Symmetrical Components 2

Sequence impedances

Although the following focuses on loads, the results apply equally well to lines, or lines and loads. Read these notes together with sections 12.6 and 12.9 of text.

Consider the Y-connected balanced load, Fig. 1. It is grounded through an impedance Z_n. Because of this, the neutral point may not be at the same potential as the ground.
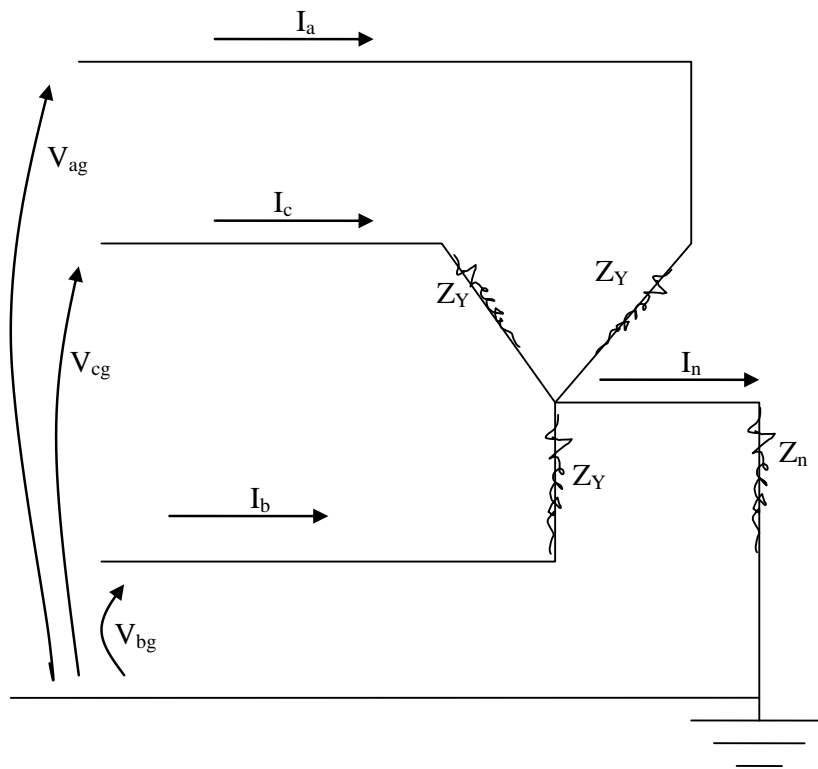

Fig. 1

Let's use KVL to write the voltage equations for the three phase-to-ground voltages V_{ag}, V_{bg}, and V_{cg} as a function of the line currents and the load impedances. Assume no mutual coupling between phases.

$$V_{ag} = Z_Y I_a + Z_n I_n$$

$$V_{bg} = Z_Y I_b + Z_n I_n \qquad (1)$$

$$V_{cg} = Z_Y I_c + Z_n I_n$$

We can replace I_n if we apply KCL at the junction node at the center of the Y.

$$I_n = I_a + I_b + I_c \qquad (2)$$

Substitution of (2) into (1) yields:

$$V_{ag} = Z_Y I_a + Z_n \left(I_a + I_b + I_c \right)$$

$$V_{bg} = Z_Y I_b + Z_n \left(I_a + I_b + I_c \right) \qquad (3)$$

$$V_{cg} = Z_Y I_c + Z_n \left(I_a + I_b + I_c \right)$$

Expanding the Z_n through and then collecting terms with like currents yields:

$$V_{ag} = \left(Z_Y + Z_n \right) I_a + Z_n I_a + Z_n I_b$$

$$V_{bg} = Z_n I_a + \left(Z_Y + Z_n \right) I_b + Z_n I_c \qquad (4)$$

$$V_{cg} = Z_n I_a + Z_n I_b + \left(Z_Y + Z_n \right) I_c$$

Now let's write these voltage equations in matrix form.

$$\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix} = \begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad (5)$$

In compact notation, eq. (5) is:

$$\underline{V}_{abc} = \underline{Z}_{abc}\underline{I}_{abc} \quad\quad (6)$$

Now I pose the following question....

If we can somehow find a way to transform eq. (5) into an equation that relates sequence voltages $\underline{V}_S$ on the LHS to sequence currents $\underline{I}_S$ on the RHS, what will the impedance matrix look like?

To answer this question, we need to derive eq. (7).

$$\underline{V}_S = \underline{Z}_S\underline{I}_S \quad\quad (7)$$

We refer to the impedance matrix, $\underline{Z}_S$, that relates sequence voltages to sequence currents, as the sequence impedance matrix.

