{i}$$

$$\textit{Constraints } (3)\text{–}(10) \tag{j}$$

Binary: S_{yktb}, x_{yktb} on the previous page

where:

$v(y) = \dfrac{N_y + 1 - y}{40}$ is the residual value factor for each year.

Nomenclature for the above model follows:

$y/s/g/h$	Year/load step/node/generation type number
$k/t/b$	Transmission type/arc number/branch index
$N_y/N_s/N_g$	Number of year/load step/node in the model
N_h/N_k	Number of generation/transmission type
N_t/N_b	Number of candidate arcs/parallel branches
N_{inv}	Set of years which allow transmission expansion
η_0	Efficiency of existing transmission system
η_{kt}	Efficiency of type *k* new transmission on arc *t*
E_r	Average energy price (M$/GWhr)
r	Discount rate: 0.02
Δ_s	Time duration for step *s* in each year (hour)
$v(y)$	Residual value factor for year *y*
P_{ysgh}	Generation output of type *h* unit on node *g* during year *y* step *s* (GW)
D_{ysg}	Active load on node *g* during year *y* step *s* (GW)
A	Incidence matrix
CG_{gh}	Type *h* unit production cost on node *g* (M$/GWhr)
CT_{kt}	Type *k* transmission investment cost on arc *t* (M$)
x_{yktb}	Number of type *k* circuits invested on arc *t* branch *b* during year *y*
S_{yktb}	Accumulative number of type *k* circuits invested on arc *t* branch *b* until year *y*
B_{yst}	Total power flow on arc *t* on year *y* step *s* (GW)
B_{yst0}	Branch flow on existing transmission on arc *t* year *y* step *s* (GW)
B_{ystkb}	Branch flow on arc *t* type *k* transmission branch *b* on year *y* step *s* (GW)
CF_{gh}	Renewable capacity factor for type *h* unit on *g*
PC_{ygh}	Generation capacity of type *h* unit on node *g* during year *y* (GW)
θ_{ysg}	Voltage angle on bus *g* on year *y* step *s* (radians)
$X0_{ty}$	Reactance of existing transmission on arc *t* year *y*
X_{tkb}	Reactance of type *k* circuit addition on arc *t* branch *b*
UB_{ysktb}	Disjunctive coefficient for year *y* step *s* type *k* trans-mission arc *t* branch *b*
G	A large number
TC_{kt}	Type *k* transmission loadability on arc *t* (GW)
$TC0_t$	Existing transmission capacity on arc *t* (GW)
l_{mt}	Investment equivalent distance on arc *t* (mile)
l_{at}	Actual route distance on arc *t* (mile)
l_0	Typical distance between AC substations (mile)
w_k	Linear coefficient between loss and distance for type *k* circuit (mile^{-1})

S_k	Type *k* circuit Surge Impedance Loading (SIL) (GW)
$f(l_{at})$	Approximation function of St. Clair Curve
μ_g	Location-specified reserve requirement for node *g*

There are five interesting features in regard to how the above model was used.

1. It was implemented using Benders decomposition, where the master problem contains all binary investment decision variables, and each operational sub-problem contains only continuous variables (generation dispatch) for each year.
2. Generation investment is identified in advance. Any generation expansion planning model may be used to do this; in our case, we utilized an application called NETPLAN [23].
3. A "candidate selection algorithm" was deployed to limit the number of possible transmission candidates.
4. N-1 security was checked after each transmission design and if violations occurred, constraints were generated and the design repeated.
5. The approach was applied to design a transmission overlay for the US assuming a high-renewable future. A 62 node model was utilized; existing interregional transmission was modeled. Although this is an interesting approach and does serve to illustrate the power of the model, it is very much an atypical application as most transmission design problems would only look to identify and design one or at most a few transmission circuits at a time. In this overlay application, we identify and design and entire subsystem.

Figure 11 below represents the overall modeling approach. Figure 12 illustrates results of applying the modeling process for a single "future" scenario.

