

# **Introduction to Economics: Producer's Surplus**

## **1.0 Introduction**

Recall our objective in this introduction to microeconomics is to gain some understanding of the following concepts:

- Supply and demand functions
- Consumers surplus
- Producers surplus
- Market efficiency
- Market equilibrium
- Competitive equilibrium

We have discussed demand functions and consumers surplus. Now we take a similar view of the producer, focusing on supply functions and producers surplus. As before, much of the below material was adapted from standard economics textbooks [1, 2, 3].

## **2.0 Supply functions and Producer's Surplus**

A supply curve characterizes the manner in which the supply of a good will change as its price changes, holding constant all other factors that influence producers' willingness or ability to supply the good. Figure 1 illustrates a supply curve for MP3 players, where  $q$  denotes number of players supplied.

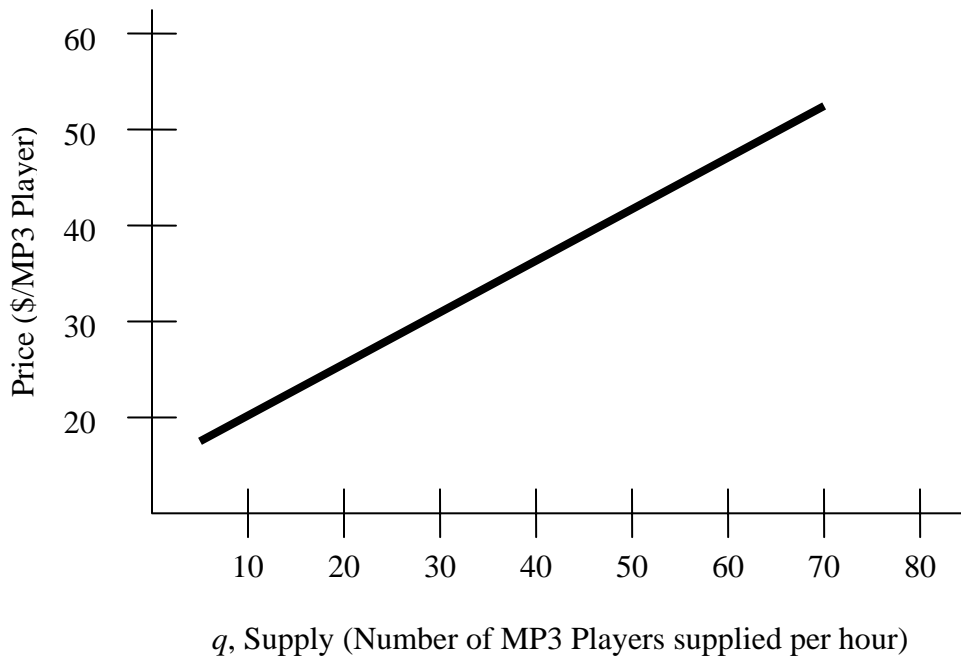


Fig. 1: Supply curve for MP3 Players

Note the supply curve indicates that the supply will increase as the price increases. This is the case for most kinds of goods. When this is the case, we say that the supply is elastic, i.e., it will change with price.

What does the supply curve for electric energy produced by a wind turbine look like? On an hour by hour basis, it appears as in Fig. 2, since the wind turbine owner will supply whatever amount of energy s/he has at whatever price the market will pay. Here,  $P$  denotes the number of kWhrs supplied.

