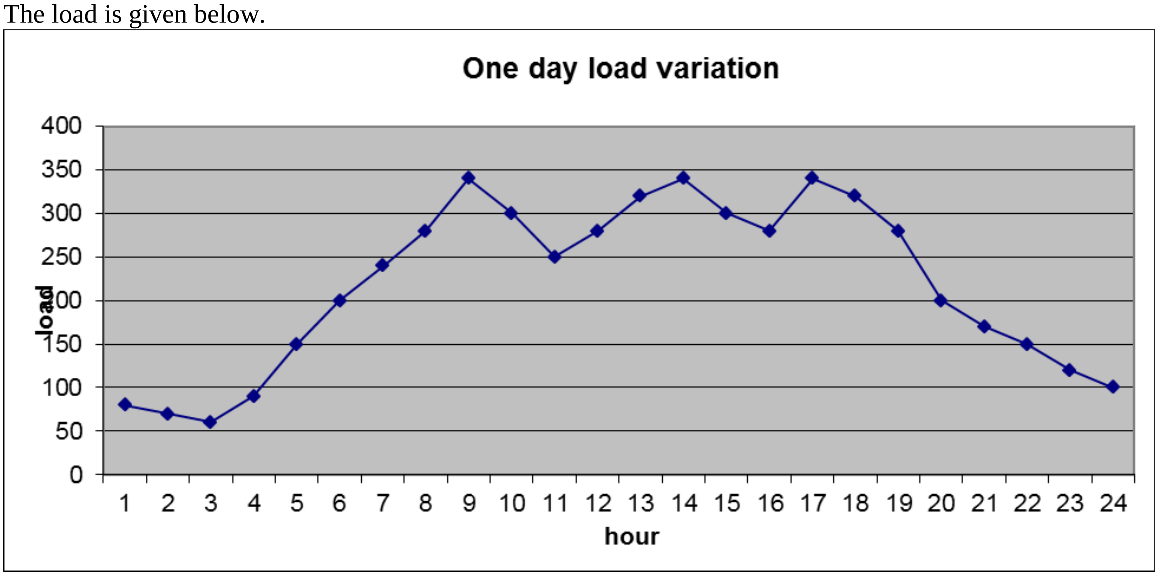
EE 458 EXAM 2, Fall 2019, Name: ____

Instructions: 75 minutes allowed. Work each problem on a separate sheet of paper with your name on it. Close book, closed notes, calculator allowed. The answer to problem #1 should have been worked before the exam period, brought into the exam, and turned in with the exam at the end of the period.

1. (20 pts) Beginning from the CPLEX file provided, provide plots of dispatch vs. time for all three units. **Give total cost.** Then implement the below ramp rate constraints for all three units such that that $MxInc_i$=$MxDec_i$=0.3 pu.
   Note that this affects constraints (7) and (8) in our formulation. Provide plots of dispatch vs. time for all three units comparing the "with ramp rate constraints" and "without" cases. Give total cost.

Solution:
The load is given below.



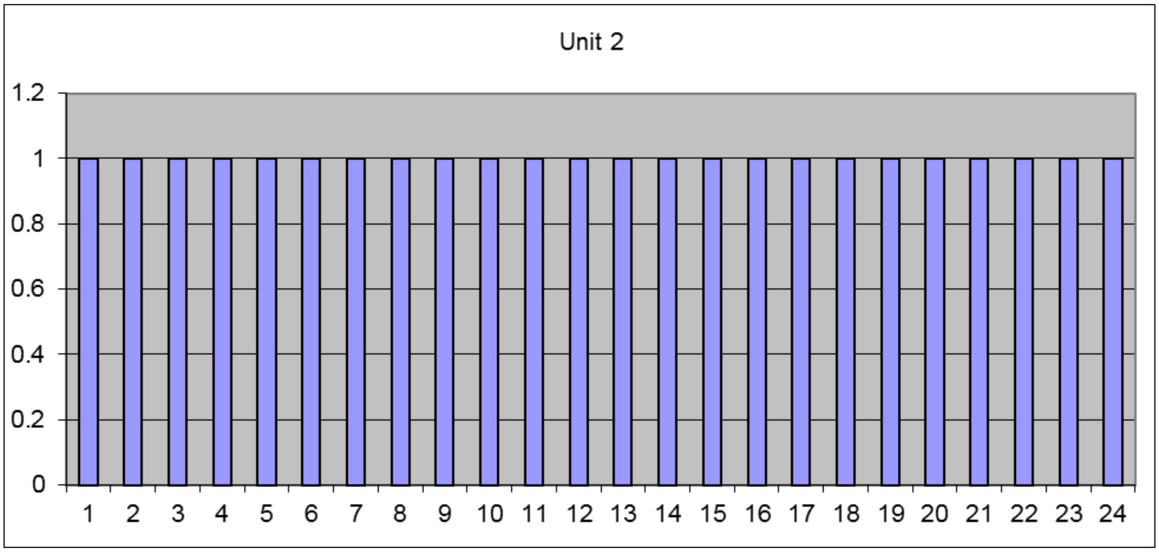
The ramp rate constraints were implemented using code as below (23 different times):

```
g11t+g12t+g13t-g11u-g12u-g13u<=0.3
g11u+g12u+g13u-g11t-g12t-g13t<=0.3
```