To derive eq. (7), consider what we have: eq. (6), repeated here for convenience.

$$\underline{V}_{abc} = \underline{Z}_{abc}\underline{I}_{abc} \qquad (6)$$

Recall that $\underline{V}_{abc}=\underline{A}\underline{V}_S$ and $\underline{I}_{abc}=\underline{A}\underline{I}_S$. Substituting into eq. (6) yields:

$$\underline{A}\underline{V}_S = \underline{Z}_{abc}\underline{A}\underline{I}_S \qquad (8)$$

Now pre-multiply both sides by $\underline{A}^{-1}$. This is:

$$\underline{A}^{-1}\underline{A}\underline{V}_S = \underline{A}^{-1}\underline{Z}_{abc}\underline{A}\underline{I}_S \qquad (9)$$

The left hand side is just $\underline{V}_S$.

$$\underline{V}_S = \underline{A}^{-1}\underline{Z}_{abc}\underline{A}\underline{I}_S \qquad (10)$$

Comparison of (10) with (7) indicates that the sequence impedance matrix, $\underline{Z}_S$, is given by

$$\underline{Z}_S = \underline{A}^{-1}\underline{Z}_{abc}\underline{A} \qquad (11)$$

So what does $\underline{Z}_S$ look like? We know all three elements of eq. (11) so why don't we do the matrix math and find out…

$$\underline{Z}_S = \underline{A}^{-1} \underline{Z}_{abc} \underline{A}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} Z_Y + Z_n & Z_n & Z_n \\ Z_n & Z_Y + Z_n & Z_n \\ Z_n & Z_n & Z_Y + Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

Multiplying the two right-hand matrices:

$$\underline{Z}_S = \begin{bmatrix} Z_S^0 \\ Z_S^+ \\ Z_S^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} Z_Y + 3Z_n & Z_Y & Z_Y \\ Z_Y + 3Z_n & \alpha^2 Z_Y & \alpha Z_Y \\ Z_Y + 3Z_n & \alpha Z_Y & \alpha^2 Z_Y \end{bmatrix}$$

Now multiply the remaining matrices:

$$\underline{Z}_S = \begin{bmatrix} Z_S^0 \\ Z_S^+ \\ Z_S^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 3Z_Y + 9Z_n & 0 & 0 \\ 0 & 3Z_Y & 0 \\ 0 & 0 & 3Z_Y \end{bmatrix} = \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$

Plugging this expression into eq. (10)…

$$\underline{V}_S = \underline{A}^{-1} \underline{Z}_{abc} \underline{A} \underline{I}_S = \underline{Z}_S \underline{I}_S$$

$$
\begin{bmatrix} V_{ag}^0 \\ V_{ag}^+ \\ V_{ag}^- \end{bmatrix} = \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} \quad (12)
$$

Now this is an amazing thing…
        all off-diagonal terms are zero!

What does this mean?

It means that
▪ the only current that determines the zero sequence voltage is the zero sequence current.
▪ the only current that determines the positive sequence voltage is the positive sequence current.
▪ the only current that determines the negative sequence voltage is the negative sequence current.
This is the case whenever the impedance matrix is diagonal, with off-diagonals all 0.

We say that the three equations represented by the matrix relation are *uncoupled* in that no variable (current) appears in more than one equation.

So these 3 uncoupled equations are:

$$V_{ag}^0 = \left(Z_Y + 3Z_n\right)I_a^0$$

$$V_{ag}^+ = Z_Y I_a^+$$

$$V_{ag}^- = Z_Y I_a^- \qquad\qquad (13)$$

The really nice thing about these 3 equations is that they represent 3 separate and distinct SINGLE PHASE CIRCUITS!!!!

Therefore we can just apply EE 303 per-phase analysis to analyze them. Fig. 2 illustrates the single phase circuits.