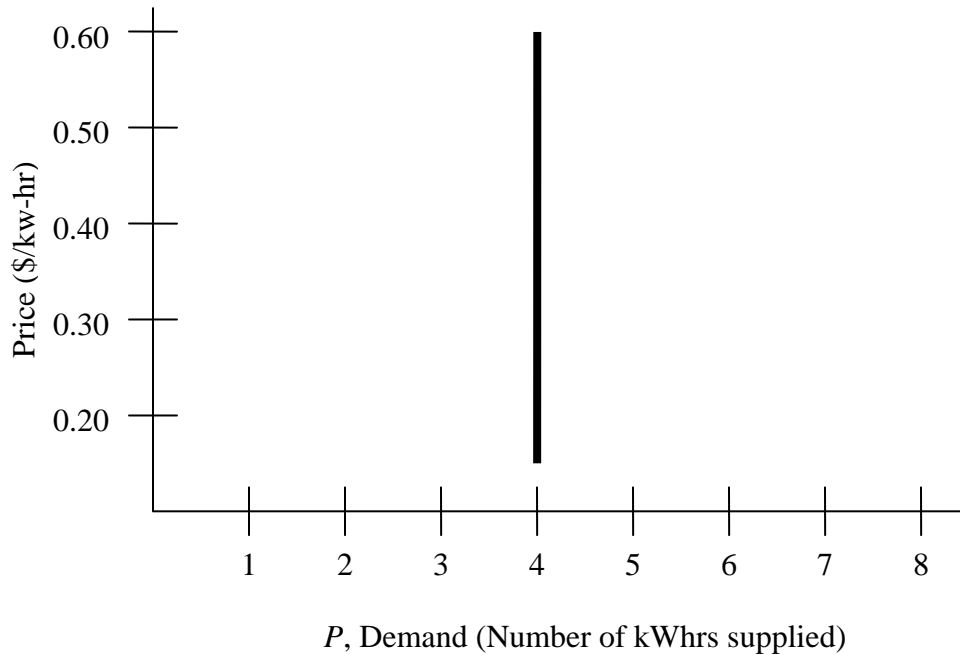


Fig. 2: Supply curve for a wind turbine

This is an example of an inelastic supply, i.e., a supply that is insensitive to price. However, most suppliers behave elastically. Figure 3 illustrates such a supplier where,

- below 20 \$/MWhr, they will shut down their power plant
- at 20 \$/MWhr, they will produce, supplying up to 4 MW in an hour;
- if the price goes to 30 \$/MWhr, they will produce up to 7 MW in an hour;
- if the price goes up to 50 \$/MWhr, they will produce up to 50 MW in an hour.

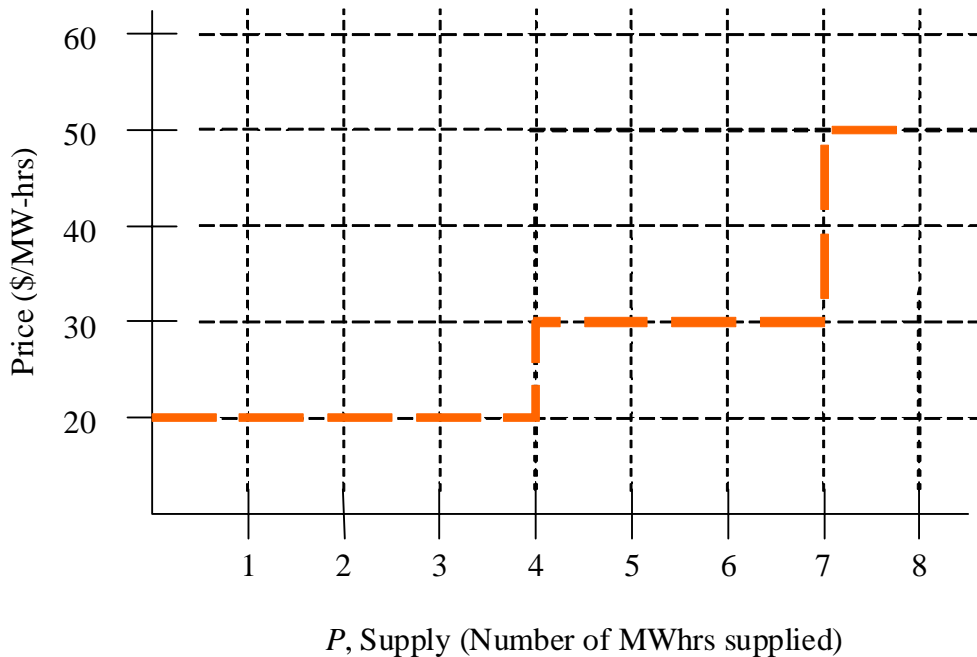


Fig. 3: Elastic Supply

Let's consider now<sup>1</sup> that we have the hourly cost function for a power producer  $C$  as a function of the amount of power it produces  $P$ . Two cautions here:

