

Introduction to Economics: Consumer's Surplus

1.0 Introduction

Economics is clearly important to electricity markets, because it was economic principles that motivated them in the first place. And so it is appropriate in a course like EE 458 to introduce some basic economic thinking. If you have already taken a course in economics, then some of this will be a review for you. Much of the below material was adapted from standard economics textbooks [1, 2, 3].

We begin our discussion of economics by distinguishing between 2 major sub-fields: microeconomics & macroeconomics. One immediately feels from the words themselves that microeconomics relates to “small” and macroeconomics relates to “large.” But that is not a complete picture.

More appropriately, microeconomics focuses on the decisions of individual units (firms, companies, households, individual consumers) within an economy; macroeconomics focuses on the behavior of entire economies, looking at, for example, determination of total investment and consumption, how central banks manage money and interest rates, what causes international financial crises.

Our interest is microeconomic, i.e., we want to see, for an electricity market,

- how individual prices are set
- what it means to have an efficient electricity market
- how an electricity market can achieve efficiency simply from the self-interested actions of its individual agents.

We do not have time in this course to treat this topic as you would in a course dedicated to microeconomics (at ISU, it is called Econ 101), but we will establish the following essential concepts:

- Supply and demand functions
- Consumers surplus
- Producers surplus
- Market efficiency
- Market equilibrium
- Competitive equilibrium

In this set of notes, we will look at the first 2 topics.

2.0 Demand functions and Consumer Surplus

A demand curve characterizes the manner in which the demand of a good will change as its price changes, holding constant all other factors that influence consumers' willingness or ability to pay for the good. Figure 1 illustrates a demand curve for MP3 players, where x denotes number of players demanded.

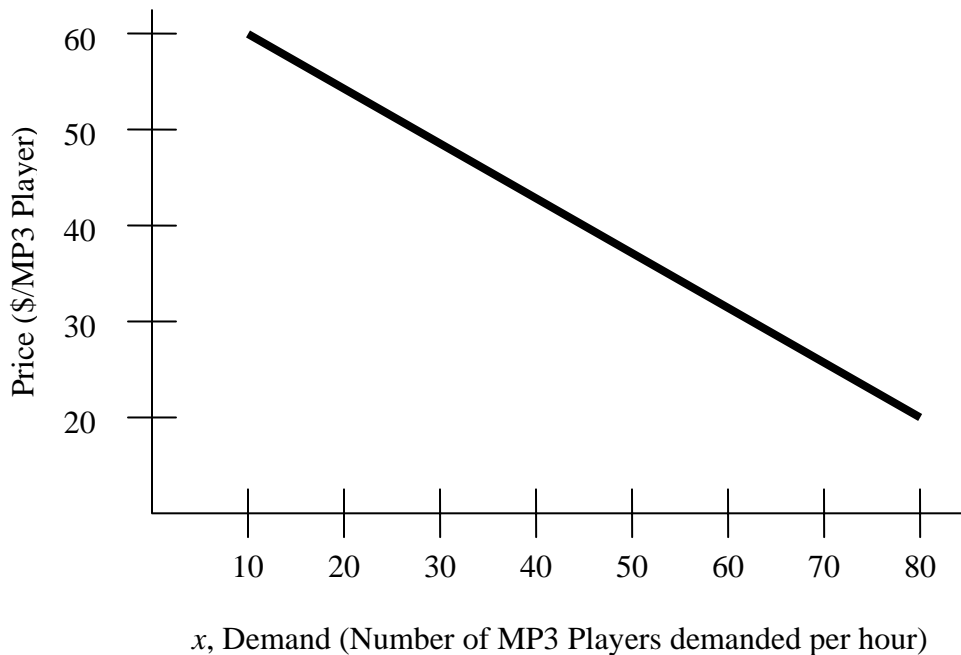


Fig. 1: Demand curve for MP3 Players

Note the demand curve indicates that the demand will increase as the price decreases. This is the case for most kinds of goods. When this is the case, we say that the demand is elastic, i.e., it will change with price.

What does the demand curve for electric energy that you and I use in our homes look like? On an hour by hour basis, it appears as in Fig. 2, since we will consume the demand we want, independent of the price. Here, x denotes the number of kWhrs demanded.

