Unit Commitment 3 – Illustrations

1.0 Introduction

Recall our UC problem formulation. However, there was an error in it in that the shutdown variable x_{ij} only appeared in the inequality $z_{it} \geq z_{it-1} - x_{it}$. Therefore, if z_{it} and z_{it-1} are both zero (which means that a unit is down for two consecutive periods and often happens), the variable x_{it} can be either 1 or 0. But in this case, we need it to be forced to 0, otherwise, if it is 1, it is saying that a shutdown unit is being shutdown, which makes no sense.

To address this issue, we will add a term corresponding to shutdown costs in the objective. Although many UC programs ignore shutdown costs, there are some, and we will model them here.

Our new model is therefore as follows:

$$\min \quad \underbrace{\sum_t \sum_i z_{it} F_i}_{\text{Fixed Costs}} + \underbrace{\sum_t \sum_i g_{it} C_i}_{\text{Production Costs}} + \underbrace{\sum_t \sum_i y_{it} S_i}_{\text{Startup Costs}} + \underbrace{\sum_t \sum_i x_{it} H_i}_{\text{Shutdown Costs}} \qquad (1)$$

subject to

power balance	$\sum_i g_{it} = D_t = \sum_i d_{it}$	$\forall\, t,$	(2)
reserve	$\sum_i r_{it} = SD_t$	$\forall\, t,$	(3)
min generation	$g_{it} \geq z_{it} MIN_i$	$\forall\, i, t,$	(4)
max generation	$g_{it} + r_{it} \leq z_{it} MAX_i$	$\forall\, i, t,$	(5)
max spinning reserve	$r_{it} \leq z_{it} MAXSP_i$	$\forall\, i, t,$	(6)
ramp rate pos limit	$g_{it} \leq g_{it-1} + MxInc_i$	$\forall\, i, t,$	(7)
ramp rate neg limit	$g_{it} \geq g_{it-1} - MxDec_i$	$\forall\, i, t,$	(8)
start if off-then-on	$z_{it} \leq z_{it-1} + y_{it}$	$\forall\, i, t,$	(9)
shut if on-then-off	$z_{it} \geq z_{it-1} - x_{it}$	$\forall\, i, t,$	(10)
normal line flow limit	$\sum_i a_{ki}(g_{it} - d_{it}) \leq MxFlow_k$	$\forall\, k, t,$	(11)

security line flow limits $\sum_i a_{ki}^{(j)}(g_{it} - d_{it}) \le MxFlow_k^{(j)} \quad \forall\, k, j, t,$ (12)

where the decision variables are:

- g_{it} is the MW produced by generator i in period t,
- r_{it} is the MW of spinning reserves from generator i in period t,
- z_{it} is 1 if generator i is dispatched during t, 0 otherwise,
- y_{it} is 1 if generator i starts at beginning of period t, 0 otherwise,
- x_{it} is 1 if generator i shuts at beginning of period t, 0 otherwise,

Other parameters are

- D_t is the total demand in period t,
- SD_t is the spinning reserve required in period t,
- F_{it} is fixed cost ($/period) of operating generator i in period t,
- C_{it} is prod. cost ($/MW/period) of operating gen i in period t;
- S_{it} is startup cost ($) of starting gen i in period t.
- H_{it} is shutdown cost ($) of shuting gen i in period t.
- $MxInc_i$ is max ramprate (MW/period) for increasing gen i output
- $MxDec_i$ is max ramprate (MW/period) for decreasing gen i output
- a_{ij} is linearized coefficient relating bus i injection to line k flow
- $MxFlow_k$ is the maximum MW flow on line k
- $a_{ki}^{(j)}$ is linearized coefficient relating bus i injection to line k flow under contingency j,
- $MxFlow_k^{(j)}$ is the maximum MW flow on line k under contingency j

The above problem statement is identical to the one given in [**Error! Bookmark not defined.**] with the exception that here, we have added eqs. (11) and (12).

➔The addition of eq. (11) alone provides that this problem is a transmission-constrained unit commitment problem.

➔ The addition of eqs. (11) and (12) together provides that this problem is a security-constrained unit commitment problem.

2.0 Example Data

Here, we provide some data to use in solving this problem.