1. Be careful to distinguish between price  $p$  and power produced  $P$ .
2. You can equivalently consider  $C$  to be in \$/hr, and  $P$  to be in MW, or you can consider  $C$  to be \$, and power  $P$  to be MW-hr. Since we previously considered demand  $x$  to be MW-hr, we will consider  $P$  to also be MW-hr.

We will assume  $C$  to be convex.

<sup>1</sup> The remainder of this section was adapted from notes developed by Oscar Volij of Iowa State University.

The value  $C(P)$  is the amount of money the producer needs to spend in order to produce  $P$  MWhr. And we know its derivative is  $C'$ , the incremental cost, or, the marginal cost of energy. It represents the rate at which the cost of production increases due to a small increase in the production of energy.

Consider that the producer faces an energy price of  $p$  and needs to find the production level which maximizes profits.

Profits are given by the amount the producer is paid for producing  $P$  MWhr less the amount it costs the producer to produce  $P$  MWhr. The amount the producer is paid when the price is  $p$  is  $pP$ . The amount it costs is  $C(P)$ .

Note, in contrast to the consumer's resource, money, the producer's resource is energy,  $P$ . Therefore, whereas the consumer had a constraint on money, the producer has a constraint on energy. That constraint is  $0 \leq P \leq P_{max}$ .

Therefore, the producer needs to solve the following problem:

$$\begin{aligned} & \max_P \{pP - C(P)\} \\ & \text{s.t. } 0 \leq P \leq P_{\max} \end{aligned} \quad (1)$$

In our approach to inequality constrained optimization problems, we first solve the unconstrained problem and then check our solution. Following this approach, we apply first order conditions to the expression inside the curly brackets to obtain:

$$\frac{\partial F(pP - C(P))}{\partial P} = p - \frac{\partial C(P)}{\partial P} = 0 \Rightarrow \frac{\partial C(P)}{\partial P} = p \quad (2)$$

And so, when the producer sees a price  $p$ , s/he should produce the amount of energy  $P$  which results in

$$C'(P) = p \quad (3)$$

That is, the marginal cost of energy should be equal to the price of energy in order for the producer to maximize profits. One can also interpret (3) as the inverse supply function: for each possible quantity  $P$ , it gives us the market price of energy  $p$  that would induce the producer to sell exactly  $P$ .

Example 1: Let  $C(P) = P^2$ . Find the inverse supply function and the supply function.

$$C(P) = P^2$$

$$C'(P) = 2P$$

and so the inverse supply function is

$$p = 2P$$

To obtain the supply function, we solve for  $P$ :

$$P(p) = \frac{p}{2}$$

I have plotted the 2 functions in Figs. 1 and 2 below.