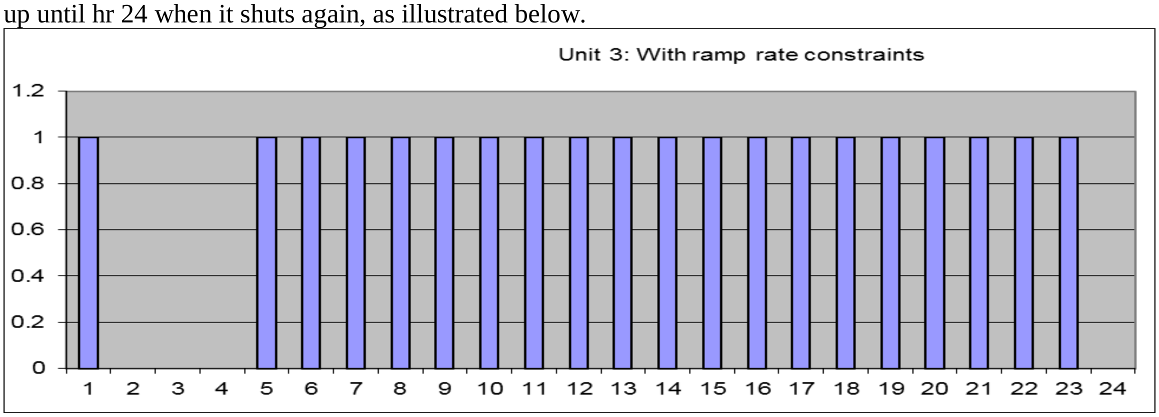
for t=2,…,24 where u=t-1.

The total cost for the problem without the ramp rate constraints (UC24b.lp) is $66867.95. The total cost for the problem with the ramp rate constraints (UC24c.lp) is $66957.05.