We illustrate using an example that utilizes the same system we have been using in our previous notes, where we had 3 generator buses in a 4 bus network supplying load at 2 different buses, but this time we will model each generator with the ability to submit 3 offers.

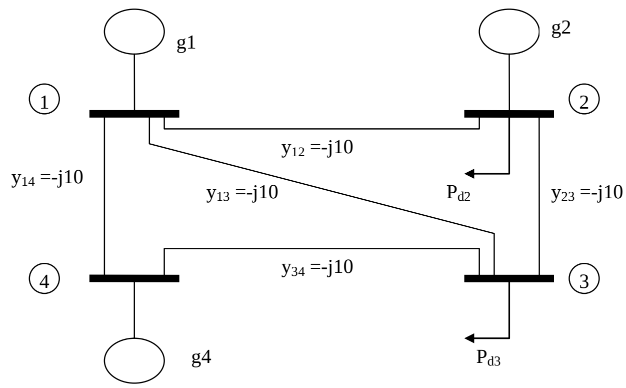


Fig. 1: One line diagram for example system

The offers, in terms of fixed costs, production costs, and corresponding min and max generation limits are as follows

Production costs (in $/pu-hr):

Unit, k	Fixed costs ($/hr)	Startup Costs ($)	Shutdown Costs ($)	Production Costs ($/pu-hr)		
				g_{k1t}	g_{k2t}	g_{k4t}
1	50	100	20	1246	1307	1358
2	50	100	20	1129	1211	1282
4	50	100	20	1183	1254	1320

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leq \begin{bmatrix} g_{11t} \\ g_{12t} \\ g_{13t} \\ g_{21t} \\ g_{22t} \\ g_{23t} \\ g_{41t} \\ g_{42t} \\ g_{43t} \end{bmatrix} \leq \begin{bmatrix} 0.50 \\ 0.60 \\ 0.40 \\ 0.35 \\ 0.60 \\ 0.20 \\ 0.45 \\ 0.50 \\ 0.40 \end{bmatrix}, \forall t$$

The UC problem is for a 24 hour period, with loading data given as below. Figure 2, the load curve, illustrates variation of load with time over the 24 hour period.

Hour, t	Load, D_t (pu)
1	1.50
2	1.40
3	1.30
4	1.40
5	1.70
6	2.00
7	2.40
8	2.80
9	3.20
10	3.30
11	3.30
12	3.20
13	3.20
14	3.30
15	3.35
16	3.40
17	3.30
18	3.30
19	3.20
20	2.80
21	2.30
22	2.00
23	1.70
24	1.60

