Name: ____

EE 458, Economic Systems for Electric Power Planning,
Exam 1, Spring 2019, Dr. McCalley, 50 minutes
Closed book, Closed Notes, No Calculator

1. (25 pts) Answer the questions below. Some data which may or may not help you in answering these questions is as follows: 1BTU=1054.85joules; 2200lbs=1MetricTon
   a. (5 pts) The maximum efficiency of a thermal unit is 30%; compute the corresponding heat rate.
   Solution: This could be computed either from the second expression below or the third.

$$H \text{ (in MBTU/MWhr)} = \underbrace{\frac{\left(10^6 \frac{\text{watts}}{\text{MW}} \times 3600 \frac{\text{sec}}{\text{hr}}\right)}{\eta \times 10^6 \frac{\text{BTU}}{\text{MBTU}} 1054.85 \frac{\text{joules}}{\text{BTU}}}}_{\text{SECOND EXPRESSION}} = \underbrace{\frac{3.4128}{\eta}}_{\text{THIRD EXPRESSION}} = \frac{3.4128}{0.30} = 11.376 \text{ MBTU/MWhr}$$

   b. (2 pts) Would a more efficient unit have a higher or lower heat rate than that computed in part (a)?
   Solution:
   Lower
   c. (5 pts) Consider a thermal unit with a heat rate of 10 MBTU/MWhr. Compute the fuel input when the power is 10 MW.
   Solution: R=H*P=(10MBTU/MWhr)(10MW)=100 MBTU/hr
   d. (5 pts) A certain thermal unit has a heat rate of 10000BTU/kWhr when generating 50 MW; its fuel cost is $2.00/MBTU. Compute the cost rate.
   Solution:
   C=R*K=H*P*K=(10MBTU/MWhr)*50MW*($2.00/MBTU)=$1000/hr.
   e. (5 pts) A certain thermal unit has a heat rate of 10MBTU/MWhr. It uses a type of coal having a CO_2 emissions content of 210lbs/MBTU. Compute the emissions per unit energy generated in units of Metric tons per MWhr.
   Solution:

$$210 \frac{lbs}{MBTU} \times \frac{10MBTU}{MWhr} \times \frac{1MT}{2200lbs} = 0.9545 MT / MWhr$$

   f. (3 pts) In 2015, California emitted 0.275 MT of CO_2 for every MWhr produced in the state. This rate is about a fourth of what a typical coal plant emits. Give three reasons for this:
   Solution:
      (i) California thermal plants use only natural gas; none use coal.
      (ii) Many of California's natural gas plants are combined cycle units which get much higher efficiencies than single-cycle plants.
      (iii) California also gets significant amounts of electric energy from renewables (hydro, wind, solar) which produce no CO_2 at all.

2. (30 pts) Generator cost rate functions, in $/hr, for a three unit system are given as

$$C_1(P_1) = 0.004P_1^2 + 5.3P_1 + 500$$
$$C_2(P_2) = 0.006P_2^2 + 5.5P_2 + 400$$
$$C_3(P_3) = 0.009P_3^2 + 5.8P_3 + 200$$

Limits on the generation levels are $200 \le P_1 \le P_{1,MAX}$, $150 \le P_2 \le 350$, $100 \le P_3 \le 225$. The three generators must supply a total demand of 975 MW.

(a) (5 pts) Express the objective function for the cost minimization problem.
(b) (5 pts) Express the LaGrangian function assuming no constraints are binding.
(c) (5 pts) Identify the first order conditions assuming no constraints are binding.
(d) (5 pts) Form the linear matrix equation necessary to solve the unconstrained optimization problem.
(e) (5 pts) The solution to the unconstrained optimization problem is $P_1 = 482.9MW$, $P_2 = 305.3MW$, $P_3 = 186.5MW$. Assume $P_{1,MAX}$=500 MW. Compute lambda.
(f) (5 pts) Now assume that $P_{1,MAX}$=450 MW. Form the linear matrix equation necessary to solve the next iteration of getting the solution to this problem.