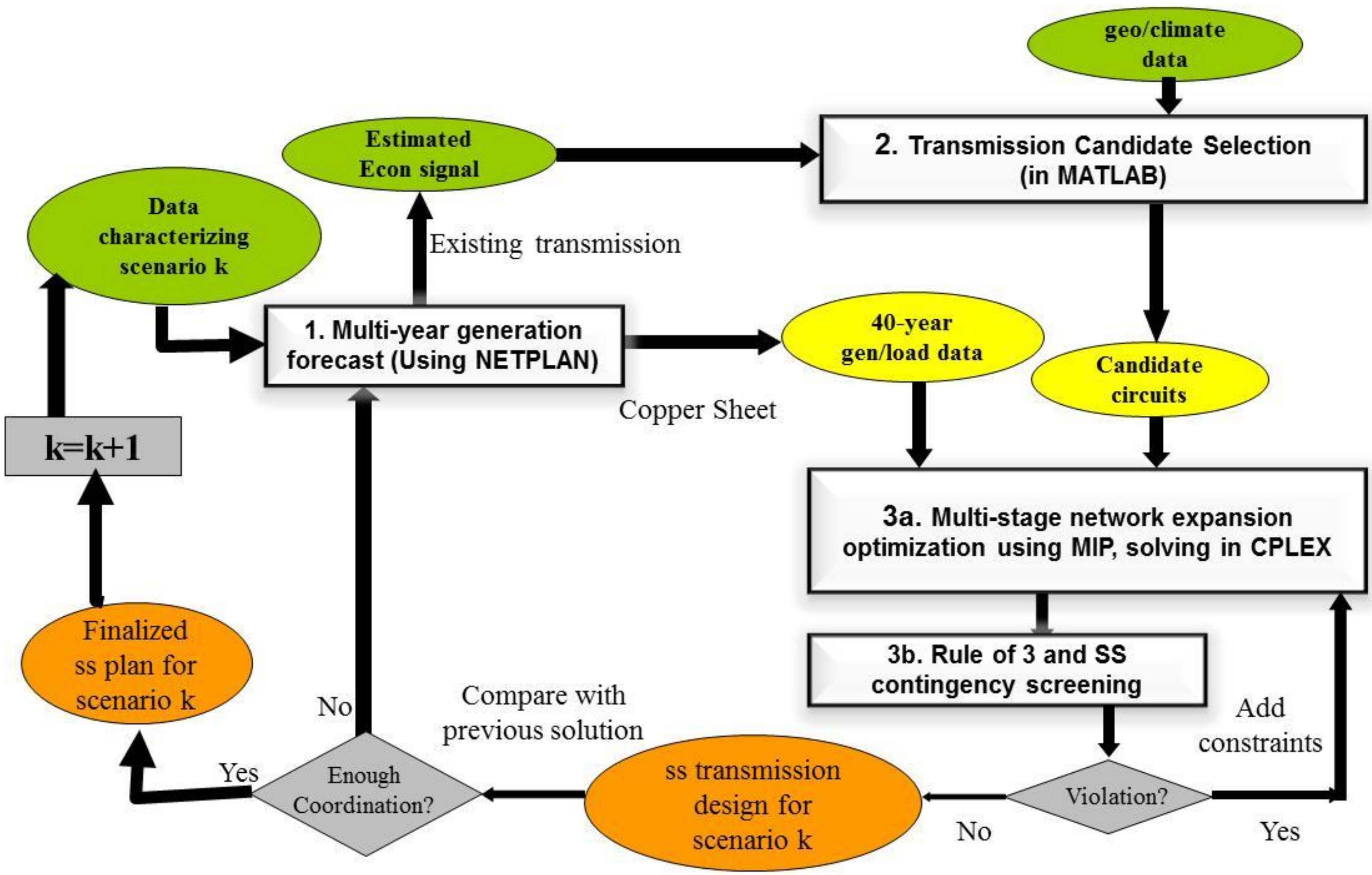


Figure 11: Overall modeling process

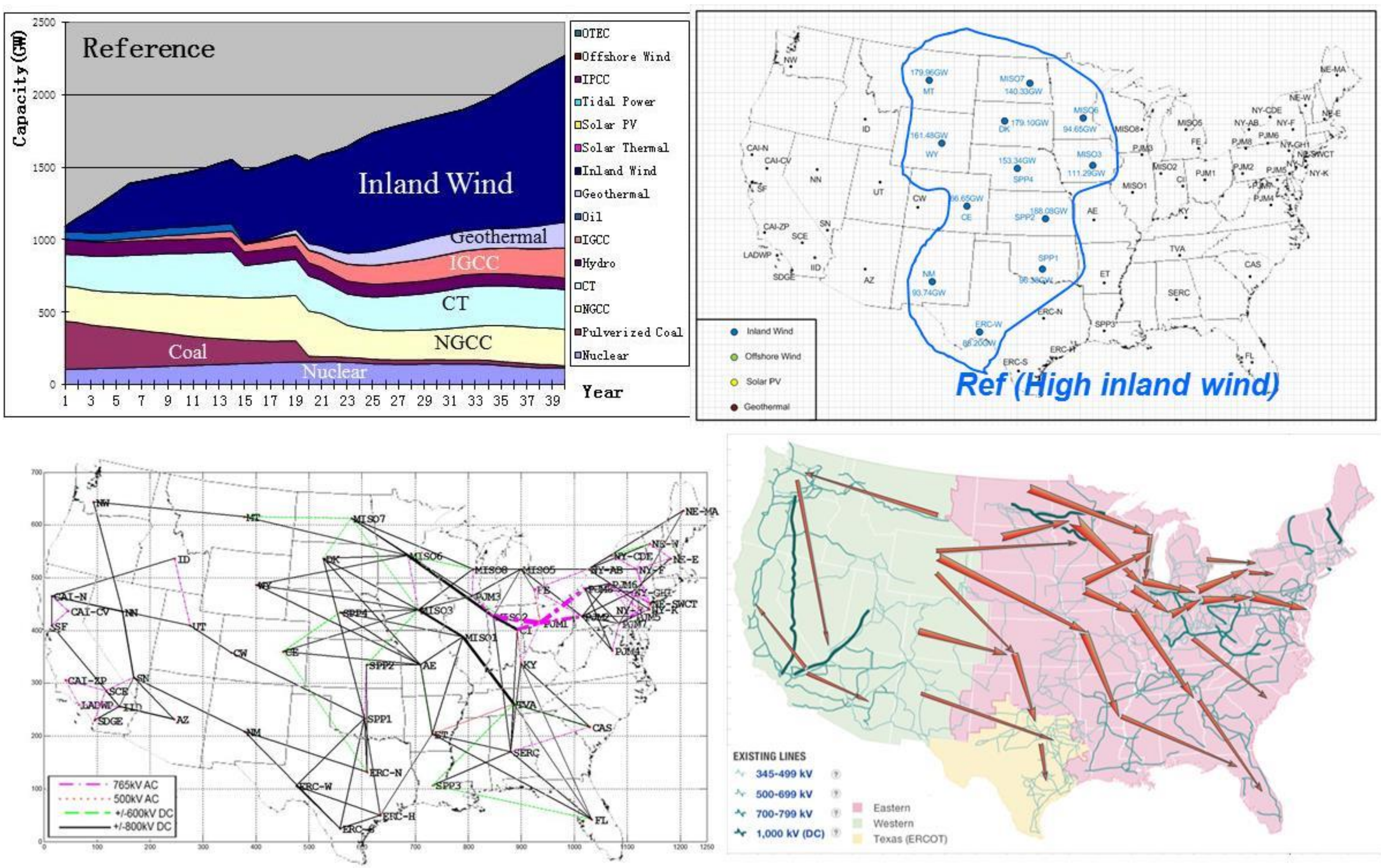


Figure 12: Result of modeling process for designing a US transmission overlay (high-wind case)

Final comment:

One last thing or comment to mention is that, from my experience of taking EE 552 course, I think it is relatively easier to understand the math/engineering part of the transmission planning (optimization problem), however, the cost/benefit analysis part, which finally justifies the transmission expansion plans, can be rather difficult to follow. To understand all kinds of benefit measurements, a very clear understanding of roles and viewpoints of different parties (WECC, ISO/RTO, Utility, IPP, etc) are essentially needed.

Appendix A: TEP Formulation with Nonnegative Variables; the model of Wang [13]

A formulation where branch flow variables must always be positive is given by Wang [13]. Such a formulation was at one time valuable because some linear programming (LP) solvers required all variables to be non-negative.

Each branch must be assigned a direction so that it has a "begin" node and an "end" node. All branches, existing and candidate, are modeled with the below nomenclature. "Candidate" nodes (new substations) can also be included[4].

All of the below variable definitions should also have dependence on *t*, in order to indicate that there is a unique set of variables and corresponding equality constraints for each time period *t*. For now, we omit writing this dependence but leave it to the reader to remember that it is there.

- Two variables for each branch flow:
  - P_b is the flow on branch b if that flow is in the defined direction.
  - $P_b^{'}$ is the flow on branch b if that flow is opposite to the defined direction.