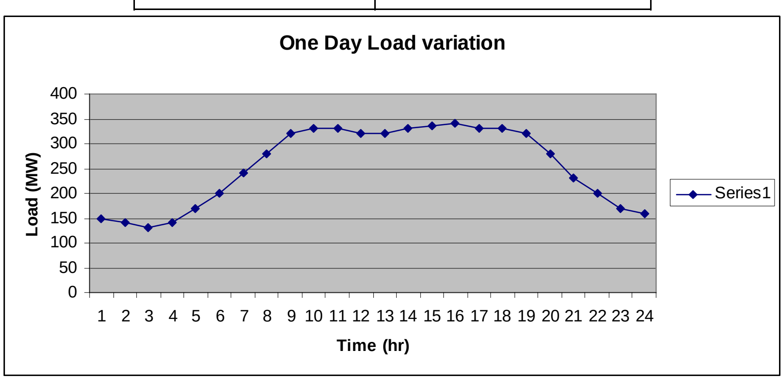


Fig. 2: Load curve

2.0 Example – 4 hours

For this solution, we will only include startup and shutdown constraints. In order to illustrate all data entered, we will analyze only the first four hours. The CPLEX code to do this is given below.

```
minimize
  50 z11 +50 z12 + 50 z13 + 50 z14
 +50 z21 +50 z22 + 50 z23 +50 z24
 +50 z41 +50 z42 + 50 z43 +50 z44
 +1246 g111 + 1307 g121 + 1358 g131
  +1129 g211 + 1211 g221 + 1282 g231
  +1183 g411 + 1254 g421 + 1320 g431
 +1246 g112 + 1307 g122 + 1358 g132
  +1129 g212 + 1211 g222 + 1282 g232
  +1183 g412 + 1254 g422 + 1320 g432
 +1246 g113 + 1307 g123 + 1358 g133
  +1129 g213 + 1211 g223 + 1282 g233
  +1183 g413 + 1254 g423 + 1320 g433
 +1246 g114 + 1307 g124 + 1358 g134
  +1129 g214 + 1211 g224 + 1282 g234
  +1183 g414 + 1254 g424 + 1320 g434
 +100 y12 + 100 y13 +100 y14
 +100 y22 + 100 y23 +100 y24
 +100 y42 + 100 y43 +100 y44
 +20 x12 + 20 x13 +20 x14
 +20 x22 + 20 x23 + 20 x24
 +20 x42 + 20 x43 +20 x44
subject to
 loadhr1: g111+g121+g131+g211+g221+g231+g411+g421+g431=1.5
 loadhr2: g112+g122+g132+g212+g222+g232+g412+g422+g432=1.4
 loadhr3: g113+g123+g133+g213+g223+g233+g413+g423+g433=1.3
 loadhr4: g114+g124+g134+g214+g224+g234+g414+g424+g434=1.4
 initialu1: z11=0
 initialu2: z21=1
 initialu4: z41=1
 starthr21u1: z12-z11-y12<=0
 starthr32u1: z13-z12-y13<=0
 starthr43u1: z14-z13-y14<=0
 starthr21u2: z22-z21-y22<=0
 starthr32u2: z23-z22-y23<=0
 starthr43u2: z24-z23-y24<=0
 starthr21u4: z42-z41-y42<=0
 starthr32u4: z43-z42-y43<=0
 starthr43u4: z44-z43-y44<=0
 shuthr21u1: z12-z11+x12>=0
 shuthr32u1: z13-z12+x13>=0
 shuthr43u1: z14-z13+x14>=0
 shuthr21u2: z22-z21+x22>=0
 shuthr32u2: z23-z22+x23>=0
 shuthr43u2: z24-z23+x24>=0
 shuthr21u4: z42-z41+x42>=0
 shuthr32u4: z43-z42+x43>=0
 shuthr43u4: z44-z43+x44>=0
```

g111 - 0.5 z11<= 0
g112 - 0.5 z12<= 0
g113 - 0.5 z13<= 0
g114 - 0.5 z14<= 0
g121 - 0.6 z11<= 0
g122 - 0.6 z12<= 0
g123 - 0.6 z13<= 0
g124 - 0.6 z14<= 0
g131 - 0.4 z11<= 0
g132 - 0.4 z12<= 0
g133 - 0.4 z13<= 0
g134 - 0.4 z14<= 0

g211 - 0.35 z21<= 0
g212 - 0.35 z22<= 0
g213 - 0.35 z23<= 0
g214 - 0.35 z24<= 0
g221 - 0.6 z21<= 0
g222 - 0.6 z22<= 0
g223 - 0.6 z23<= 0
g224 - 0.6 z24<= 0
g231 - 0.2 z21<= 0
g232 - 0.2 z22<= 0
g233 - 0.2 z23<= 0
g234 - 0.2 z24<= 0

g411 - 0.45 z41<= 0
g412 - 0.45 z42<= 0
g413 - 0.45 z43<= 0
g414 - 0.45 z44<= 0
g421 - 0.5 z41<= 0
g422 - 0.5 z42<= 0
g423 - 0.5 z43<= 0
g424 - 0.5 z44<= 0
g431 - 0.4 z41<= 0
g432 - 0.4 z42<= 0
g433 - 0.4 z43<= 0
g434 - 0.4 z44<= 0

```
Bounds
 0<= g111
 0<= g112
 0<= g113
 0<= g114
 0<= g121
 0<= g122
 0<= g123
 0<= g124
 0<= g131
 0<= g132
 0<= g133
 0<= g134

 0<= g211
 0<= g212
 0<= g213
 0<= g214
 0<= g221
 0<= g222
 0<= g223
 0<= g224
 0<= g231
 0<= g232
 0<= g233
 0<= g234

 0<= g411
 0<= g412
 0<= g413
 0<= g414
 0<= g421
 0<= g422
 0<= g423
 0<= g424
 0<= g431
 0<= g432
 0<= g433
 0<= g434
Integer
 z11 z12 z13 z14
 z21 z22 z23 z24
 z41 z42 z43 z44
 y12 y13 y14
 y22 y23 y24
 y42 y43 y44
 x12 x13 x14
 x22 x23 x24
 x42 x43 x44
end
```

Result: CPLEX gives an objective function value of 7020.7 $.