Solution:

(a) $f(P_1, P_2, P_3)=C_1(P_1)+C_2(P_2)+C_3(P_3)$
(b) $F(P_1, P_2, P_3,\lambda)=f(P_1, P_2, P_3)-\lambda(P_1+P_2+P_3-975)$
(c) The KKT conditions assuming no binding constraints are:

$$\frac{\partial F}{\partial P_1} = 0.008P_1 + 5.3 - \lambda = 0$$
$$\frac{\partial F}{\partial P_2} = 0.012P_2 + 5.5 - \lambda = 0$$
$$\frac{\partial F}{\partial P_3} = 0.018P_3 + 5.8 - \lambda = 0$$
$$\frac{\partial F}{\partial \lambda} = P_1 + P_2 + P_3 - 975 = 0$$

(d)

$$\underline{\underline{\begin{bmatrix} 0.008 & 0 & 0 & -1 \\ 0 & 0.012 & 0 & -1 \\ 0 & 0 & 0.018 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} -5.3 \\ -5.5 \\ -5.8 \\ 975 \end{bmatrix}}}$$

(e)

We can use any of the $\frac{\partial F}{\partial P_i}$ to obtain λ

$$\lambda = 0.008 \cdot (482.9) + 5.3 = 0.012 \cdot (305.3) + 5.5 = 0.018 \cdot (186.8) + 5.8$$
$$= \underline{\underline{9.163\$ / MW - hr}}$$

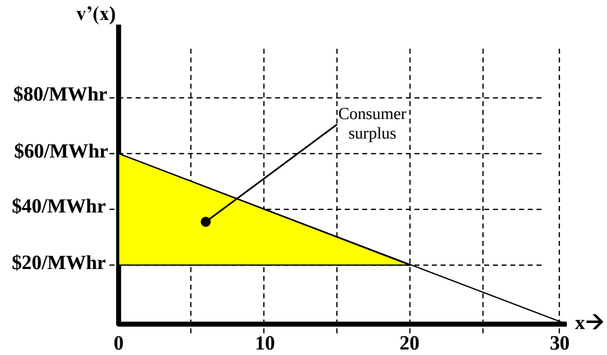
(f) Now since P_1 exceeds its limit, we need to bring in the corresponding constraint with its Lagrange Multiplier.

$$F = C_1 + C_2 + C_3 - \lambda(P_1 + P_2 + P_3 - 975) - \mu(P_1 - 450)$$

And when we apply first-order conditions, we will get

$$\begin{bmatrix} 0.008 & 0 & 0 & -1 & -1 \\ 0 & 0.012 & 0 & -1 & 0 \\ 0 & 0 & 0.018 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} -5.3 \\ -5.5 \\ -5.8 \\ 975 \\ 450 \end{bmatrix}$$

3. (25 pts) A consumer has a utility function of $v(x)=60x-x^2$.
   a. (8 pts) Find the inverse demand function.
      Solution:
      $$v(x) = 60x - x^2$$
      $$v'(x) = 60 - 2x$$
      So the inverse demand function is
      $$p = 60 - 2x$$
   b. (8 pts) Find the demand function.
      Solution:
      $$x = 30 - p/2$$
   c. (9 pts) Find the demand which maximizes consumer surplus when the price is \$20/MWhr. Calculate consumer surplus at this price and demand. Illustrate the consumer surplus on the graph below.
      Solution:
      $$x = 30 - p/2 = 30 - 20/2 = 20MWhr$$
      $$CS(p) = \int_0^{x(p)} v'(x)dx - px(p)$$
      $$= \int_0^{20} 60 - 2xdx - 20*20 = 60x - x^2\Big|_0^{20} - 400 = 60*20 - 20^2 - 400 = 1200 - 400 - 400 = 400$$



4. (20 pts) True-False:

a___T__

a. Up until the 1970's, a competitive electric power marketplace was not seriously considered because Averch-Johnson effects were thought to be outweighed by economies of scale benefits achievable by monopolistic firms.

b__T___

b. One reason why smaller power plants became more economically attractive was that the highly efficient combined cycle plants have to account for design complexities due to coupling between the combustion turbines and the heat recovery steam generators, and so tend to be lower in rating.

c__T___

c. Dr. Fred Schweppe developed the theory underlying locational marginal pricing in electric networks.

d__F___

d. Today, all electric energy used in the US is purchased through competitive electricity markets.

e__F___

e. Vertical disaggregation, the tendency of multiple electricity markets to operate independently, was a central concept underlying their success.

f__T___

f. Solution to an equality-constrained optimization problem occurs when the magnitude of the objective function gradient equals the magnitude of the equality constraint gradient.

g__T___

g. Electrical energy may be considered non-homogeneous because of differences in ramping capabilities and because generators have different network effects based on location.

h__F___

h. Any commodity market will achieve the exact same price and quantity condition as that achieved by a market run by a "benevolent dictator," as long as the agents behave autonomously and self-interestedly, and there is homogeneity of supply.

i__T___

i. All market agents become *price-takers* when the composite inverse demand and composite inverse supply curves become "flat" (zero slope) at the equilibrium point.

j__T___

j. The requirement for electricity markets to make information on system operation and conditions available (transparent) to the market agents is an attempt by the US Federal Energy Regulatory Commission (FERC) to achieve perfect competition.