We require both P_b and $P_b^{'}$ to be nonnegative, and if one of them is non-zero, the other one must be zero.

- Begin and end nodes for branch *b*:
  - B_b : This is the node from which branch *b* begins.
  - E_b : This is the node at which branch *b* ends.
- θ_{Bb} is the angle variable at the begin node of branch *b*.
- θ_{Eb} is the angle variable at the end node of branch *b*.
- P_{Dj} is the demand at node *j* (previously defined)

[4] Normally, only existing substations are included; when candidate substations need to be considered, it may be necessary to include them as "fictitiously existing" by connecting them to the existing network with at least one high-impedance line. This is a topic that needs to be developed further in these notes. But it is not conceptually difficult to do so. Indeed, recent work on designing an East Coast offshore transmission grid has used such a formulation.

- P_{Gj} is the generation at bus j (previously defined).

In addition, we make three definitions that are independent of the time period. They are:

- X_b : The branch reactance associated with branch b.
- A_e: The set of existing branches.
- A_n: The set of candidate branches (previously defined).

We now want to write the equations necessary to enforce the network flow equations while keeping our equations linear in spite of the presence of the integer decision variable associated with each candidate line.

But first recall the matrix relations for the DC load flow equations given above

$$\underline{P} = \underline{B}'\,\underline{\theta} \qquad (5)$$

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \qquad (6)$$

Equation (5) is all that is necessary to identify a unique network solution (equation (6) simply computes the resulting line flows).