CPLEX> display solution variables -

Variable Name	Solution Value
z21	1.000000
z22	1.000000
z23	1.000000
z24	1.000000
z41	1.000000
z42	1.000000
z43	1.000000
z44	1.000000
g211	0.350000
g221	0.600000
g411	0.450000
g421	0.100000
g212	0.350000
g222	0.600000
g412	0.450000
g213	0.350000
g223	0.500000
g413	0.450000
g214	0.350000
g224	0.600000
g414	0.450000

All other variables in the range 1-66 are 0.

Note that all y- and x-variables are zero, therefore there is no starting up or shutting down.

One should check that the generation in each hour equals the demand in that hour:

g211+g221+g411+g421=0.35+0.6+0.45+0.1=1.5

g212+g222+g412=0.35+0.6+0.45=1.4

g213+g223+g413=0.35+0.5+0.45=1.3

g214+g224+g414=0.35+0.6+0.45=1.4

This very simple solution was obtained as a result of the fact that the initial solution of

initialu1: z11=0
initialu2: z21=1
initialu4: z41=1

was in fact the best one for the initial loading condition, and since the loading condition hardly changed during the first four hours, there was no reason to change any of the units.

Let's try a different initial condition:

initialu1: z11=1
initialu2: z21=0
initialu4: z41=1

Result: CPLEX gives an objective function value of 7208.9 $.

CPLEX> display solution variables -

Variable Name	Solution Value
z11	1.000000
z22	1.000000
z23	1.000000
z24	1.000000
z41	1.000000
z42	1.000000
z43	1.000000
z44	1.000000
g111	0.500000
g121	0.050000
g411	0.450000
g421	0.500000
g212	0.350000
g222	0.600000
g412	0.450000
g213	0.350000
g223	0.500000
g413	0.450000

g214	0.350000
g224	0.600000
g414	0.450000
y22	1.000000
x12	1.000000

All other variables in the range 1-66 are 0.

Why was this solution more expensive?

➔ Because we initialized the solution with more expensive units, to get back to the less expensive solution, notice that the program forces unit 2 to start up (y_{22}=1) and unit 1 to shut down (x_{12}=1) at the beginning of period 2. Apparently, the additional cost of starting unit 2 ($100) and shutting down unit 1 ($20) was not enough to offset the savings associated with running the more efficient unit over the remaining three hours of the simulation.

Let's test our theory by increasing the startup costs of unit 2 from $100 to $10,000. The objective function value in this case is $7281.25 (higher than the last solution). The decision variables are:

Variable Name	Solution Value
z11	1.000000
z12	1.000000
z13	1.000000
z14	1.000000
z41	1.000000
z42	1.000000
z43	1.000000
z44	1.000000
g111	0.500000
g121	0.050000
g411	0.450000
g421	0.500000
g112	0.500000
g412	0.450000
g422	0.450000

g113	0.500000
g413	0.450000
g423	0.350000
g114	0.500000
g414	0.450000
g424	0.450000

All other variables in the range 1-66 are 0.

We observe that unit 1 was on-line the entire four hours, i.e, there was no switching, something we expected since the start-up cost of unit 2 was so very high.

2.0 Example – 24 hours

We refrain from providing the data in this case because it is extensive, having 426 variables:

72 z-variables
69 y-variables
69 x-variables
216 g-variables

Rather, we have posted the dataset on the web page under "UC24 Data" for the 04/16/08 date.

