Unit Commitment 1

1.0 Introduction

We have, at the beginning of the course, discussed the day-ahead market (DAM). In these notes, we will study the main tool used to implement the DAM, which is the security-constrained unit commitment program, or SCUC. Before doing so, however, it will be useful to review some basics about the DAM. An effective way to do this is to take a look at some descriptions given by a few industry authors. You are encouraged to review the papers from which these quotes were taken. Notice that any references made inside the quotations are given only in the bibliography of the subject paper and not in the bibliography of these notes. References made outside of the quotations are given in the bibliography of these notes.

1.1 Paper by Chow & De Mello:

Reference [1] offers an overall view of the sequence of functions used by an ISO, as given in Fig. 1. Observe that the "day-ahead scheduling" and the "real time commitment and dispatch" both utilize the SCUC.

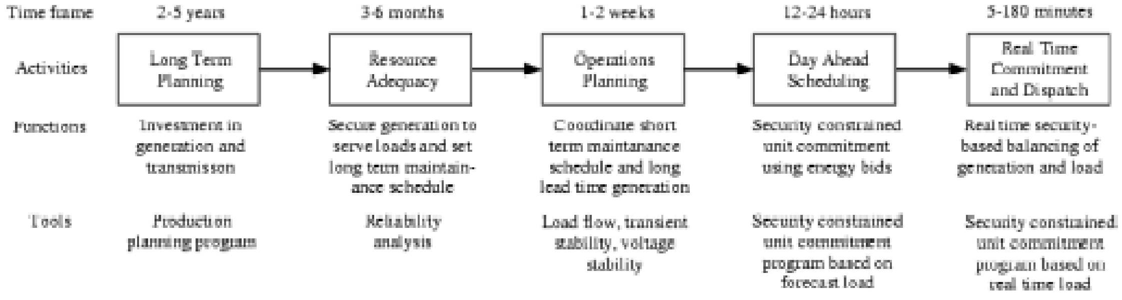

Fig. 1

They state:

"Electricity is a commodity that cannot be effectively stored and the energy-supplying generators have limits on how quickly they can be started and ramped up or down. As a result, both the supply and demand become more inelastic and the electricity market becomes more volatile and vulnerable as it gets closer to real time [34]. To achieve a stable margin as well as to maintain the system reliability, a forward market is needed to provide buyers and sellers the opportunity to lock in energy prices and quantities and the ISO to secure adequate resources to meet predicted energy demand well in advance of real time. Thus architecturally, many ISOs (e.g. PJM, ISO New England, New York ISO) take a multisettlement approach for market design…."

"The two main energy markets, each producing a financial settlement, in a multisettlement system, are the following.
1) DAM: schedules resources and determines the LMPs for the 24 h of the following day based on offers to sell and bids to purchase energy from the market participants.
2) Real-time market: optimizes the clearing of bids for energy so that the real-time system load matching and reliability requirements are satisfied based on actual system operations. LMPs are computed for settlement at shorter intervals, such as 5–10 min…."

"Fig. 6 shows the timeline of the multiple-settlement systems used in NYISO, PJM, and ISO-NE, which are typical of those used in practice. Supply and demand bids are submitted for the DAM, typically 12–24 h ahead of the real-time operation. Then the day-ahead energy prices are computed and posted, 6–12 h ahead of real-time operation…."

"The DAM typically consists of supply and demand bids on an hourly basis, usually from midnight to the following midnight. The supply bids include generation supply offers with start-up and no-load costs, incremental and decremental bids[1], and external transactions schedules. The demand bids are submitted by loads individually or collectively through load-serving entities. In scheduling the supply to meet the demand, all the operating constraints such as transmission network constraints, reserve requirements, and external transmission limits must not be violated. This process is commonly referred to as an SCUC problem, which is to determine hourly commitment schedules with the objective of minimizing the total cost of energy, start-up, and spinning at no-load while observing

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[1] Decremental bids are similar to price-sensitive demand bids. They allow a marketer or other similar entity without physical demand to place a bid to purchase a certain quantity of energy at a certain location if the day-ahead price is at or below a certain price. Incremental offers are the flip side of decremental bids.

transmission constraints and physical resources' minimum runtime, minimum downtime, equipment ramp rates, and energy limits of energy-constrained resources. Based on the commitment schedules for physical resources, SCUC is used to clear energy supply offers, demand bids, and transaction schedules, and to determine LMPs and their components at all defined price nodes including the hubs, zones, and aggregated price nodes for the DAM settlement. The SCUC problem is usually optimized using a Lagrangian relaxation (LR) or a mixed-integer programming (MIP) solver…."