We saw in (10) that the node-arc incidence matrix is useful in relating branch flows to injections. Repeating for convenience:

$$\underline{A}^T \underline{P}_B = \underline{P} \qquad (10)$$

<u>Fact A</u>: We may obtain eq. (5) from eq. (6) and (10).
To prove this, we will use (9), repeated here for convenience:

$$\underline{A}^T \underline{D}\underline{A} = \underline{B}' \qquad (9)$$

<u>Proof of Fact A</u>: From eq. (10), we have

$$\underline{A}\underline{A}^T \underline{P}_B = \underline{A}\underline{P}$$

$$\left[\underline{A}\underline{A}^T\right]^{-1} \underline{A}\underline{A}^T \underline{P}_B = \left[\underline{A}\underline{A}^T\right]^{-1} \underline{A}\underline{P}$$

$$\Rightarrow \underline{P}_B = \left[\underline{A}\underline{A}^T\right]^{-1} \underline{A}\underline{P}$$

Equating the right-hand-side of the last equation to the right-hand-side of eq. (6), we obtain:

$$\left[\underline{A}\underline{A}^T\right]^{-1} \underline{A}\underline{P} = (\underline{D} \times \underline{A}) \times \underline{\theta}$$

$$\Rightarrow \left[\underline{A}\underline{A}^T\right]\left[\underline{A}\underline{A}^T\right]^{-1} \underline{A}\underline{P} = \left[\underline{A}\underline{A}^T\right](\underline{D} \times \underline{A}) \times \underline{\theta}$$

$$\Rightarrow \underline{A}\underline{P} = \left[\underline{A}\underline{A}^T\right](\underline{D} \times \underline{A}) \times \underline{\theta} = \underline{A}\underline{A}^T \underline{D}\underline{A}\underline{\theta}$$

$$\Rightarrow \underline{A}\underline{P} = \underline{A}\left[\underline{A}^T \underline{D}\underline{A}\right]\underline{\theta}$$

From (9), we observe that the term in brackets is actually $\underline{B}$' Therefore,

$$\underline{A}\underline{P} = \underline{A}\underline{B}'\underline{\theta}$$

From the above, it must be true that

$$\underline{P} = \underline{B}'\underline{\theta}$$

which is eq. (5), and this proves Fact A, that eq. (5) may be obtained from eqs. (6) and (10).

$$\underline{P}_B = (\underline{D} \times \underline{A}) \times \underline{\theta} \qquad (6)$$

$$\underline{A}^T \underline{P}_B = \underline{P} \tag{10}$$

The significance of Fact A is that we may write the equality constraints to implement the DC load flow solution as two sets of equations, one set for eq. (10) and one set for eq. (6).

Equation (10) is power balance, i.e., the flows on all branches leaving node j less the flows on all branches entering node j equals the injected power at node j. To write eq. (10) only in terms of non-negative variables, we have:

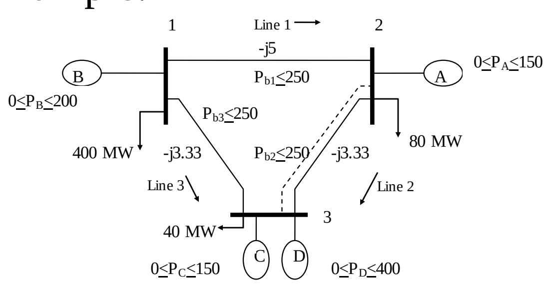
P_b is the flow on branch b if that flow is in the defined direction. P'_b is the flow on branch b if that flow is opposite to the defined direction.

$$\sum_{b:B_b=j}\left(P_b' - L_b(P_j') - P_b\right) + \sum_{b:E_b=j}\left(P_b - L_b(P_j) - P_b'\right) = P_{Dj} - P_{Gj},\ j=1,\ldots N \tag{11}$$

where $L_b(P_j')$ is the losses in branch b when the flow is opposite to the defined direction, and $L_b(P_j)$ is the losses in branch b when flow is in the defined direction. Observe that these losses are indexed (and modeled) at bus b. Also, with respect to eq. (11),

- The first summation corresponds to the flow on all branches that begin on node *j*.
- The second summation corresponds to the flow on all branches that end on node *j*.
- No branch will both end at and leave from node *j*, therefore, for any node, each branch connected to it will only appear in either the first term or the second, but not both. Furthermore, as previously indicated, P_b and P_b' cannot both be nonzero.

Example:



<u>j=1</u>:
$P'_1-L_1(P'_1)-P_1+P'_3-L_3(P_3)-P_3=P_{D1}-P_{G1}$
<u>j=2</u>:
$P'_2-L_2(P'_2)-P_2+P_1-L_1(P_1)-P'_1=P_{D2}-P_{G2}$
<u>j=3</u>:
$P_2-L_2(P_2)-P'_2+P_3-L_3(P'_3)-P'_3=P_{D3}-P_{G3}$

Equation (6) is the DC version of KVL. Writing (6) in terms of our non-negative variables, we have:

For existing branches ($b \in A_e$)

$$\theta_{B_b} - \theta_{E_b} = X_b (P_b - P_b^{'}) \qquad (12)$$

For candidate branches ($b \in A_n$):

$$\theta_{B_b} - \theta_{E_b} = X_b (P_b - P_b^{'}) + (S_b - 1)G + U_b \qquad (13)$$

$$U_b \leq 2(1 - S_b)G \qquad (14)$$

$$U_b \geq 0 \qquad (15)$$

$$S_b(t) = \sum_{n=1}^{t} Z_b(n) \qquad (16)$$

These equations need explanation, but before we give that, we introduce inequality constraints.