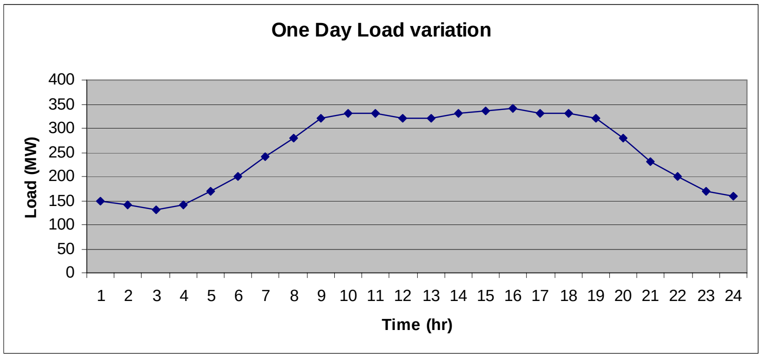
The solution was initialized at

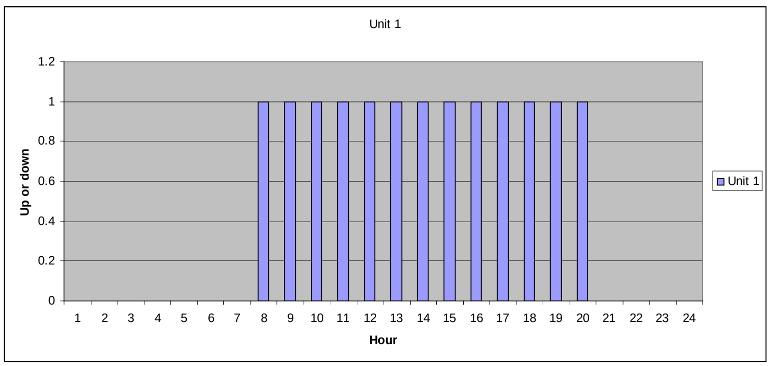
initialu1: z11=0
initialu2: z21=1
initialu4: z41=1

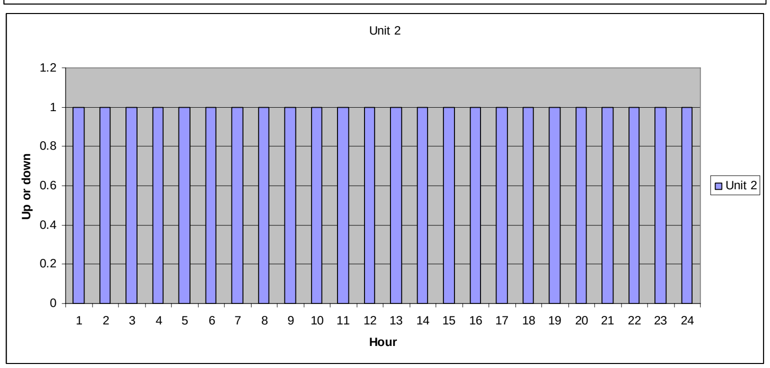
which is the most economic solution for this loading level.

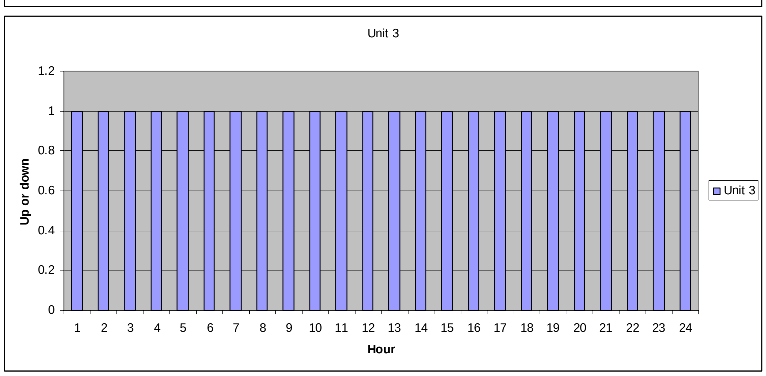
The output is most easily analyzed by using
"display solution variables -"
and then searching the output variables for y-variables and/or x-variables that are listed (and therefore 1). These variables indicate changes in the unit commitment. In studying the load curve, what kind of changes do you expect?

The result, objective value=$77667.3, shows that the only x and y variables that are non-zero are y18 and x121. This means that the changes in the unit commitment occur only for unit 1 and only at hours 18 and 21. A pictorial representation of the unit commitment through the 24 hour period is shown below.