$I_a^0 \rightarrow$

V_a^0

Z_Y

$3Z_n$

$Z_0 = Z_Y + 3Z_n$

Zero sequence network

V_a^+ $I_a^+ \rightarrow$

Z_Y

$Z_S^+ = Z_Y$

Positive sequence network

V_a^- $I_a^- \rightarrow$

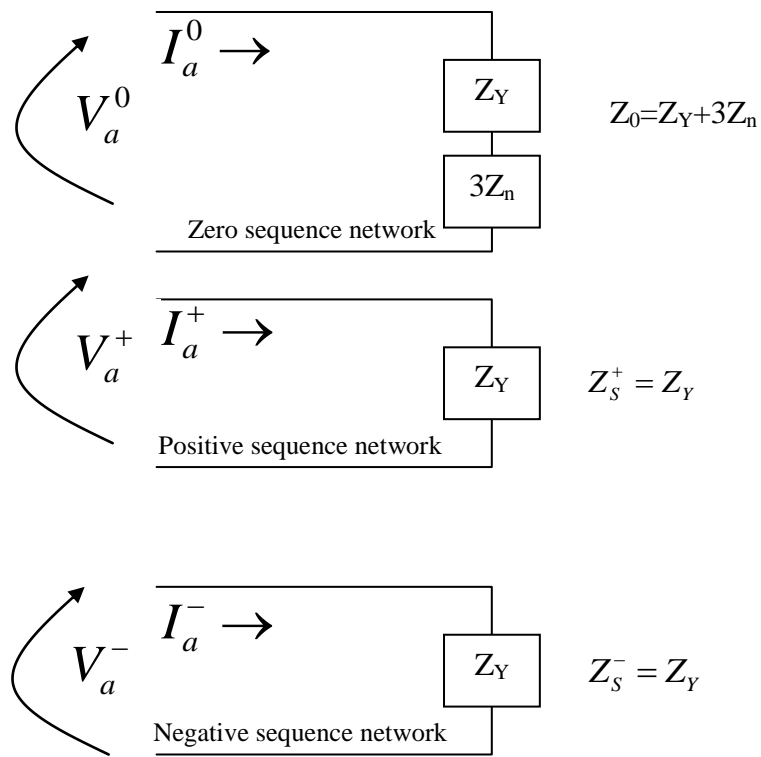Z_Y

$Z_S^- = Z_Y$

Negative sequence network

Fig. 2

Some questions:

1. Why doesn't the neutral impedance appear in the positive & negative sequence networks?

   Because the positive and negative sequence networks contain balanced currents only, and balanced currents sum to 0 and therefore do not contribute to flow in the neutral.

2. Why do we have $3Z_n$ in the zero sequence network instead of just Z_n?

   Recall $I_A + I_B + I_C = I_n$. We defined $I_a^0 = \frac{1}{3} I_n$. So $I_n = 3 I_a^0$ actually flows in the Z_n. But our zero sequence network has only I_a^0 flowing. Therefore, to obtain the correct voltage drop seen in the neutral conductor with a flow of only I_a^0, we model the zero-sequence impedance as $3Z_n$. Then the voltage drop is $3 I_a^0 Z_n$, as it should be .

3. What do these three networks look like if the neutral is solidly grounded (no neutral impedance)?

   Positive and negative sequence networks are the same. Zero sequence is the same except $Z_n=0$.

4. What do these three networks look like if the neutral is ungrounded (floating)?

   Positive and negative sequence networks are the same. Zero sequence has an open circuit, which means $I_n = I_a^0 = 0$.

5. What is benefit of the SC transformation?

   <u>Answer</u>: If the load (or line, or load and line) is symmetric (so that $\underline{Z}_S$ is diagonal), then the three networks will decouple and we can analyze an unbalanced situation with three separate per-phase analyses.

6. What if the load (or line, or load and line) is not symmetric?

Look at this case closely.

Consider a general a-b-c impedance matrix as given below (consistent with the matrix of eq. 3.40, pg. 473, in text)

$$\underline{Z}_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \tag{14}$$

This is a general impedance matrix in that
- The diagonals Z_{aa}, Z_{bb}, and Z_{cc} may differ.
- The off-diagonals Z_{ab}, Z_{ac}, Z_{bc} may differ.

The above occurs when the phase geometry is not equilateral & transposition is not used.

Recall eq. (11), repeated here for convenience:

$$\underline{Z}_S = \underline{A}^{-1} \underline{Z}_{abc} \underline{A} \tag{11}$$

Then we compute $\underline{Z}_S$ using the a-b-c impedance matrix of eq. (14):

$$\underline{Z}_S = \underline{A}^{-1} \underline{Z}_{abc} \underline{A} = \begin{bmatrix} Z_S^0 & Z_S^{0+} & Z_S^{0-} \\ Z_S^{+0} & Z_S^+ & Z_S^{+-} \\ Z_S^{-0} & Z_S^{-+} & Z_S^- \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ab} & Z_{bb} & Z_{bc} \\ Z_{ac} & Z_{bc} & Z_{cc} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