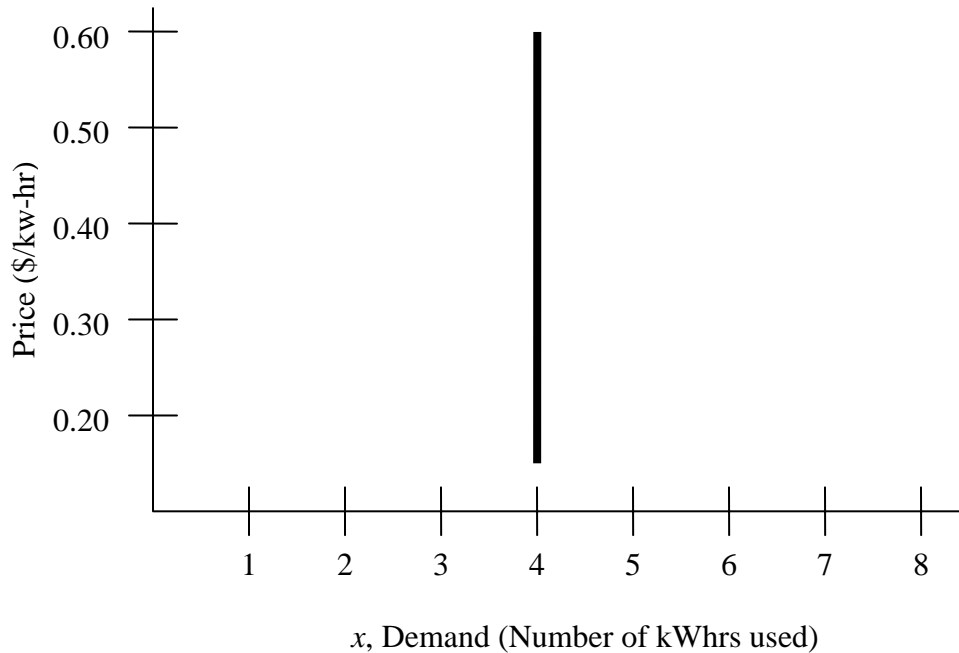


Fig. 2: Demand curve for Residential Energy

This is an example of an inelastic demand, i.e., a demand that is insensitive to price. However, we may also consider that there exist some electricity consumers, particularly in industry, that behave elastically. Figure 3 illustrates such a consumer where,

- above 40 \$/MWhr, they will shut down all production and only power the security lights, requiring only 1 MWhr.
- at 40 \$/MWhr or below, they will produce, requiring 7 MWhrs.
- if the price goes to 20 \$/MWhr or lower, they will initiate a separate production facility requiring an additional MWhr for a total of 8 MWhrs.