For existing branches ($b \in A_e$)

$$P_b + P_b^{'} \leq P_{b,\max} \qquad (17)$$

For candidate branches ($b \in A_n$):

$$P_b + P_b^{'} \leq S_b P_{b,\max} \qquad (18)$$

We also need to constrain the generation levels:

$$P_{Gj} \leq P_{Gj,\max} \quad , j=1,\ldots N \qquad (19)$$

And finally we constrain all variables to be non-negative:

$$P_{Gj}, P_b, P_b^{'}, \theta_j \geq 0 \qquad (20)$$

Recall that $Z_b(t)$ is the binary decision variable that indicates branch b is installed in period t ($Z_b(t)=1$) or not ($Z_b(t)=0$), and $S_b(t)$ is the binary variable that indicates whether branch b has been installed during any period $1, \ldots, t$ ($S_b(t)=1$) or not ($S_b(t)=0$). The U_b is a continuous fictitious variable included in the vector of decision variables.

When $S_b=1$ (branch b is in), then eqs. (13, 14, 15) reduce to

$$\theta_{B_b} - \theta_{E_b} = X_b(P_b - P_b^{'}) + U_b \qquad (13a)$$

$$U_b \le 0 \qquad (14a)$$

$$U_b \ge 0 \qquad (15a)$$

Equation (13a) is just the line flow equation for branch b, because eqs. (14a) and (15a) constrain U_b to be exactly zero.

When $S_b=0$ (branch b is out), then (18) and (20) force P_b and P_b' to be zero, and eq. (13) reduces to

$$\theta_{B_b} - \theta_{E_b} = -G + U_b \qquad (13b)$$

and eqs. (13, 14) reduce to

$$U_b \le 2G \qquad (14b)$$

$$U_b \ge 0 \qquad (15b)$$

Notice that since (14b) and (15b) allow $0 \le U_b \le 2G$, the right hand side of (13b) can vary from -G (when $U_b=0$) to G (when $U_b=2G$). Thus, as long as the angular difference

$$\theta_{B_b} - \theta_{E_b}$$

lies in a closed interval $[-G,G]$, (e.g., -2π to 2π), there always exists a variable U_b such that eqs. (13b, 14b, and 15b) hold. That is

if the value of G is large enough, eqs. (13b, 14b, 15b) put no restriction on the angular variables.

This is desirable in the case of $S_b=0$ since in this case, branch b has not been included in the network!

We could also choose $G=1000$, and the procedure would work.

Model Summary (We include notational dependence on *t* here)

Minimize:

$$C = C_E + C_I =$$

$$\sum_{t=1}^{N_T}\sum_{j=1}^{N} \varsigma^{t-1} C_j(t) P_{Gj}(t) + \sum_{t=1}^{N_T}\sum_{b\in A_n} \varsigma^{t-1} K_b(t) Z_b(t) \qquad (4)$$

Need to include loss terms in the equality constraint.

Subject to:

Equality Constraints:

$$\sum_{b:B_b=j} P_b^{'} - P_b + \sum_{b:E_b=j} P_b - P_b^{'} = P_{Dj} - P_{Gj},\ j = 1,\ldots N \qquad (11)$$

For existing branches ($b \in A_e$)

$$\theta_{B_b}(t) - \theta_{E_b}(t) = X_b(P_b(t) - P_b^{'}(t)) \qquad (12)$$

For candidate branches ($b \in A_n$):

$$\theta_{B_b}(t) - \theta_{E_b}(t)$$
$$= X_b(P_b(t) - P_b^{'}(t)) + (S_b(t) - 1)G + U_b(t), \qquad (13)$$

$$U_b(t) \le 2(1 - S_b(t))G \qquad (14)$$

$$U_b(t) \ge 0 \qquad (15)$$

$$S_b(t) = \sum_{n=1}^{t} Z_b(n) \qquad (16)$$

Inequality constraints:

For existing branches ($b \in A_e$)

$$P_b(t) + P_b^{'}(t) \le P_{b,\max} \qquad (17)$$

For candidate branches ($b \in A_n$):

$$P_b(t) + P_b^{'}(t) \le S_b(t) P_{b,\max} \qquad (18)$$

For generation levels:

$$P_{Gj}(t) \le P_{Gj,\max} \qquad (23)$$

Non-negativity:

$$P_{Gj}(t), P_b(t), P_b^{'}(t), \theta_j(t) \ge 0 \qquad (24)$$

Comment: If the planning horizon contains only 1 period ($N_T=0$), then $S_b(t)=Z_b(t)$, and we may eliminate eq. (16) and replace every occurrence of $S_b(t)$ in our formulation with $Z_b(t)$.