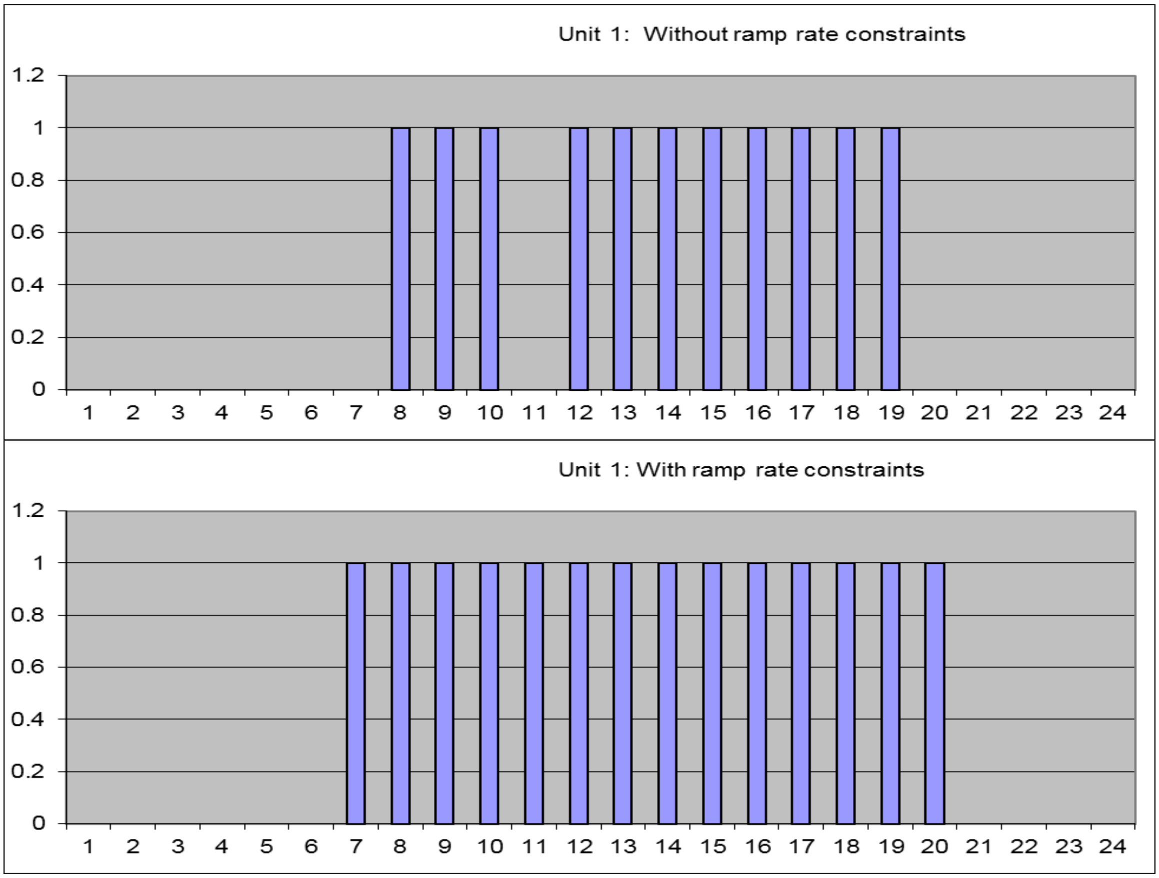
In both cases, unit 2 stays "up" the entire 24 hours – its plot is below.



In both cases, unit 3 is up at hr 1, shuts at hr 2 and remains down for hrs 3 and 4, starts at hr 5 and remains up until hr 24 when it shuts again, as illustrated below.



The plots, for unit 1, with and without ramp rate constraints, are as below, where we can tell unit 1 is initially down in both situations. However, without ramp rate constraints, it starts at hr 8, shuts at hr 11, starts at hr 12, and shuts at hr 20. With ramp rate constraints, it starts at hr 7 and shuts at hr 21.



2. (31 pts) Consider the linear program (LP) given below.

$$\max F = 5x_1 + 4x_2 + 3x_3$$

Subject to

$$\text{Constraint 1: } 2x_1 + 3x_2 + x_3 \le 5$$

$$\text{Constraint 2: } 4x_1 + x_2 + 2x_3 \le 11$$

$$\text{Constraint 3: } 3x_1 + 4x_2 + 2x_3 \le 8$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

a. (4 pts) Rewrite the above LP in equality form.
Solution:

$\max F - 5x_1 - 4x_2 - 3x_3 = 0$

$2x_1 + 3x_2 + x_3 + x_4 = 5$

$4x_1 + x_2 + 2x_3 + x_5 = 11$

$3x_1 + 4x_2 + 2x_3 + x_6 = 8$

$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$

b. (4 pts) Identify the decision and slack variables in the equality form of the LP.
Solution:
Decision variables are x_1, x_2, x_3.
Slack variables are x_4, x_5, x_6

c. (4 pts). Identify basic and non-basic variables for the equality form of the LP.
Solution:
Basic variables are x_4, x_5, x_6. Nonbasic variables are x_1, x_2, x_3.

d. (4 pts) Write the initial tableau for the LP.
Solution:

Basic variable	Eq. #	Coefficients of							Right side
		F	x_1	x_2	x_3	x_4	x_5	x_6	
F	0	1	-5	-4	-3	0	0	0	0
x_4	1	0	2	3	1	1	0	0	5
x_5	2	0	4	1	2	0	1	0	11
x_6	3	0	3	4	2	0	0	1	8

e. (3 pts) Identify the entering variable in the 1^{rst} iteration of the simplex method.
Solution: The entering variable is x_1 because it improves the objective at the highest rate, i.e., it has the largest (most negative) coefficient in the objective function.