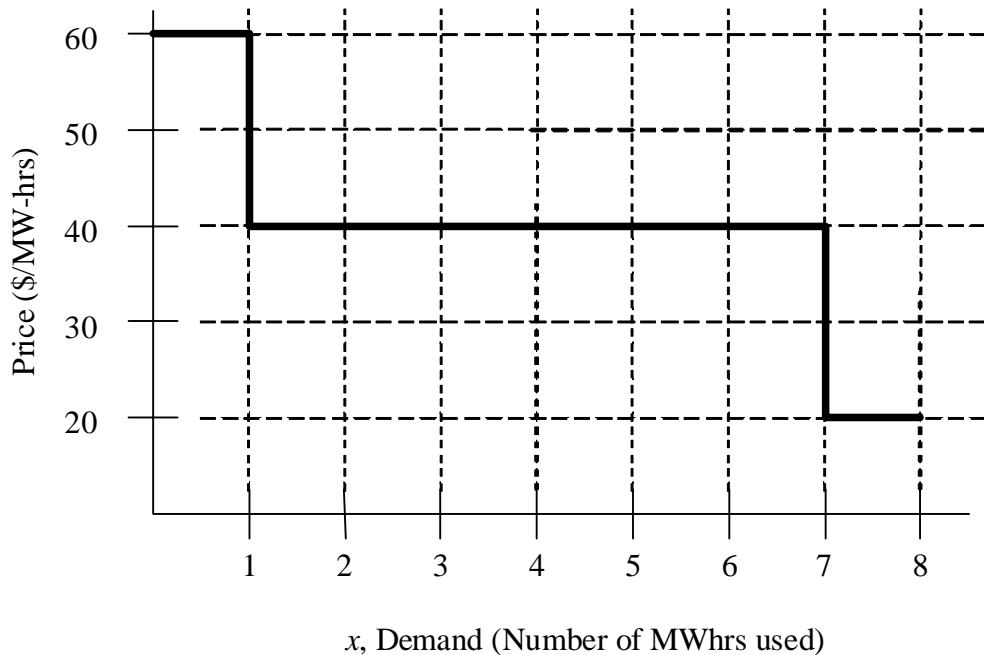


Fig. 3: Elastic Demand

Let's consider now¹ that the industrial consumer is able to quantify the satisfaction or happiness of the plant manager, denoted by U , as a function of the amount of energy the plant consumes in an hour x , and the money that it has at the end of that hour, y . A given bundle (x,y) provides a level of satisfaction expressed by the so-called *utility function* for the plant, given by

$$U(x, y) = v(x) + y \quad (1)$$

where $v(x)$ quantifies, in dollars, the satisfaction associated with the amount of energy consumed, x . It is the utility function for energy. We will require $v(x)$ to be an increasing and concave function, and $v(0)=0$.

¹ The remainder of this section was adapted from notes developed by Oscar Volij of Iowa State University.

Let's assume that the company has an amount of money m to use for the hour, and that it faces an energy price of p . This means there exists the following constraint on what the company can do during the hour:

$$y + px = m \quad (2)$$

The company has to find the amount of energy to consume that maximizes its satisfaction $U(x,y)$, but it must do so subject to the constraint identified in (2), that is, the company wants to solve the following problem:

$$\begin{aligned} \max_{x,y} \{v(x) + y\} \\ \text{s.t. } y + px = m \end{aligned} \quad (3)$$

One way to solve this is to by forming the Lagrangian:

$$F(x, y, \lambda) = v(x) + y - \lambda(y + px - m)$$

and then applying first order conditions:

$$\begin{aligned} \frac{\partial F(x, y, \lambda)}{\partial x} &= \frac{\partial v(x)}{\partial x} - \lambda p = 0 \Rightarrow \frac{\partial v(x)}{\partial x} = \lambda p \\ \frac{\partial F(x, y, \lambda)}{\partial y} &= 1 - \lambda = 0 \Rightarrow \lambda = 1 \\ \frac{\partial F(x, y, \lambda)}{\partial \lambda} &= y + px - m = 0 \end{aligned} \tag{4}$$

From the first two equations above, we obtain

$$v'(x) = p \tag{5}$$

Another way to solve this is to substitute the constraint into the objective function, eliminating y , resulting in:

$$\max_x \{U(x)\} = \max_x \{v(x) + m - px\} \tag{6}$$

Taking first derivatives with respect to x , and setting to 0 yields:

$$v'(x) - p = 0 \Rightarrow v'(x) = p \tag{7}$$

Recalling that $v(x)$ expresses utility (satisfaction) for x , then $v'(x)$ expresses the change in utility per unit change in x , otherwise known as the *marginal utility* for energy. It provides an upper bound to what the company should be willing to pay for one more unit of energy. If they have to pay more, then it means that the value of what they can do with that energy will be less than what they have to pay for it.

And so that we see that (5) expresses the following condition to guarantee that the company will maximize their total utility $U(x,y)$:

the willingness to pay for each additional unit of energy should equal the price paid for that energy.

In other words, when the upper bound to what the company is willing to pay for one more unit of energy exactly equals the price they pay for that energy, their total utility (satisfaction from energy plus the remaining money) will be maximized.

Here is one more articulation of the above idea [3]:

It always pays the consumer to buy more of any commodity whose marginal utility (measured in money) exceeds its price (the marginal cost of the commodity to the consumer), and less of any commodity whose marginal utility is less than its price. When possible, the consumer should buy the quantity of each good at which price (p) and marginal utility v' are exactly equal, because only these quantities will maximize the net total utility the consumer gains from the purchases, given the fact that the money available must be divided among all the goods purchased.

We call (5) the inverse demand function, i.e., it expresses price as a function of demand.

The demand function, then, expresses demand as a function of price.

Example 1: Let $v(x) = 60x - x^2$. Find the inverse demand function and the demand function.

$$v(x) = 60x - x^2$$

$$v'(x) = 60 - 2x$$

and so the inverse demand function is

$$p = 60 - 2x$$

To obtain the demand function, we solve for x :

$$x(p) = 30 - \frac{p}{2}$$

I have plotted the 2 functions in Figs. 1 and 2 below.