Appendix B: The model of Bahiense [15]

$$Min_{\{x,\, f,\, g,\, \theta\}} \sum_{t \in \mathrm{H}inv} \beta(t) c_I x(t)$$

Subject to

$$\sum_{k=(i,j)\, j \in \Omega i} f_k(t) - g_i(t) = d_i(t), \qquad i = 1, n \quad \forall t \in \mathrm{H} \tag{a}$$

$$f_k(t) - \gamma 0_k(\theta_i(t) - \theta_j(t)) = 0,$$
$$k = (i, j),\ j \in \Omega_i^0,\ i = 1, n \quad \forall t \in \mathrm{H} \tag{b}$$

$$-M_k(1 - S_k(t)) \le f_k(t) - \gamma_k(\theta_i(t) - \theta_j(t)) \le M_k(1 - S_k(t)),$$
$$k = (i, j),\ j \in \Omega_i^+,\ i = 1, n \quad \forall t \in \mathrm{H} \tag{c}$$

$$S(t) = \sum_{i \in \mathrm{H}inv, i \le t} x(i) \tag{d}$$

$$-f\,0_k^{\max}(t) \le f_k(t) \le f\,0_k^{\max}(t),$$
$$k = (i, j),\ j \in \Omega_i^0,\ i = 1, n \quad \forall t \in \mathrm{H} \tag{e}$$

$$-f_k^{\max} S_k(t) \le f_k(t) \le f_k^{\max} S_k(t),$$
$$k = (i, j),\ j \in \Omega_i^+,\ i = 1, n \quad \forall t \in \mathrm{H} \tag{f}$$

$$0 \le g_i(t) \le g_i^{\max}(t), \qquad i = 1, n \quad \forall t \in \mathrm{H} \tag{g}$$

$$\theta_{ref}(t) = 0 \tag{h}$$

$$x(t), S(t) \in \{0,1\}^m \tag{i}$$

Nomenclature for this model is provided below:

t:	Time step
n:	Number of nodes
m :	Number of candidate circuits
H:	Planning time horizon (set of time steps)
H_{inv}:	Set of Investment time steps within H
Ω_i^0:	Set of existing circuits connected to bus i, i=1, n
Ω_i^+:	Set of candidate circuits connected to bus i, i=1, n

Ω_i:	The union of Ω_i^0 and Ω_i^+
$f(t)$:	Vector of flows on step t (existing and candidates)
$f0\max(t)$:	Vector of circuit capacities on step t (existing)
$f\max$:	Vector of circuit capacities (candidates)
$g(t)$:	Vector of bus generations on step t
$g\max(t)$:	Vector of bus generation capacities on step t
$d(t)$:	Vector of bus active loads
$\theta(t)$:	Vector of bus voltage angles in radians on step t
$x(t)$:	Investment decision binary vector on step t
$S(t)$:	Accumulate investment decision vector on step t
cI :	Vector of unit investment cost of candidates
co :	Vector of unit generation production cost
$\gamma 0$:	Vector of circuit susceptance (existing)
γ:	Vector of circuit susceptance (candidates)
M:	Vector of penalty factors of candidate circuits
$\beta(t)$:	Discount factor for step t

Equation (a) represents the nodal power balance; (b) and (c) represent Kirchhoff's Voltage Law for existing and candidate circuits, respectively; (d) is the relationship between transmission investment on each investment time step t and accumulative investment until time step t (note S and x are vectors and therefore they have no subscripts); (e) and (f) are transmission capacity constraints for existing and candidate circuits, respectively; (g) is the generation output limits; (h) sets reference bus voltage angle to be 0; and (i) defines investment variables to be binary variable.

The nomenclature for this model is clearly different from the nomenclature of the other models presented in these notes. We observe that the disjunctive part of the model presented in this appendix (Appendix B) is similar to the disjunctive part of the model presented in the main body of these notes. However, the disjunctive part of the model presented in Appendix A differs.

In the model presented here, in Appendix B, which we refer to as Bahiense's model, the disjunctive relation for candidate branches is

$$-M_k(1-S_k(t)) \le f_k(t) - \gamma_k(\theta_i(t)-\theta_j(t)) \le M_k(1-S_k(t)),$$

$$k=(i,j),\ j\in\Omega_i^+,\ i=1,n \quad \forall t\in \mathrm{H} \qquad \text{(c)}$$

In the model of Appendix A, which we refer to as "Wang's model," the disjunctive relation for candidate branches is given as:

$$\theta_{B_b}(t) - \theta_{E_b}(t)$$
$$= X_b(P_b(t) - P_b^{'}(t)) + (S_b(t) - 1)G + U_b(t), \tag{13}$$
$$U_b(t) \le 2(1 - S_b(t))G \tag{14}$$
$$U_b(t) > 0 \tag{15}$$

We want to show that these two models are equivalent. To do so, we first observe in Bahiense's model, (25e,f) allow the flow variable to be negative, in contrast to Wang's model where we prevented this by utilizing two variables for flow P_b and P_b'. This was done in Wang's model because the LP solver used for that model was "standard" in that it did not allow negative decision variables, whereas the LP solver used for Bahiense's model allows it and then performs a variable transformation internally to satisfy its LP solver. And so we will write Wang's model as if it were to be used by Bahiense's solver, i.e.,

$$\theta_{B_b}(t) - \theta_{E_b}(t)$$
$$= X_b P_b(t) + (S_b(t) - 1)G + U_b(t), \tag{13}$$
$$U_b(t) \le 2(1 - S_b(t))G \tag{14}$$
$$U_b(t) > 0 \tag{15}$$