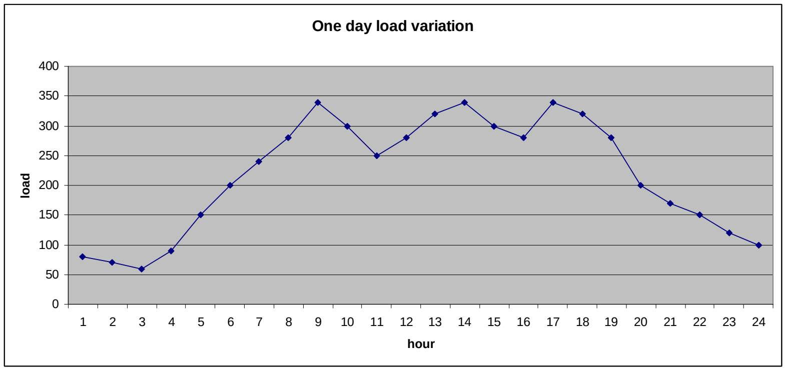


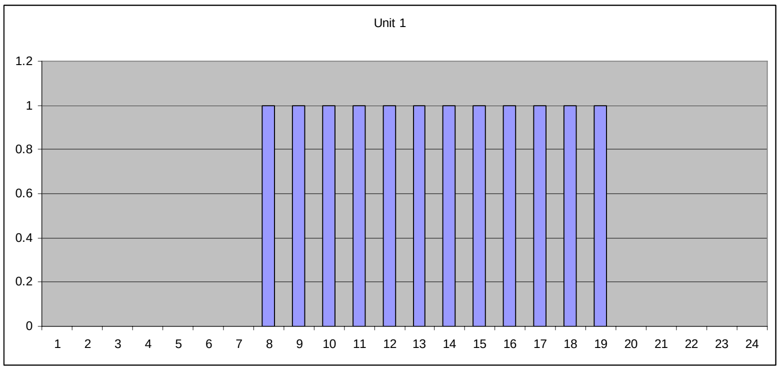


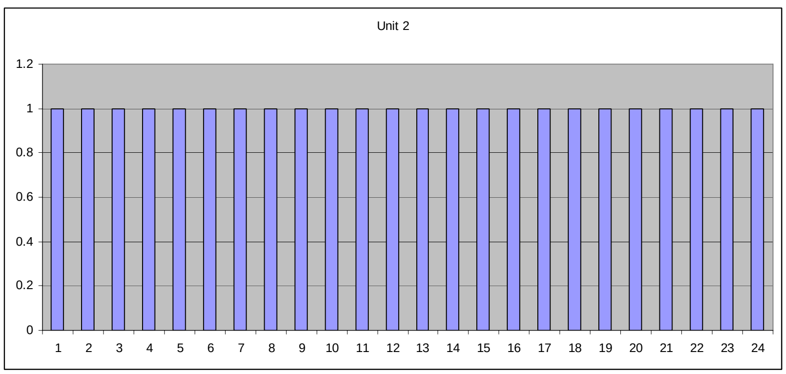


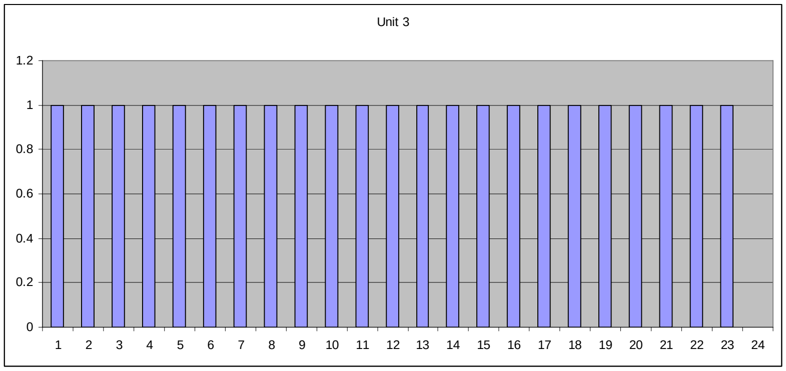


To perform additional investigation, the load curve was modified as shown below (UC24a.lp). All other data remained as before. The result, with objective function value of $, shows that the only x and y variables that are non-zero are y18, x120, and x424. A pictorial representation of the UC through the 24 hour period is shown below.