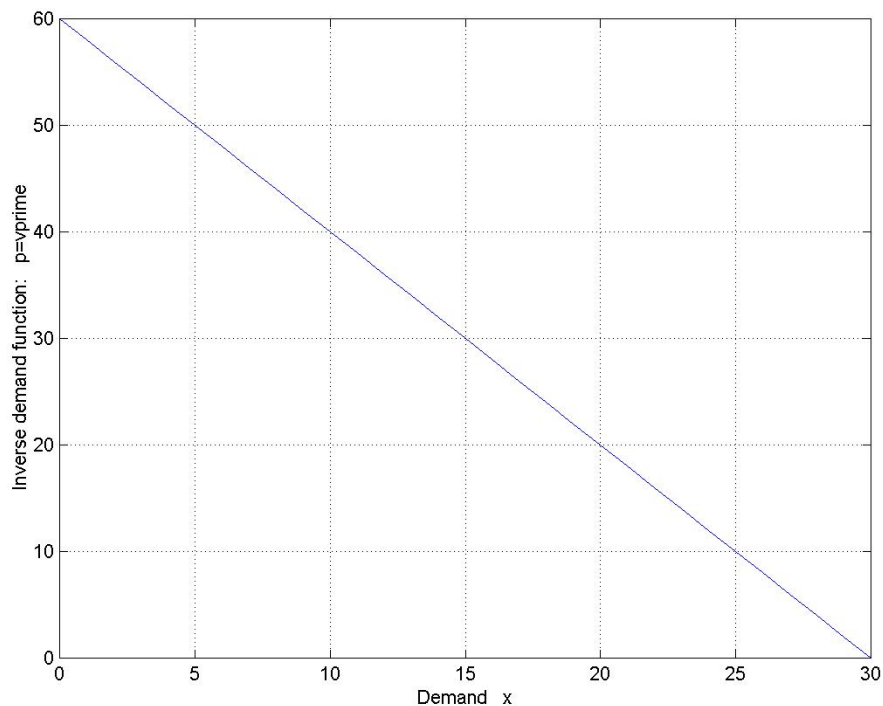


Fig 1: Inverse demand function

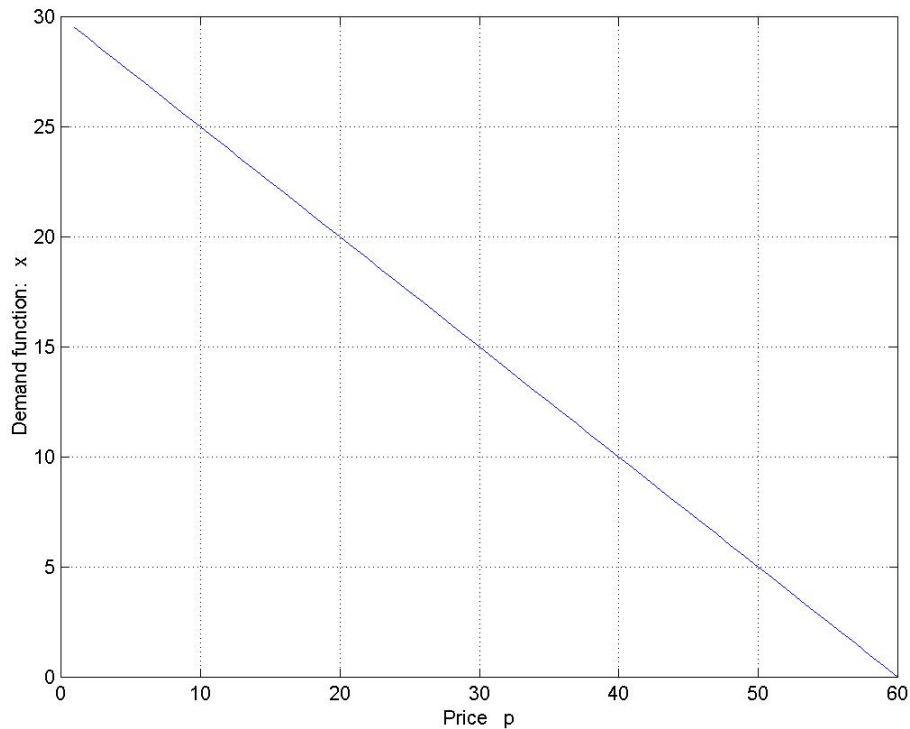


Fig. 2: Demand Function

Question: Consider that the company has a choice of participating in the electric energy market or not. Will it benefit by doing so?