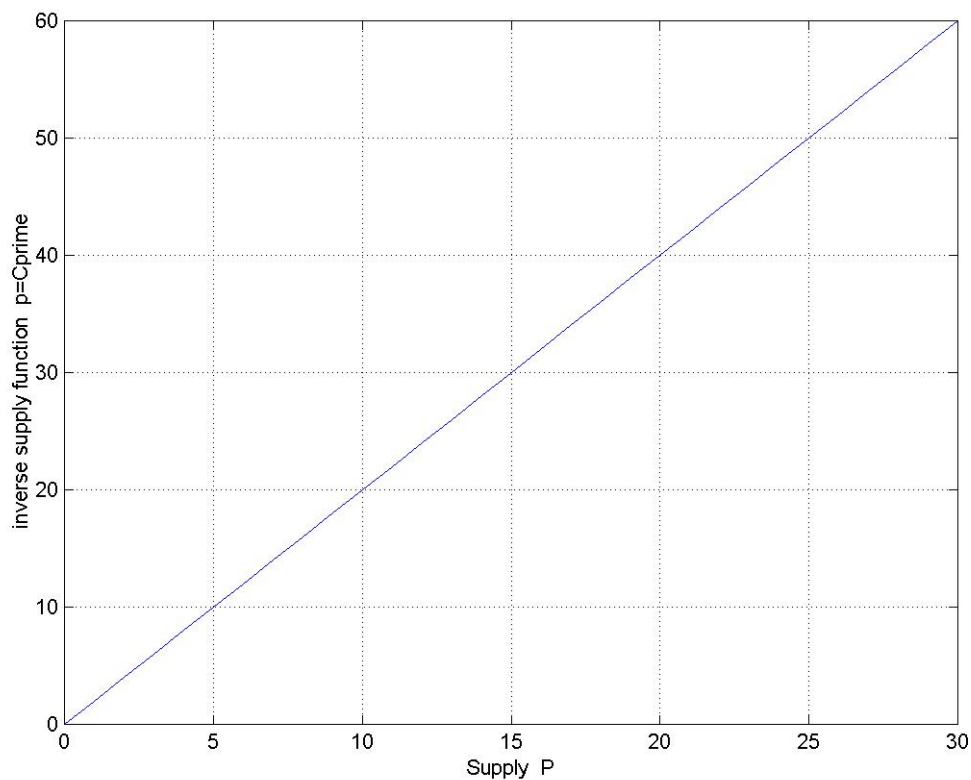


Fig 1: Inverse supply function

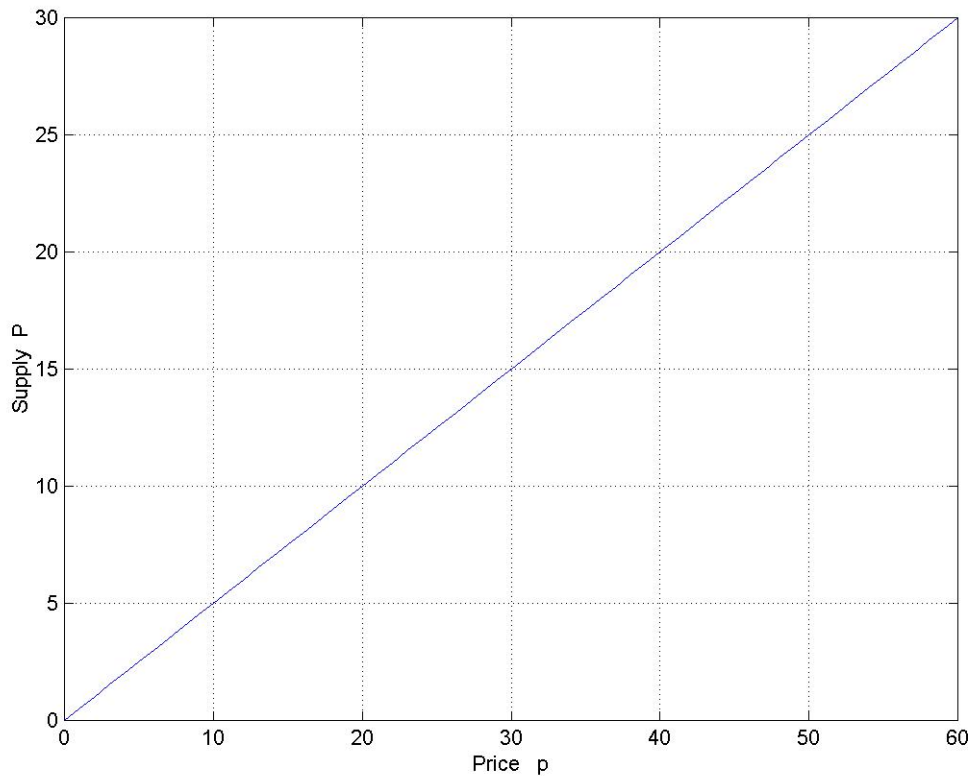


Fig. 2: Supply Function

Question: Consider that the producer has a choice of participating in the electric energy market or not. Will it benefit by doing so?