"A critical part of the DAM is the bid-in loads, which is a day-ahead forecast of the real-time load. The load estimate depends on the season, day type (weekday, weekend, holiday), and hour of the day. Most ISOs have sophisticated load forecasting programs, some with neural network components [36], [37], to predict the day-ahead load to within 3%–5% accuracy and the load forecasts are posted. LSEs with fully hedged loads through long-term bilateral contracts tend to bid in the amount corresponding to the ISO predicted loads. Some other LSEs may bid in loads that are different from those posted by the ISO. In such cases, if the LSE bid load exceeds the ISO load, the LSE bid load is taken as the load to be dispatched. Otherwise, the ISO load will supersede the LSE bid load and the SCUC will commit generators to supply the ISO forecasted load in a reliability stage. Then the generation levels of the committed generators will be allocated to supply LSE bid loads. Committing extra generators outside the DAM will be treated as uplifts and be paid by the LSEs…."

1.2 Paper by Papalexopoulos:

Reference [2] states:

"The Must Offer Waiver (MOW) process is basically a process of determining which Must Offer units should be committed in order to have enough additional capacity to meet the system energy net short which is the difference between the forecast system load and the Day-Ahead Market energy schedules. This commitment process ensures that the resulting unit schedule is feasible with respect to network and system resource constraints. Mathematically, this can be stated as a type of a SCUC problem [3]. The objective is to minimize the total start up and minimum load costs of the committed units while satisfying the power balance constraint, the transmission interface constraints, and the system resource constraints, including unit inter-temporal constraints…."

"The most popular algorithms for the solutions of the unit commitment problems are Priority-List schemes [4], Dynamic Programming [5], and Mixed Integer Linear Programming [6]. Among these approaches the MILP technique has achieved significant progress in the recent years [7]. The MILP methodology has been applied to the SCUC formulation to solve this MOW problem. Recent developments in the implementation of MILP-based algorithms and careful attention to the specific problem formulation have made it possible to meet accuracy and performance requirements for solving such large scale problems in a practical competitive energy market environment. In this section the MILP-based SCUC formulation is presented in detail…."

1.3 Paper by Ott:

Reference [3] states:
"In addition to the LMP concept, the fundamental design objectives of the PJM day-ahead energy market are: 1) to provide a mechanism in which all participants have the opportunity to lock in day-ahead financial schedules for energy and transmission; 2) to coordinate the day-ahead financial schedules with system reliability requirements; 3) to provide incentive for resources and demand to submit day-ahead schedules; and 4) to provide incentive for resources to follow real-time dispatch instructions…."

1.4 Paper by AREVA and PJM:

Reference [4] states:
"As the operator of the world's largest wholesale market for electricity, PJM must ensure that market-priced electricity flows reliably, securely and cost-effectively from more than 1100 Generating resources to serve a peak load in excess of 100,000 MW. In doing so, PJM must balance the market's

needs with thousands of reliability-based constraints and conditions before it can schedule and commit units to generate power the next day. The PJM market design is based on the Two Settlement concept [4]. The Two-Settlement System provides a Day-ahead forward market and a real-time balancing market for use by PJM market participants to schedule energy purchases, energy sales and bilateral contracts. Unit commitment software is used to perform optimal resource scheduling in both the Day-ahead market and in the subsequent Reliability Analysis…."