The answer to this question depends on the difference between its total utility when it participates and its total utility when it does not, where total utility is given by (1), repeated here for convenience:

$$U(x, y) = v(x) + y \quad (1)$$

Let's denote the total utility when

- it participates in the market: $U_{par}(x, y)$,

- it does not participate in the market: $U_{not}(x, y)$

If it does participate, and the energy price is p , then it obtains an amount of energy equal to $x(p)$, and the amount of money it will have is, by (2), $y=m-px(p)$. Then the company's total utility, i.e., its (x,y) bundle, is given by:

$$U_{par}(x, y) = U(x(p), m - px(p)) \quad (8)$$

If it does not participate, then it receives no electric energy, and it has m dollars. And so the company's total utility, i.e., its (x,y) bundle, is given by $U(0, m)$.

$$U_{not}(x, y) = U(0, m) \quad (9)$$

The difference, then, is given by the *consumer surplus* at price p . We will denote the consumer surplus as $CS(p)$. It is expressed as the difference between (8) and (9):

$$\begin{aligned} CS(p) &= U_{par}(x, y) - U_{not}(x, y) \\ &= U(x(p), m - px(p)) - U(0, m) \end{aligned} \quad (10)$$

Using (1) above, we can express (10) as

$$CS(p) = \underbrace{v(x(p)) + m - px(p)}_{U(x(p), m - px(p))} - \underbrace{(v(0) + m)}_{U(0, m)} \quad (11)$$

Observing the addition and subtraction of m ,

$$CS(p) = v(x(p)) - px(p) - v(0) \quad (12)$$

Rearranging,

$$CS(p) = v(x(p)) - v(0) - px(p) \quad (13)$$

Recall the fundamental theorem of calculus:

$$F(b) - F(a) = \int_a^b F'(x)dx$$

This allows us to write the first two terms of (13) as

$$v(x(p)) - v(0) = \int_0^{x(p)} v'(x)dx \quad (14)$$

Substitution of (14) into (13) results in

$$CS(p) = \int_0^{x(p)} v'(x)dx - px(p) \quad (15)$$

Equation (15) has a nice graphical interpretation illustrated by Fig. 3, where we assume a price p^* results in a demand x^* .

- Observe that the first term of (15) corresponds to the area under the $p=v'(x)$ curve from 0 to x^* , which is the entire area shaded by vertical lines.
- The second term of (15) corresponds to the lower “box,” not shaded by horizontal lines, lower side 0: x^* and left-hand-side 0: p^* .

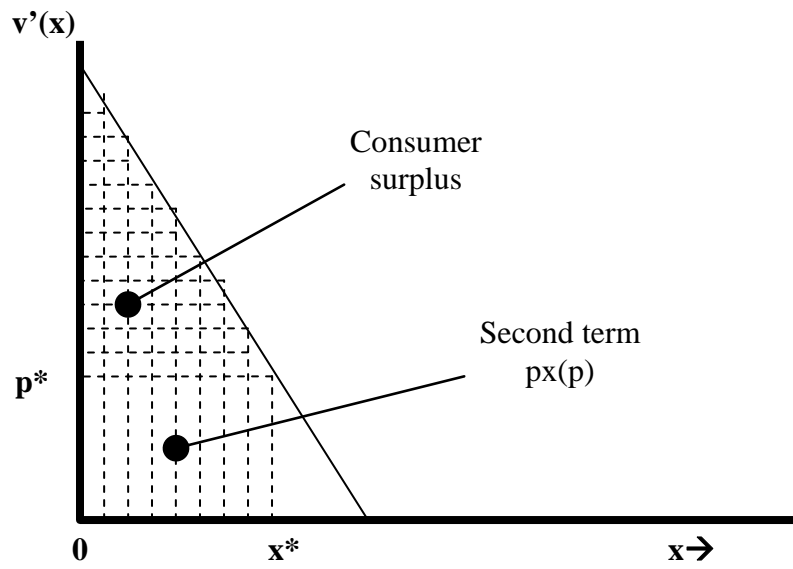


Fig. 3: Consumer surplus

The consumer surplus is the gap, or difference, between the total utility of the energy and its total market value. The surplus arises because we “get more than we pay for.”