We will not go through the detailed matrix multiplication here but instead will just provide the expressions for each of the 9 terms in the $\underline{Z}_S$ matrix, as follows:

$$Z_S^0 = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc}\right) \quad (13)$$

$$Z_S^+ = Z_S^- = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc}\right) \quad (14)$$

$$Z_S^{0+} = Z_S^{-0} = \frac{1}{3}\left(Z_{aa} + \alpha^2 Z_{bb} + \alpha Z_{cc} - \alpha Z_{ab} - \alpha^2 Z_{ac} - Z_{bc}\right)(15)$$

$$Z_S^{0-} = Z_S^{+0} = \frac{1}{3}\left(Z_{aa} + \alpha Z_{bb} + \alpha^2 Z_{cc} - \alpha^2 Z_{ab} - \alpha Z_{ac} - Z_{bc}\right)(16)$$

$$Z_S^{+-} = \frac{1}{3}\left(Z_{aa} + \alpha^2 Z_{bb} + \alpha Z_{cc} + 2\alpha Z_{ab} + 2\alpha^2 Z_{ac} + 2Z_{bc}\right)(17)$$

$$Z_S^{-+} = \frac{1}{3}\left(Z_{aa} + \alpha Z_{bb} + \alpha^2 Z_{cc} + 2\alpha^2 Z_{ab} + 2\alpha Z_{ac} + 2Z_{bc}\right)(18)$$

For our 0+- circuits to be decoupled (and thus obtain the advantage of symmetrical component decomposition – see question 5 above), the off-diagonal elements of $\underline{Z}_S$ must be 0.

So what are the conditions for the off-diagonal elements of $\underline{Z}_S$ to be 0?

We obtain these conditions by setting (15)-(18) to 0 (making Z_S off-diagonal elements 0) and solving them simultaneously.

We will not go through this pain here. Rather, you should be able to inspect eqs. (15)-(18) and notice that for them to be 0, it must be true that

$$Z_{aa} = Z_{bb} = Z_{cc} \qquad (19)$$

(Diagonal phase impedances must be equal)

$$Z_{ab} = Z_{ac} = Z_{bc} \qquad (20)$$

(Offdiagonal phase impedances must be equal).

Under the conditions imposed by eqs. (19) and (20), it will be the case that the 0+- impedances are given by

$$Z_S^{0+} = Z_S^{+0} = Z_S^{0-} = Z_S^{-0} = Z_S^{+-} = Z_S^{-+} = 0 \quad (21)$$

$$Z_S^0 = Z_{aa} + 2Z_{ab} \quad (22)$$

$$Z_S^+ = Z_S^- = Z_{aa} - Z_{ab} \quad (23)$$

Check it for the example we worked above corresponding to Fig. 1:

$$\underline{Z}_{abc} = \begin{bmatrix} \mathbf{Z}_Y + \mathbf{Z}_n & \mathbf{Z}_n & \mathbf{Z}_n \\ \mathbf{Z}_n & \mathbf{Z}_Y + \mathbf{Z}_n & \mathbf{Z}_n \\ \mathbf{Z}_n & \mathbf{Z}_n & \mathbf{Z}_Y + \mathbf{Z}_n \end{bmatrix}$$

$$\underline{Z}_S = \begin{bmatrix} Z_Y + 3Z_n & 0 & 0 \\ 0 & Z_Y & 0 \\ 0 & 0 & Z_Y \end{bmatrix}$$

Some additional observations from this work:

- By eq. (14), repeated here for convenience,

$$Z_S^+ = Z_S^- = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} - Z_{ab} - Z_{ac} - Z_{bc}\right) \quad (14)$$

the positive and negative sequence impedances are always equal, independent of whether the load is symmetric or not. This is true for transmission lines, cables, and transformers[1].

- By eq. (13), repeated here for convenience,

$$Z_S^0 = \frac{1}{3}\left(Z_{aa} + Z_{bb} + Z_{cc} + 2Z_{ab} + 2Z_{ac} + 2Z_{bc}\right) \quad (13)$$

the 0-sequence impedance Z_S^0 is not equal to the positive and negative sequence impedances of a symmetrical load unless the off-diagonal phase impedances $Z_{ab}=Z_{ac}=Z_{bc}$ are 0.

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[1] It is not true for rotating machines because positive sequence currents, rotating in the same direction as the rotor, produce fluxes in the rotor iron differently than the negative sequence currents which rotate in the opposite direction as the rotor.