"As the market was projected to more than double its original size, PJM identified the need to develop a more robust approach for solving the unit commitment problem. The LR algorithm was adequate for the original market size, but as the market size increased, PJM desired an approach that had more flexibility in modeling transmission constraints. In addition, PJM has seen an increasing need to model Combined-cycle plant operation more accurately. While these enhancements present a challenge to the LR formulation, the use of a MIP formulation provides much more flexibility. For these reasons, PJM began discussion with its software vendors, in late 2002, concerning the need to develop a production grade MIP-based approach for large-scale unit commitment problems…."

"The Day-ahead market clearing problem includes next-day generation offers, demand bids, virtual bids and offers, and bilateral transactions schedules. The objective of the problem is to minimize costs subject to system constraints. The Day-ahead market is a financial market that provides participants an operating plan with known compensation: If their generation (or load) is the same in the real-time market, their revenue (or cost) is the same. Compensation for any real-time deviations is based on real-time prices, providing participants with opportunities to improve profit (or reduce cost) if they have flexibility to adjust their schedules…."

"In both problems, unit commitment accepts data that define bids (e.g., generator constraints, generator costs, and costs for other resources) and the physical system (e.g., load forecast, reserve requirements, security constraints). In real time, the limited responsiveness of units and additional physical data (e.g., state estimator solution, net-interchange forecast) further constrains the unit commitment problem."

"The Unit Commitment problem is a large-scale non-linear mixed integer programming problem. Integer variables are required for modeling: 1) Generator hourly On/Off-line status, 2) generator Startups/Shutdowns, 3) conditional startup costs (hot, intermediate & cold). Due to the large number of integer variables in this problem, it has long been viewed as an intractable optimization problem. Most existing solution methods make use of simplifying assumptions to reduce the dimensionality of the problem and the number of combinations that need to be evaluated. Examples include priority-based methods, decomposition schemes (LR) and stochastic (genetic) methods. While many of these schemes have worked well in the past, there is an increasing need to solve larger (RTO-size) problems with more complex (e.g. security) constraints, to a greater degree of accuracy. Over the last several years, the number of units being scheduled by RTOs has increased dramatically. PJM started with about 500 units a few years ago, and is now clearing over 1100 each day. MISO cases will be larger still…."

"The classical MIP implementation utilizes a Branch and Bound scheme. This method attempts to perform an implicit enumeration of all combinations of integer variables to locate the optimal solution. In theory, the MIP is the only method that can make this claim. It can, in fact, solve non-convex problems with multiple local minima. Since the MIP methods utilize multiple Linear Programming (LP) executions, they have benefited from recent advances in both computer hardware and software [6]…"

"This section presents results from using the CPLEX 7.1 and CPLEX 9.0 MIP solvers on a large-scale RTO Day Ahead Unit Commitment problem. This problem has 593 units and a 48 hour time horizon…."

2.0 The UC problem (in words)

The unit commitment problem is solved over a particular time period *T*; in the day-ahead market, the time period is usually 24 hours. It is articulated in [4], in words, as follows:

1. Min Objective=UnitEnergyCost+StartupCost+TransactionCost+
VirtualBidCost+DemandBidCost+Wheeling Cost

Subject to:

2. Area Constraints:
    a. Demand + Net Interchange
    b. Spinning and Operating Reserves
3. Zonal Constraints:
    a. Spinning and Operating Reserves
4. Security Constraints
5. Unit Constraints:
    a. Minimum and Maximum Generation limits
    b. Reserve limits
    c. Minimum Up/Down times
    d. Hours up/down at start of study
    e. Must run schedules
    f. Pre-scheduled generation schedules
    g. Ramp Rates
    h. Hot, Intermediate, & Cold startup costs
    i. Maximum starts per day and per week
    j. Maximum Energy per day and per study length

We describe the objective function and the various constraints in what follows.