The answer to this question depends on the difference between its profits when it participates and its profits utility when it does not, where profits are  $pP - C(P)$ . given as the objective in (1).

If it does participate, and the energy price is  $p$ , then it produces  $P$ , it gets paid  $pP$ , and it incurs a cost  $C(P)$ .

If it does not participate, then it produces 0, it gets paid 0, and it incurs a cost  $C(0)$ . The difference in profits is therefore the producer's surplus:

$$\begin{aligned}
 PS(p_0) &= pP - C(P) - (0 - C(0)) \\
 &= pP - (C(P) - C(0)) \\
 &= pP - \int_0^P C'(P)dP \quad (4)
 \end{aligned}$$

where the last step is taken according to the Fundamental Theorem of Calculus.

Equation (4) has a nice graphical interpretation illustrated by Fig. 3, where we assume a price  $p^*$  results in a supply  $P^*$ .

- The first term of (4) corresponds to the total “box,” shaded by vertical lines, lower side 0: $P^*$  and left-hand-side 0: $p^*$ .
- The second term of (4) corresponds to the area under the  $p=C'(x)$  curve from 0 to  $P^*$ , which is the area shaded by horizontal lines.

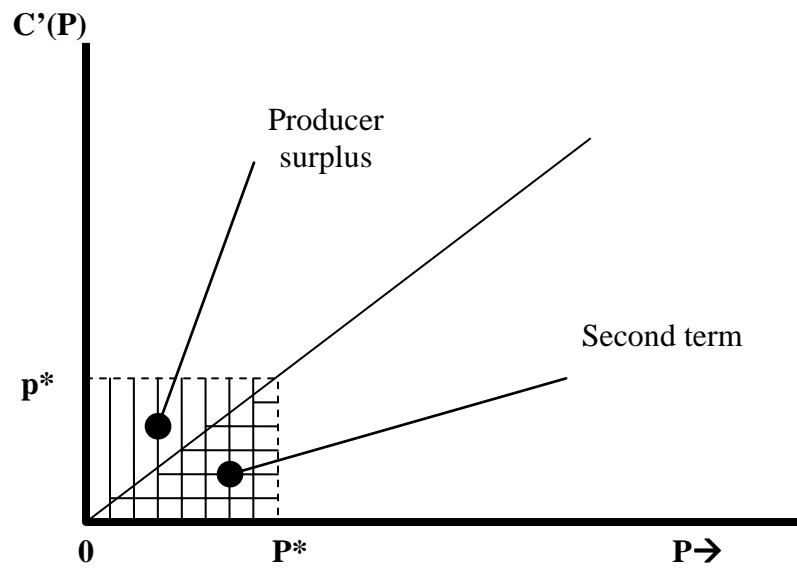


Fig. 3: Producer surplus

Whereas consumer surplus may have been a little bit of a new idea for us (“you get more than you pay for”), the interpretation of producer surplus, difference in profits, should be pretty familiar.

Question: Suppose the price of energy is increased from  $p_0$  to  $p_1$ . What happens to the producer’s surplus?

When the price is  $p_0$ , the producer supplies  $P_0$  and the producer’s surplus is, from (4):

$$PS(p_0) = p_0 P_0 - \int_0^{P_0} C'(P) dP \quad (5)$$

When the price is  $p_1$ , the producer supplies  $P_1$  and the producer surplus is, from (4):

$$PS(p_1) = p_1 P_1 - \int_0^{P_1} C'(P) dP \quad (6)$$

So we want to compute the amount the producer surplus has increased. Subtracting (6) from (5) to get the change in producer surplus, we have:

$$\begin{aligned} PS(p_1) - PS(p_0) &= p_1 P_1 - \int_0^{P_1} C'(P) dP - p_0 P_0 + \int_0^{P_0} C'(P) dP \\ &= p_1 P_1 - p_0 P_0 + \int_0^{P_0} C'(P) dP - \int_0^{P_1} C'(P) dP \quad (7) \end{aligned}$$

