Homework 4, EE 458, Fall 2019

1. As in our example 1 and 2 of the notes called "Intro to Economics: Consumer Surplus," let $v(x) = 60x - x^2$, with $p = 60 - 2x$ and $x(p) = 30 - \frac{p}{2}$. Recall that in example 2, we computed the consumer surplus at p=30, x=15, to be 225.
   a. How much energy is obtained when p=40?
      Solution: x=30-40/2=10 units.
   b. Compute the consumer surplus when the price is p=40.

$$CS(p) = \int_0^{x(p)} v'(x)dx - px(p) = 60x - x^2\Big|_0^{10} - 40(10)$$
$$= 60(10) - (100) - 40(10) = 600 - 100 - 400 = 100$$

   c. What is the change in consumer surplus when the price is raised from 30 to 40?
      From the notes:

$$CS(p) = \int_0^{15} 60 - 2xdx - 30*15 = 60x - x^2\Big|_0^{15} - 450$$
$$= 60*15 - 15^2 - 450$$
$$= 900 - 225 - 450 = 675 - 450 = 225$$

      Therefore, the change in consumer surplus when price is raised from 30 to 40 is
      CS(40)-CS(30) = 100-225 = -125.
   d. Determine the loss due to buying the old amount of energy (15) at the higher price. If the consumer loses this amount of money, who gets it?
      Solution: From the notes, we recall that:

$$CS(p_1) - CS(p_0) = \underbrace{\int_{x_0}^{x_1} v'(x)dx - p_1(x_1 - x_0)}_{\substack{\text{Change in CS due to change in} \\ \text{utility from buying amount } x_1\text{-}x_0 \\ \text{at price } p_1}} + \underbrace{(p_0 - p_1)x_0}_{\substack{\text{Change in CS due to} \\ \text{rchange in price from} \\ \text{buying old amount } x_0}}$$

      So it is the second term that we want:

$$(p_0 - p_1)x_0 = (30 - 40)15 = -150$$

      So a -150 change in CS is a "loss" of 150.

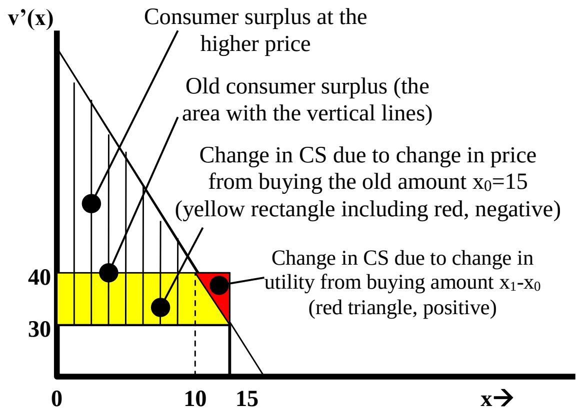
      The seller (producer) would get the money.

   e. Determine the change in CS due to change in utility from buying amount x_1-x_0 at price p_1.
      Solution: This is the first term:

$$\int_{15}^{10} 60 - 2xdx - 40(10 - 15) = 60x - x^2\Big|_{15}^{10} - 40(-5)$$
$$= 60(10) - 100 - 60(15) + 225 + 200 = 600 - 100 - 900 + 225 + 200 = 25$$

      As a check, we sum the answer from (d) to the answer from (e):
      -150+25=-125
      and see that it agrees with (c).
   f. Draw a graph of the inverse demand function and illustrate on it:

- The change in consumer surplus from buying the old amount of energy at the higher price;
- The change in consumer surplus due to change in utility due to the decreased purchased amount;
- The consumer surplus at the higher price.



Two comments about the red triangle in the above figure.

1. The calculation for that area is given more explicitly by

$$\underbrace{\int_{x_0}^{x_1} v'(x)dx - p_1\left(x_1 - x_0\right)}_{\substack{\text{Change in CS due to change in} \\ \text{utility from buying amount } x_1\text{-}x_0 \\ \text{at price } p_1}} = \left\{\int_{15}^{10} 60 - 2x dx\right\} - \left[40\left(10-15\right)\right] = \left\{60x - x^2 \Big|_{15}^{10}\right\} - \left[40(-5)\right]$$

$$= \left\{60(10) - 100 - 60(15) + 225\right\} - \left[-200\right] = \left\{600 - 100 - 900 + 225\right\} - \left[-200\right]$$

$$= \left\{-175\right\} - \left[-200\right] = 25$$

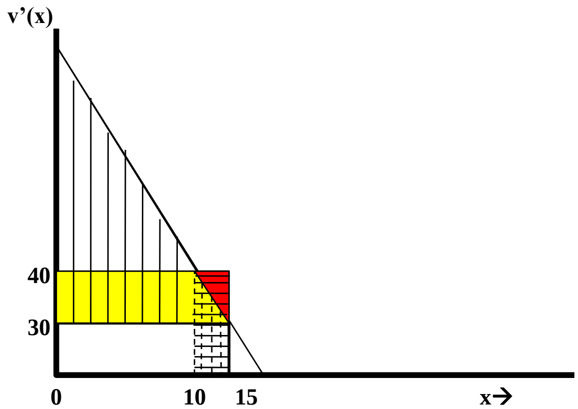
2. The calculation is given by:

$$\underbrace{\int_{x_0}^{x_1} v'(x)dx - p_1\left(x_1 - x_0\right)}_{\substack{\text{Change in CS due to change in} \\ \text{utility from buying amount } x_1\text{-}x_0 \\ \text{at price } p_1}}$$

We redraw this below, where we observe that the calculation corresponds to

- (first term) area under the v'(x) curve (vertical dashed lines), which is -175 LESS
- (second term) the area of the $p_1 \times (x_1 - x_0)$ rectangle (horizontal solid lines) which is -200

It is clear from inspecting the below diagram that the difference between the first and second term is the red triangle (the part of the area that has horizontal lines only). This difference is the "change in CS due to change in utility from *buying amount x_1-x_0 at price p_1.*" The fact that it is positive (+25) is explained by the fact that *$x_1-x_0=10-15=-5$* is *negative*; thus, it is as if this particular part of the change in CS (the red triangle) is due to *selling amount $x_0-x_1=15-10=+5$ at price p_1*.


v'(x)
40
30
0
10
15
x→