f. (3 pts) Identify the leaving variable in the 1^{rst} iteration of the simplex method.
Solution:
To identify the leaving variable, we need to identify the variable that hits 0 first as the entering variable is increased, as dictated by one of the constraint equations. For each equation that contains the entering variable, the leaving variable is the basic (nonzero) variable for which the ratio of

RHS/Variable Coefficient

is minimum. Inspecting the above table, the possibilities are

5/2=2.5
11/4=2.75
8/3=2.67

And so the leaving variable is the basic variable in the first equation, which is x_4.

g. (3 pts) Perform the full elimination step necessary to obtain the new complete tableau (for the next iteration)
Solution: First, divide the pivot row (first row in this case) by the pivot element (2). This results in the following:

Basic variable	Eq. #	Coefficients of							Right side
		F	x_1	x_2	x_3	x_4	x_5	x_6	
F	0	1	-5	-4	-3	0	0	0	0
x_1	1	0	1	3/2	1/2	1/2	0	0	5/2
x_5	2	0	4	1	2	0	1	0	11
x_6	3	0	3	4	2	0	0	1	8

Now eliminate x_1 from all other equations (including the objective function), add an appropriate multiple of the pivot row to each other row. The results is:

Basic variable	Eq. #	Coefficients of							Right side
		F	x_1	x_2	x_3	x_4	x_5	x_6	
F	0	1	0	7/2	-1/2	5/2	0	0	25/2
x_1	1	0	1	3/2	1/2	1/2	0	0	5/2
x_5	2	0	0	-5	0	-2	1	0	1
x_6	3	0	0	-1/2	1/2	-3/2	0	1	1/2

h. (3 pts) Identify the value of the objective function following the first iteration, and indicate evidence regarding whether this solution is optimal or not.
Solution: The value of the objective function is 25/2=12.5.
It is not optimal because not all coefficients in the objective function row of the tableau are positive, i.e., there is one coefficient in the objective function row in the Tableau that is negative.

i. (3 pts) Assume the tableau you obtained following part (g) is optimal (whether it is or isn't). Identify how much the objective function would change if the right-hand side of constraint 1 was increased from 5 to 6.
Solution:
The coefficients of the slack variables in the objective function expression of the final tableau (the one that corresponds to the optimal solution) give the improvement in the objective for a unit increase in the right-hand-sides of the corresponding constraints. Therefore, 5/2=2.5 is the answer.
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3. (15 pts) A linear program is below. We refer to this problem as the primal.

$$\max F = 5x_1 + 4x_2 + 3x_3$$

Subject to

$$2x_1 + 3x_2 + x_3 \le 5$$
$$4x_1 + x_2 + 2x_3 \le 11$$
$$3x_1 + 4x_2 + 2x_3 \le 8$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0$$

a. (7 pts) Obtain the dual problem.
Solution:

$$\min G = 5\lambda_1 + 11\lambda_2 + 8\lambda_3$$

Subject to

$$2\lambda_1 + 4\lambda_2 + 3\lambda_3 \ge 5$$
$$3\lambda_1 + \lambda_2 + 4\lambda_3 \ge 4$$
$$\lambda_1 + 2\lambda_2 + 2\lambda_3 \ge 3$$
$$\lambda_1 \ge 0, \lambda_2 \ge 0, \lambda_3 \ge 0$$

b. (8 pts) The solution of the above primal problem is (2,0,1) with optimal objective function value of 13. In using the simplex method, the coefficients of the primal slack variables in the final tableaux for the primal problem are (1,0,1).
   i. What is the optimal objective function value of the dual problem?
      Solution: 13
   ii. What is the solution of the dual problem, i.e., what are the values of the decision variables for the dual problem?
      Solution: $(\lambda_1, \lambda_2, \lambda_3)=(1,0,1)$
   iii. The dual problem also has its own dual variables. What are the values of these variables?
      Solution: (2,0,1)
   iv. What are the coefficients of the slack variables in the final tableaux for the dual problem?
      Solution: (2,0,1)

4. (16 pts) A linear program optimal power flow (LPOPF) for a four-bus system is solved. The following information characterizes the bids, offers, and LMPs for this system. Branches 1, 2, 4, and 5 have infinite transmission capacity. Branch 3 is constrained to 16MW.