2.1 Objective function

a. *UnitEnergyCost*: This is the total costs of supply over *T*, based on the supply offers made, in \$/MWhr.

b. *StartupCost*: This is the total cost of starting units over *T*, based on the startup costs

c. *TransactionCost*: Transactions are bilateral agreements made outside the market. Transaction cost for a particular transaction is the difference between nodal prices of transaction sink and source nodes, multiplied by the MW value of the transaction. So TransactionCost is the total transaction costs over *T*.

d. *VirtualBidCost*: Purely financial energy bids and offers made to arbitrage between the day ahead and real time market prices.

e. *DemandBidCost*: This is the total "cost" of demand over *T*, based on the demand bids made, in $/MWhr.

f. *WheelingCost*: I do not find this defined in the PJM materials but assume this is the transmission service cost associated with non-firm transactions.

Revenue from transaction sales, virtual bids and demand bids are added as negative costs so that by minimizing the objective the profit is maximized. For Day Ahead studies, this results in a large negative objective cost.

2.2 Area constraints

a. *Demand + Net Interchange*: The area demand plus the exports from the area (which could be negative, or imports).

b. *Spinning and Operating Reserves*: The spinning reserve is the amount of generation capacity $\Sigma(P_{gmax,k}-P_{gen,k})$ in MW that is on-line and available to produce energy within 10 minutes. Operating reserve is a broader term: the amounts of generating capacity scheduled to be available for specified periods of an Operating Day to ensure the security of the control area. Generally, operating reserve includes primary (which includes spinning) and secondary reserve, as shown in Fig. 2.

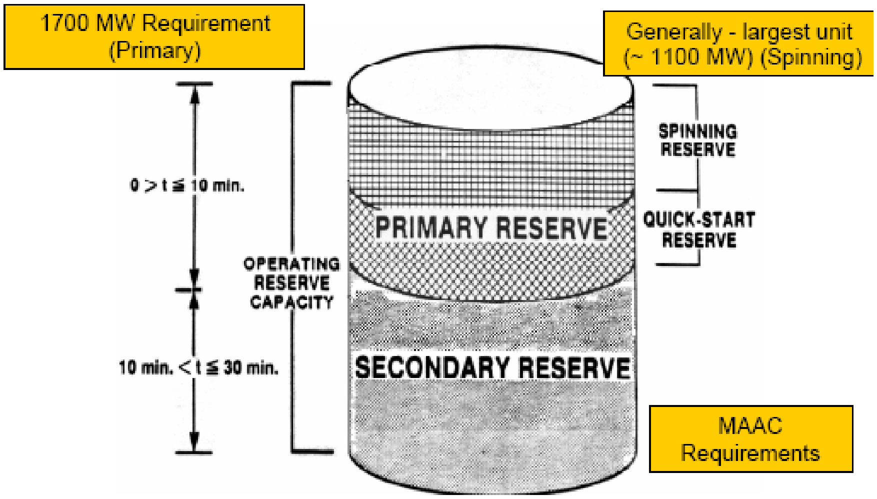


Fig. 2 [5]

2.3 Zonal constraints

Some regions within the control area, called zones, may also have spinning and operating reserve constraints, particularly if transmission interconnecting that region with the rest of the system is constrained.

2.4 Security constraints

These include constraints on branch flows under the no-contingency condition and also constraints on branch flows under a specified set of contingency conditions. The set is normally a subset of all N-1 contingencies.

2.5 Unit constraints

a. *Minimum and Maximum Generation limits*: Self explanatory.

b. *Reserve limits*: The spinning, primary, and/or secondary reserves must exceed some value, or some percentage of the load.

c. *Minimum Up/Down times*: Units that are committed must remain committed for a minimum amount of time. Likewise, units that are de-committed must remain down for a minimum amount of time. These constraints are due to the fact that thermal units can undergo only gradual temperature changes.

d. *Hours up/down at start of study*: The problem must begin at some initial time period, and it will necessarily be the case that all of the units will have been either up or down for some number of hours at that initial time period. These hours need to be accounted for to ensure no unit is switched in violation of its minimum up/down times constraint.

e. *Must run schedules*: There are some units that are required to run at certain times of the day. Such requirements are most often driven by network security issues, e.g., a unit may be required in order to supply the reactive needs of the network to avoid voltage instability in case of a contingency, but other factors can be involved, e.g., steam supply requirements of co-generation plants.