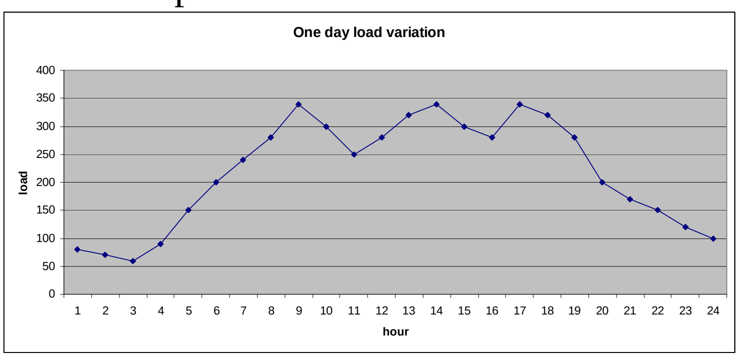


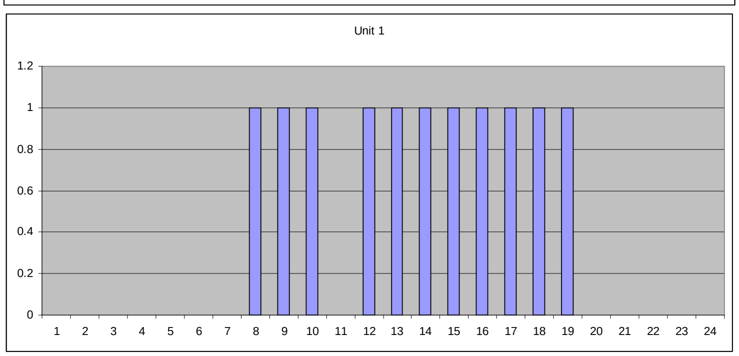


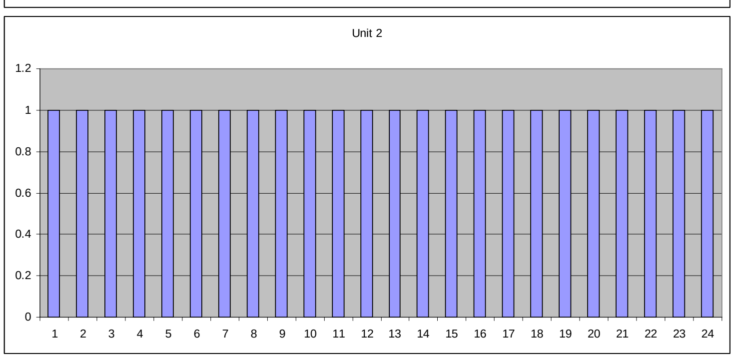


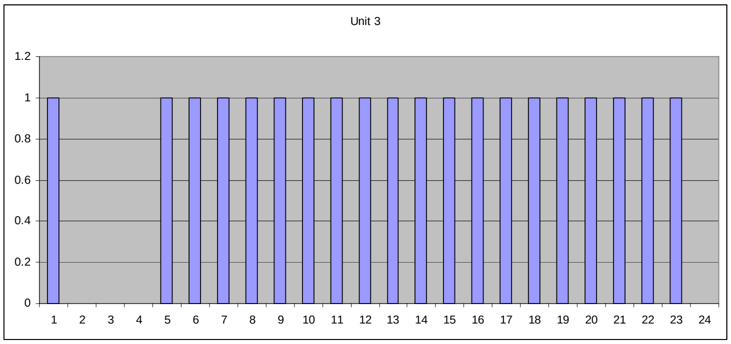


In a last investigation, the load curve remained modified, and startup costs were reduced to $10, shutdown costs reduced to $2. All other data remained as before (UC24b.lp). The result, with objective function value of $66,867.95, shows that the only x and y variables that are non-0 are y18, x112, y45, x111, x120, x42, x424. A pictorial representation of the UC through the 24 hour period is shown below.








Unit 3
1.2
1
0.8
0.6
0.4
0.2
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24