The reason we “get more than we pay for” is because the price we pay is equal to the value to us of the last unit of energy (the marginal value). Figure 4 discretizes Fig. 3 to illustrate.

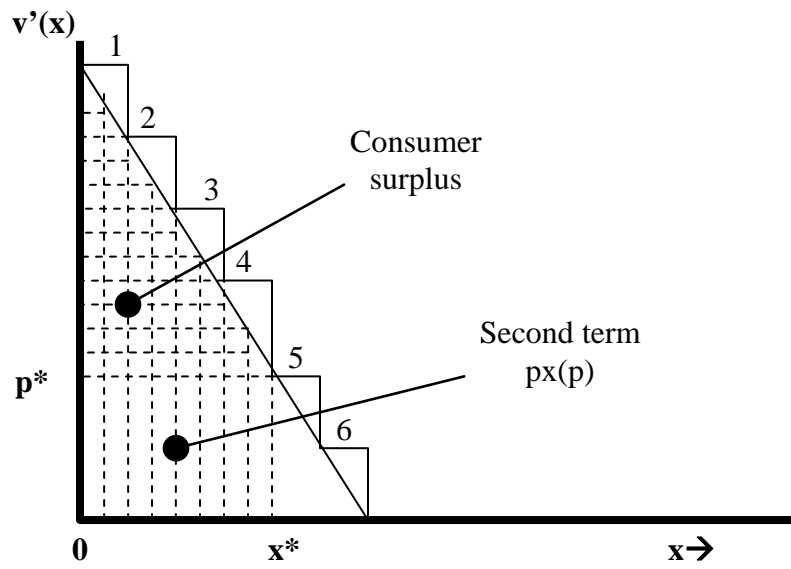


Fig. 4: Consumer surplus

When we purchase the first unit (step 1) at price p^* , we are very happy because that first unit is worth a lot more to us than what we pay, as indicated by its high value of $v'(x)$. The purchases of the second, third, and fourth units also make us happy (although not quite as happy as the first unit made us 😊).

But when we purchase the fifth unit, it provides us with a level of satisfaction that we assess is exactly equal to the price we paid. We will want that unit (because otherwise we will be giving up some surplus), but we want no additional unit beyond that. The inverse demand curve is also a way of rethinking the *law of diminishing marginal product* which we defined in our notes on Cost Curves to be as follows:

For almost all processes, the rate of increase in

output decreases as the input increases, assuming other inputs are fixed.

The above statement is really from the producer's perspective. But it may be restated from the consumer's perspective, where it is called the *law of diminishing marginal utility*:

Additional units of a commodity are worth less and less to a consumer in money terms. As the individual's consumption increases, the marginal utility of each additional unit declines.

Example 2: In example 1, we were given the utility function for energy as $v(x) = 60x - x^2$ and we determined the inverse demand function as $p = 60 - 2x$ and the demand function as $x(p) = 30 - \frac{p}{2}$.

Assume that the price of energy is set at 30, so that the corresponding demand is $x = 30 - 30/2 = 15$. What is the corresponding consumer surplus?

The consumer surplus is given by

$$\begin{aligned}
CS(p) &= \int_0^{x(p)} v'(x) dx - px(p) \\
&= \int_0^{15} 60 - 2x dx - 30 * 15 = 60x - x^2 \Big|_0^{15} - 450 \\
&= 60 * 15 - 15^2 - 450 \\
&= 900 - 225 - 450 = 675 - 450 = 225
\end{aligned}$$

Question: Suppose the price of energy is dropped from p_0 to p_1 . What happens to the consumer's surplus?

When the price is p_0 , the consumer buys x_0 and the consumer surplus is, from (15):

$$CS(p_0) = \int_0^{x_0} v'(x) dx - p_0 x_0 \quad (16)$$

When the price is p_1 , the consumer buys x_1 and the consumer surplus is, from (15):

$$CS(p_1) = \int_0^{x_1} v'(x) dx - p_1 x_1 \quad (17)$$

So we want to compute the amount the consumer surplus has increased. Subtracting (16) from (17) to get the change in consumer surplus, we have:

$$\begin{aligned}
 CS(p_1) - CS(p_0) &= \int_0^{x_1} v'(x)dx - p_1x_1 - \int_0^{x_0} v'(x)dx + p_0x_0 \\
 &= \int_{x_0}^{x_1} v'(x)dx - p_1x_1 + p_0x_0
 \end{aligned} \tag{18}$$