f. *Pre-scheduled generation schedules*: There are some units that are required to generate certain amounts at certain times of the day. The simplest example of this is nuclear plants which are usually required to generate at full load all day. Import, export, and wheel transactions may also be modeled this way.

g. *Ramp Rates*: The rate at which a unit may increase or decrease generation is limited, therefore the generation level in one period is constrained to the generation level of the previous period plus the generation change achievable by the ramp rate over the amount of time in the period.

h. *Hot, Intermediate, & Cold startup costs*: A certain amount of energy must be used to bring a thermal plant on-line, and that amount of energy depends on the existing state of the unit. Possible states are: hot, intermediate, and cold. Although it costs less to start a hot unit, it is more expensive to maintain a unit in the hot state. Likewise, although it costs more to start a cold unit, it is less expensive to maintain a unit in the cold state. Whether a de-committed unit should be maintained in the hot, intermediate, or cold state, depends on the amount of time it will be off-line.

i. *Maximum starts per day and per week*: Starting a unit requires people. Depending on the number of people and the number of units at a plant, the number of times a particular unit may be started in a day, and/or in a week, is usually limited.

j. *Maximum Energy per day and per study length*: The amount of energy produced by a thermal plant over a day, or over a certain study time T, may be less than $P_{max}\times T$, due to limitations of other facilities in the plant besides the electric generator, e.g., the coal processing facilities. The amount of energy produced by a reservoir hydro plant over a time period may be similarly constrained due to the availability of water.

3.0 The UC problem (analytic statement)

The unit commitment problem is a mathematical program characterized by the following basic features.

- *Dynamic*: It obtains decisions for a sequence of time periods.
- *Inter-temporal constraints*: What happens in one time period affects what happens in another time period. So we may not solve each time period independent of solutions in other time periods.
- *Mixed Integer*: Decision variables are of two kinds:
  - Integer variables: For example, we must decide whether a unit will be up (1) or down (0). This is actually a special type of integer variable in that it is binary.
  - Continuous variables: For example, given a unit is up, we must decide what its generation level should be. This variable may be any number between the minimum and maximum generation levels for the unit.

There are many papers that have articulated an analytical statement of the unit commitment problem, more recent ones include [1, 2, 6, 7], but there are also more dated efforts that pose the problem well, although the solution method is not as effective as what we have today, an example is [8].

We provide a mathematical model of the security-constrained unit commitment problem in what follows. This model was adapted from the one given in [9, ch 1]. This model is a mixed integer linear program.

$$\min \quad \underbrace{\sum_t \sum_i z_{it} F_i}_{\text{Fixed Costs}} + \underbrace{\sum_t \sum_i g_{it} C_i}_{\text{Production Costs}} + \underbrace{\sum_t \sum_i y_{it} S_i}_{\text{Startup Costs}} \tag{1}$$

subject to

power balance $\sum_i g_{it} = D_t = \sum_i d_{it}$ $\forall t,$ (2)

reserve $\sum_i r_{it} = SD_t$ $\forall t,$ (3)

min generation $g_{it} \geq z_{it} MIN_i$ $\forall i, t,$ (4)

max generation $g_{it} + r_{it} \leq z_{it} MAX_i$ $\forall i, t,$ (5)

max spinning reserve $r_{it} \leq z_{it} MAXSP_i$ $\forall i, t,$ (6)

ramp rate pos limit $g_{it} \leq g_{it-1} + MxInc_i$ $\forall i, t,$ (7)

ramp rate neg limit $g_{it} \geq g_{it-1} - MxDec_i$ $\forall i, t,$ (8)

start if off-then-on $z_{it} \leq z_{it-1} + y_{it}$ $\forall i, t,$ (9)

shut if on-then-off $z_{it} \geq z_{it-1} - x_{it}$ $\forall i, t,$ (10)

normal line flow limit $\sum_i a_{ki}(g_{it} - d_{it}) \leq MxFlow_k$ $\forall k, t,$ (11)

security line flow limits $\sum_i a_{ki}^{(j)}(g_{it} - d_{it}) \leq MxFlow_k^{(j)}$ $\forall k, j, t,$ (12)

where the decision variables are:

- g_{it} is the MW produced by generator *i* in period *t*,
- r_{it} is the MW of spinning reserves from generator *i* in period *t*,
- z_{it} is 1 if generator *i* is dispatched during *t*, 0 otherwise,
- y_{it} is 1 if generator *i* starts at beginning of period *t*, 0 otherwise,
- x_{it} is 1 if generator *i* shuts at beginning of period *t*, 0 otherwise,

Other parameters are

- D_t is the total demand in period *t*,
- SD_t is the spinning reserve required in period *t*,
- F_{it} is fixed cost ($/period) of operating generator *i* in period *t*,
- C_{it} is prod. cost ($/MW/period) of operating gen *i* in period *t*;
- S_{it} is startup cost ($) of starting gen *i* in period *t*.
- $MxInc_i$ is max ramprate (MW/period) for increasing gen *i* output
- $MxDec_i$ is max ramprate (MW/period) for decreasing gen *i* output
- a_{ij} is linearized coefficient relating bus *i* injection to line *k* flow
- $MxFlow_k$ is the maximum MW flow on line *k*

- $a_{ki}^{(j)}$ is linearized coefficient relating bus *i* injection to line *k* flow under contingency *j*,
- $MxFlow_k^{(j)}$ is the maximum MW flow on line *k* under contingency *j*

The above problem statement is identical to the one given in [9] with the exception that here, we have added eqs. (11) and (12).
➔The addition of eq. (11) alone provides that this problem is a transmission-constrained unit commitment problem.
➔ The addition of eqs. (11) and (12) together provides that this problem is a security-constrained unit commitment problem.

One should note that our problem is entirely linear in the decision variables. Therefore this problem is a *linear* mixed integer program, and it can be compactly written as