We also recognize that susceptance γ_k is used in Bahiense's model, whereas reactance X_b is used in Wang's model. We will use the susceptance notation of Bahiense's model, i.e., $X_b=1/\gamma_b$. Substituting, we get:

$$\theta_{B_b}(t) - \theta_{E_b}(t)$$
$$= (1/\gamma_b)P_b(t) + (S_b(t) - 1)G + U_b(t), \tag{13}$$
$$U_b(t) \le 2(1 - S_b(t))G \tag{14}$$
$$U_b(t) \ge 0 \tag{15}$$

Solving (13) for $U_b(t)$, we obtain

$$\theta_{B_b}(t)-\theta_{E_b}(t)-(1/\gamma_b)P_b(t)-(S_b(t)-1)G=U_b(t) \qquad \text{(i)}$$

Imposing (14) and (15) on (i), we obtain:

$$0\le\theta_{B_b}(t)-\theta_{E_b}(t)-(1/\gamma_b)P_b(t)-(S_b(t)-1)G\le 2(1-S_b(t))G \qquad \text{(ii)}$$

Using $-(S_b(t)-1)G=(1-S_b(t))G$, (ii) becomes

$$0\le\theta_{B_b}(t)-\theta_{E_b}(t)-(1/\gamma_b)P_b(t)+(1-S_b(t))G\le 2(1-S_b(t))G \qquad \text{(iii)}$$

Subtracting $(1-S_b(t))G$ from all terms, we obtain

$$-(1-S_b(t))G\le\theta_{B_b}(t)-\theta_{E_b}(t)-(1/\gamma_b)P_b(t)\le(1-S_b(t))G \qquad \text{(iv)}$$

Multiply through by -1 and reverse the inequalities:

$$(1-S_b(t))G\ge-(\theta_{B_b}(t)-\theta_{E_b}(t))+(1/\gamma_b)P_b(t)\ge-(1-S_b(t))G \qquad \text{(v)}$$

Rewrite (v), switching the left and right bounds:

$$-(1-S_b(t))G\le-(\theta_{B_b}(t)-\theta_{E_b}(t))+(1/\gamma_b)P_b(t)\le(1-S_b(t))G \qquad \text{(vi)}$$

Rearrange, and compare to (25c):

$$-G(1-S_b(t))\le(1/\gamma_b)P_b(t)-(\theta_{B_b}(t)-\theta_{E_b}(t))\le G(1-S_b(t)) \qquad \text{(vii)}$$

$$-M_k(1-S_k(t))\le f_k(t)-\gamma_k(\theta_i(t)-\theta_j(t))\le M_k(1-S_k(t)), \qquad \text{(25c)}$$

and we see that the effects of $S_b(t)$ is the same as the effect of $S_k(t)$. That is, consider when they are both 1 (the circuit is "in"), then we have:

$$0\le(1/\gamma_b)P_b(t)-(\theta_{B_b}(t)-\theta_{E_b}(t))\le 0$$

$$0\le f_k(t)+\gamma_k(\theta_i(t)-\theta_j(t)\le 0$$

which are equivalent, i.e., they both require the middle term to equal 0, thus forcing the flow to equal the angular difference across the line multiplied by the line susceptance.

Now consider when both $S_b(t)$ and $S_k(t)$ are 0 (the circuit is "out"), then we have:

$$-G\le(1/\gamma_b)P_b(t)-(\theta_{B_b}(t)-\theta_{E_b}(t))\le G$$

$$-M_k\le f_k(t)+\gamma_k(\theta_i(t)-\theta_j(t)\le M_k$$

If G and M_k are both chosen to be large positive numbers, then the last two equations have the same effect, which is to have no effect, since they allow the flow $P_b(t)$ (or $f_k(t)$) directly between two nodes to be completely unconstrained by the DC power flow expression (product of reactance and angular difference) associated with those two nodes, as it is if the two nodes are not connected.

➔These two models are equivalent, i.e., they are just different representations of the same "disjunctive" modeling approach.

References

[1] D. Brooks, “Transmission planning risk considerations,” presentation slides, Eastern Interconnection States Planning Council (EISPC) Meeting, October 8, 2013.

[2] A. Gaikwad and K. Carden, “Probabilistic Risk Analysis: A consideration of risks in transmission planning,” presentation slides, Eastern Interconnection States Planning Council (EISPC) Meeting, October 8, 2013.

[3] Midcontinent Independent System Operator (MISO) website, “MISO Transmission Expansion Plan (MTEP), Overview.” Accessed 4/4/2024. Available: www.misoenergy.org/planning/transmission-planning/mtep/#t=10&p=0&s=&sd=.