Offers and bids :

s_{g2}=\$12.11/MWhr s_{d2}=\$13.00/MWhr

s_{g4}=\$12.54/MWhr s_{d3}=\$12.00/MWhr

s_{g1}=\$13.07/MWhr

Generation and demand levels, branch flows, and LMPs:

P_{g1}=50.67 MW	P_{b1}=-58.57MW	LMP_1=\$13.07/MWhr
P_{g2}=150 MW	P_{b2}=46.67MW	LMP_2=\$13.00/MWhr
P_{g4}=180 MW	P_{b3}=16MW	LMP_3=\$13.12/MWhr
P_{d2}=180.67 MW	P_{b4}=121.3MW	LMP_4=\$13.09/MWhr
P_{d3}=200 MW	P_{b5}=62.67MW	

a. (4 pts) Compute the total amount paid to the generators.

Solution:

$$Payment_{g1} = P_{g1} \times LMP_1 = 50.67MW \times 13.07\$/MWhr = 662.21\$/hr$$
$$Payment_{g2} = P_{g2} \times LMP_2 = 150MW \times 13.00\$/MWhr = 1950.00\$/hr$$
$$Payment_{g4} = P_{g4} \times LMP_4 = 180MW \times 13.09\$/MWhr = 2356.74\$/hr$$

Note: small error in first and third calculations above, should be 662.26 and 2356.20
Total payments to gens: 662.26+1950.00+2356.20=4968.46\$/hr.

b. (4 pts) Compute the total amount paid by the loads.
Solution:

$$Payment_{d2} = P_{d2} \times LMP_2 = 180.667MW \times 13.00\$/MWhr = 2348.67\$/hr$$
$$Payment_{d3} = P_{d3} \times LMP_2 = 200MW \times 13.12\$/MWhr = 2623.40\$/hr$$

Note: small error in first and second calculation above, should be 2348.71 and 2624.
Total payments by loads: 2348.71+2624.00=4972.71\$/hr,

c. (4 pts) Compute the congestion charges.
Solution:
   Congestion Charges = 4972.71-4968.46=\$4.25

d. (4 pts) Compute the dual variable corresponding to the constraint on the branch 3 flow.
Solution:
   Congestion Charges=\$4.25=λ(16)➔λ=4.25/16=\$0.27/MWhr

5. (6 pts) In class, we computed shift factors for our four bus, five branch system, associated with a +1 injection at bus 2 and a -1 injection at bus 3. The shift factor vector was computed to be

$$\begin{bmatrix} t_{1,23} \\ t_{2,23} \\ t_{3,23} \\ t_{4,23} \\ t_{5,23} \end{bmatrix} = \begin{bmatrix} 0.125 \\ -0.375 \\ 0.625 \\ 0.125 \\ 0.25 \end{bmatrix}$$

Compute the change in flow on branch 3 for a "double shift" of +2 injection at bus 2 compensated by a -2 injection at bus 1, together with a -2 injection at bus 3 compensated by a +2 injection at bus 1.

Solution:
This "double shift" amounts to a +2 injection at bus 2 compensated by a -2 injection at bus 3, which is exactly what the above shift factors give. But we must multiply them by 2. So the change in flow on branch 3 will be:
$\Delta P_{branch3}$=2(0.625)=1.25pu.

6. **(12 pts) True-False**
   a. _T_ A linear program is a convex programming problem.
   b. _F_ In an LP solution, a dual variable relates the change in the objective function to a change in the right-hand-side of the constraint corresponding to the dual variable according to $\lambda_i = \Delta F^*/\Delta b_i$ for any value of Δb_i.
   c. _T_ The optimal solution to a linear program is always at a point where active constraints intersect (corner points).
   d. _F_ The situation where a unit is up in hour 1, starts in hour 1, and is up in hour 2 cannot occur in the SCUC because it violates the constraint $z_{it} \leq z_{it-1} + y_{it}$.
   e. _F_ The presence of inter-temporal constraints provides the means for an effective solution strategy by solving each time period independently.
   f. _F_ In order to avoid double payment to suppliers, the day-ahead market is financially binding, but the real-time market is not.