Now I know a trick, and that is to add and subtract p_1x_0 to the above expression, resulting in:

$$CS(p_1) - CS(p_0) = \int_{x_0}^{x_1} v'(x)dx - p_1x_1 + p_0x_0 + p_1x_0 - p_1x_0 \tag{19}$$

Let's rearrange the terms:

$$CS(p_1) - CS(p_0) = \int_{x_0}^{x_1} v'(x)dx - p_1x_1 + p_1x_0 + p_0x_0 - p_1x_0$$

$$CS(p_1) - CS(p_0) = \int_{x_0}^{x_1} v'(x)dx - (p_1x_1 - p_1x_0) + (p_0x_0 - p_1x_0) \tag{20}$$

Finally, let's factor p_1 from the second term and x_0 from the third term, resulting in

$$CS(p_1) - CS(p_0) = \underbrace{\int_{x_0}^{x_1} v'(x)dx - p_1(x_1 - x_0)}_{\text{Additional CS due to increased utility from buying old amount } x_0} + \underbrace{(p_0 - p_1)x_0}_{\text{Additional CS due to savings from reduction in price from buying old amount } x_0} \tag{21}$$

Equation (21) says that the change in the consumer's surplus can be decomposed into two parts:

- The savings from the reduction in price that we enjoy from buying the old amount (yellow) and
- The increased utility due to the additional purchased amount (red).

We can see why if we return to our picture:

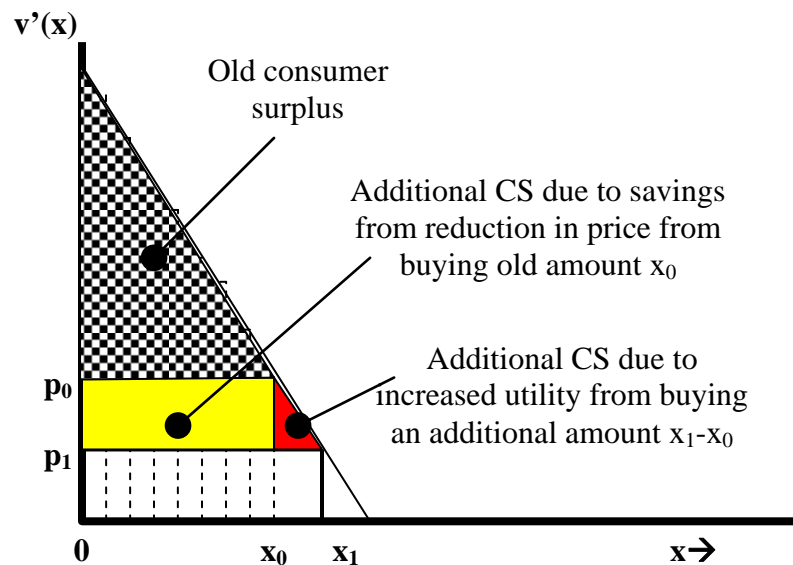


Fig. 5: Change in consumer surplus due to price drop

Additional homework problem (Problem 4):

As in our example 1 and 2, let $v(x) = 60x - x^2$, with

$p = 60 - 2x$ and $x(p) = 30 - \frac{p}{2}$. Recall that in example

2, we computed the consumer surplus at $p=30$, $x=15$, to be 225.

- a. How much energy is obtained when $p=40$?

- b. Compute the consumer surplus when the price is 40.
- c. What is the change in consumer surplus when the price is raised from 30 to 40?
- d. Determine the loss due to buying the new amount of energy at the higher price. If the consumer loses this amount of money, who gets it?
- e. Determine the decreased utility due to the decreased purchased amount.
- f. Draw a graph of the inverse demand function and illustrate on it:
 - The loss due to buying the new amount of energy at the higher price;
 - The decreased utility due to the decreased purchased amount;
 - The consumer surplus at the higher price.

[1] P. Samuelson and W. Nordhaus, "Economics," 17th edition, McGraw-Hill, 2001.

[2] A. Dillingham, N. Skaggs, and J. Carlson, "Economics: Individual Choice and its Consequences," Allyn and Bacon, 1992.

[3] W. Baumol and A. Blinder, "Economics: Principles and Policy," Dryden Press, 1994.