[4] D. Osborn, “MISO’s transmission planning processes.” Accessed 4/4/2024. Available: https://www.cne.cl/wp-content/uploads/2017/04/Dale-Osborn.pdf.

[5] The National Electrical Manufacturers Association, “Siting transmission corridors – a real life game of chutes and ladders,” [Online]. Available: www.nema.org/docs/default-source/advocacy-document-library/nema_chutesandladder_2017_revised-4web.pdf.

[6] J. McCalley and Q. Zhang, “Macrogrids in the Mainstream,” a report prepared for the Americans for a Clean Energy Grid, December, 2020. [Online]. Available: https://cleanenergygrid.org/macro-grids-mainstream/.

[7] B. Earley, “FERC considering changes to transmission incentives,” in Inside Energy & Environment, a publication of Covington & Burling LLP, March 25, 2020. [Online]. Available: www.insideenergyandenvironment.com/2020/03/ferc-considering-changes-to-transmission-incentives/.

[8] “Coordination of Federal Authorizations for Electric Transmission Facilities,” US Department of Energy Regulation, 2018 Edition, CreateSpace Publishing. [Online]. Available: www.barnesandnoble.com/w/coordination-of-federal-authorizations-for-electric-transmission-facilities-the-law-library/1129053398.

[9] A. Klass and J. Rossi, “Reconstituting the federalism battle in energy transportation,” Harvard Environmental Law Revie, Vol. 41, 2017, pp. 423-492. [Online]. Available: https://harvardelr.com/wp-content/uploads/sites/12/2017/08/KlassRossi_final.pdf.

[10] FERC Staff, “Report on Barriers and Opportunities for High Voltage Transmission: A Report to the Committees on Appropriations of Both Houses of Congress Pursuant to the 2020 Further Consolidated Appropriations Act,” June, 2020. [Online]. Available: https://cleanenergygrid.org/wp-content/uploads/2020/08/Report-to-Congress-on-High-Voltage-Transmission_17June2020-002.pdf.

[11] TEPCC Webpages, https://www.wecc.biz/TEPPC/Pages/Default.aspx.

[12] “2015 TEPPC Study Program,” Draft for TAS Review, Studies Work Group, April 16, 2015. This document was available at the below URL but is no longer (as of 4/4/2024). www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&cad=rja&uact=8&ved=0ahUKEwjW8rODzPfLAhWIbSYKHfxQDl8QFggcMAA&url=https%3A%2F%2Fwww.wecc.biz%2FAdministrative%2F150417_TEPPC_2015_Study_Program_Draft.docx&usg=AFQjCNFBtsH4neDNynwbD29mGsaNXzrMCw

[13] X. Wang and J. McDonald, “Modern Power System Planning,” McGraw Hill Book Company, London, 1994.

[14] R. Villasana, “Transmission network planning using linear and mixed linear integer programming,” Ph.D. dissertation, Rensselaer Polytechnic Institute, 1984.

[15] L. Bahiense, G. C. Oliveira, M. Pereira, and S. Granville, “A mixed integer disjunctive model for transmission network expansion,” *Power Systems, IEEE Transactions on*, vol. 16, no. 3, pp. 560–565, Aug. 2001.

[16] R. Romero, A. Monticelli, A. Garcia and S. Haffner, "Test systems and mathematical models for transmission network expansion planning," in Proc. 2002 IEE Generation Transmission Distribution, vol. 149, No. 1.
[17] J. M. Areiza, G. Latorre, R. D. Cruz, and A. Villegas, “Classification of publications and models on transmission expansion planning,” *IEEE Trans. Power Syst.*, vol. 18, no. 02, pp. 938–946, May 2003.
[18] Y. Li and J. McCalley, “A Decimal-Binary Disjunctive Model for Value-based Bulk Transmission Expansion Planning,” under development.
[19] R. Pletka, J. Khangura, A. Rawlins, E. Waldren, and D. Wilson, “Capital Costs for Transmission and Substations - Updated Recommendations for WECC Transmission Expansion Planning,” B&V Project No. 181374, prepared for the Western Electric Coordinating Council (WECC), February, 2014, prepared by Black and Veatch, available at www.wecc.biz/Reliability/2014_TEPPC_Transmission_CapCost_Report_B+V.pdf.
[20] R. Dunlop, R. Gutman, and P. Marchenko, “Analytical Development of Loadability Characteristics for EHV and UHV Transmission Lines,” IEEE Transactions on Power Apparatus and Systems, Vol. PAS-98, No. 2, March/April 1979.
[21] Y. Li, “Transmission design and optimization at the national level,” Ph.D. Dissertation, Iowa State University, 2014.
[22] B. B. Chakrabarti; C. Edwards; C. Callaghan; S. Ranatunga, “Alternative loss model for the New Zealand electricity market using SFT,” IEEE Power and Energy Society General Meeting, 2011
Pages: 1 - 8, DOI: 10.1109/PES.2011.6039794.
[23] E. Ibanez. (2011b) ‘A multiobjective optimization approach to the operation and investment of the national energy and transportation systems,’ Doctoral thesis, Iowa State University, Ames, Iowa.