$$\min \underline{c}^T \underline{x}$$

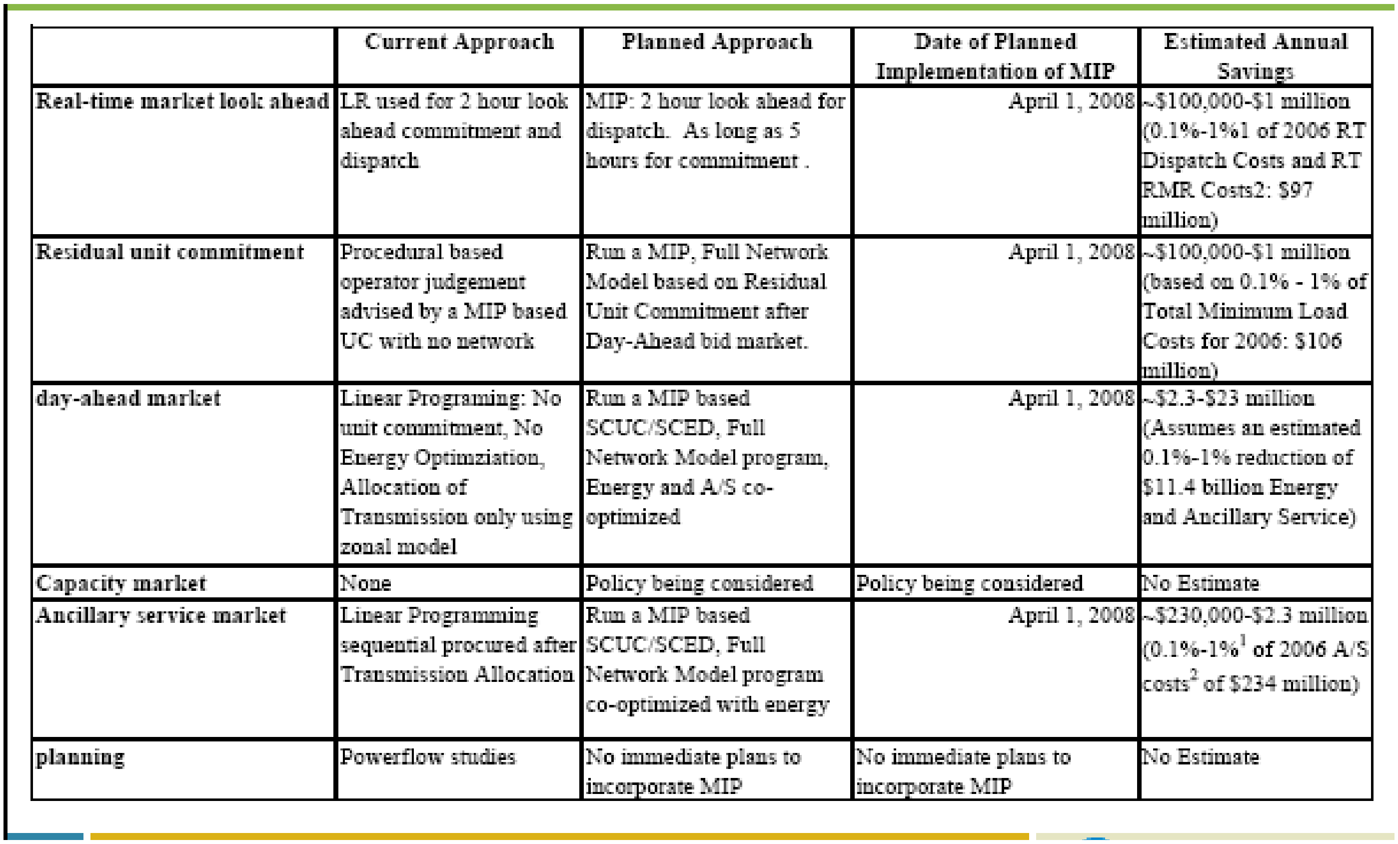
$$\text{Subject to}$$

$$\underline{A}\underline{x} \leq \underline{b}$$

In the next set of notes, we will investigate how to solve such problems. There have four basic methods used in the past few years:

- Priority list methods
- Dynamic programming
- Lagrangian relaxation
- Branch and bound

The last method, branch and bound, is what the industry means when it says "MIP." It is useful to understand that the chosen method can have very large financial implications. This point is well-made in the following chart [10].



	Current Approach	Planned Approach	Date of Planned Implementation of MIP	Estimated Annual Savings
Real-time market look ahead	LR used for 2 hour look ahead commitment and dispatch	MIP: 2 hour look ahead for dispatch. As long as 5 hours for commitment .	April 1, 2008	~$100,000-$1 million (0.1%-1%1 of 2006 RT Dispatch Costs and RT RMR Costs2: $97 million)
Residual unit commitment	Procedural based operator judgement advised by a MIP based UC with no network	Run a MIP, Full Network Model based on Residual Unit Commitment after Day-Ahead bid market.	April 1, 2008	~$100,000-$1 million (based on 0.1% - 1% of Total Minimum Load Costs for 2006: $106 million)
day-ahead market	Linear Programing: No unit commitment, No Energy Optimziation, Allocation of Transmission only using zonal model	Run a MIP based SCUC/SCED, Full Network Model program, Energy and A/S co-optimized	April 1, 2008	~$2.3-$23 million (Assumes an estimated 0.1%-1% reduction of $11.4 billion Energy and Ancillary Service)
Capacity market	None	Policy being considered	Policy being considered	No Estimate
Ancillary service market	Linear Programming sequential procured after Transmission Allocation	Run a MIP based SCUC/SCED, Full Network Model program co-optimized with energy	April 1, 2008	~$230,000-$2.3 million (0.1%-1%[1] of 2006 A/S costs[2] of $234 million)
planning	Powerflow studies	No immediate plans to incorporate MIP	No immediate plans to incorporate MIP	No Estimate

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[10] M. Rothleder, presentation to the Harvard Energy Policy Group, Dec 7, 2007.