Let's reverse the signs on the integrals:

$$PS(p_1) - PS(p_0) = p_1 P_1 - p_0 P_0 - \left( \int_0^{P_1} C'(P) dP - \int_0^{P_0} C'(P) dP \right) \quad (8)$$

because now we can combine the integrals as:

$$PS(p_1) - PS(p_0) = p_1 P_1 - p_0 P_0 - \int_{P_0}^{P_1} C'(P) dP \quad (9)$$

Now I know a trick, and that is to add and subtract  $p_1 P_0$  to the above expression, resulting in:

$$PS(p_1) - PS(p_0) = p_1 P_0 - p_1 P_0 + p_1 P_1 - p_0 P_0 - \int_{P_0}^{P_1} C'(P) dP \quad (10)$$

Let's rearrange the terms:

$$PS(p_1) - PS(p_0) = p_1 P_0 - p_0 P_0 + p_1 P_1 - p_1 P_0 - \int_{P_0}^{P_1} C'(P) dP \quad (11)$$

Now, let's factor  $P_0$  from the first two terms and  $p_1$  from the second two terms, resulting in

$$PS(p_1) - PS(p_0) = (p_1 - p_0)P_0 + p_1(P_1 - P_0) - \int_{P_0}^{P_1} C'(P) dP \quad (12)$$

Grouping the second term with the integral term,

$$PS(p_1) - PS(p_0) = (p_1 - p_0)P_0 + \left( p_1(P_1 - P_0) - \int_{P_0}^{P_1} C'(P) dP \right) \quad (13)$$

Equation (13) says that the change in the producer's surplus can be decomposed into two parts:

- The new revenues from the increase in price that the producer enjoys from selling the old amount (yellow) and
- The increased profit due to the additional sold amount (red).

We can see why if we return to our picture:

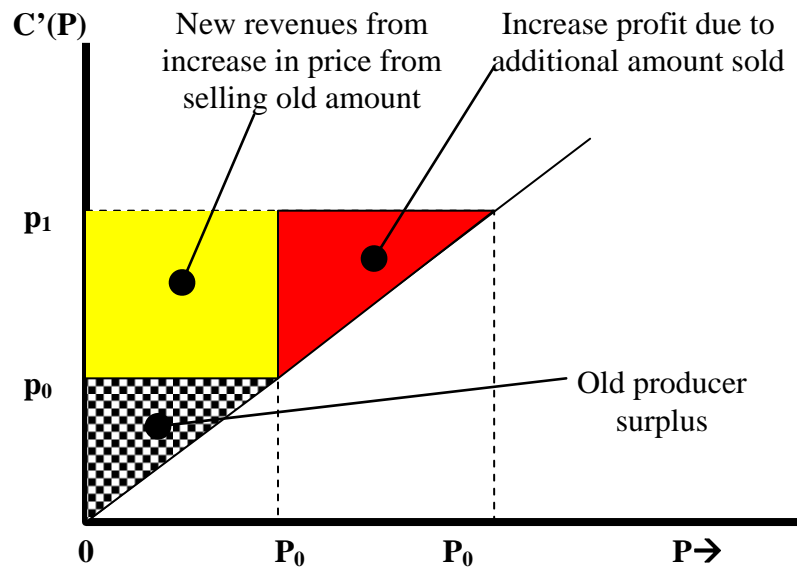


Fig. 4: Change in producer surplus due to price increase

You should consider what happens to producer's surplus when the price is decreased.

[1] P. Samuelson and W. Nordhaus, "Economics," 17th edition, McGraw-Hill, 2001.

[2] A. Dillingham, N. Skaggs, and J. Carlson, "Economics: Individual Choice and its Consequences," Allyn and Bacon, 1992.

[3] W. Baumol and A. Blinder, "Economics: Principles and Policy," Dryden Press, 1994.