Stability 3

1.0 Introduction

In our last set of notes (Stability 2), we described in great detail the behavior of a synchronous machine following a faulted condition that is cleared by protective relaying. A key figure for us was Fig. 1 (it was Fig. 15 in the previous notes).

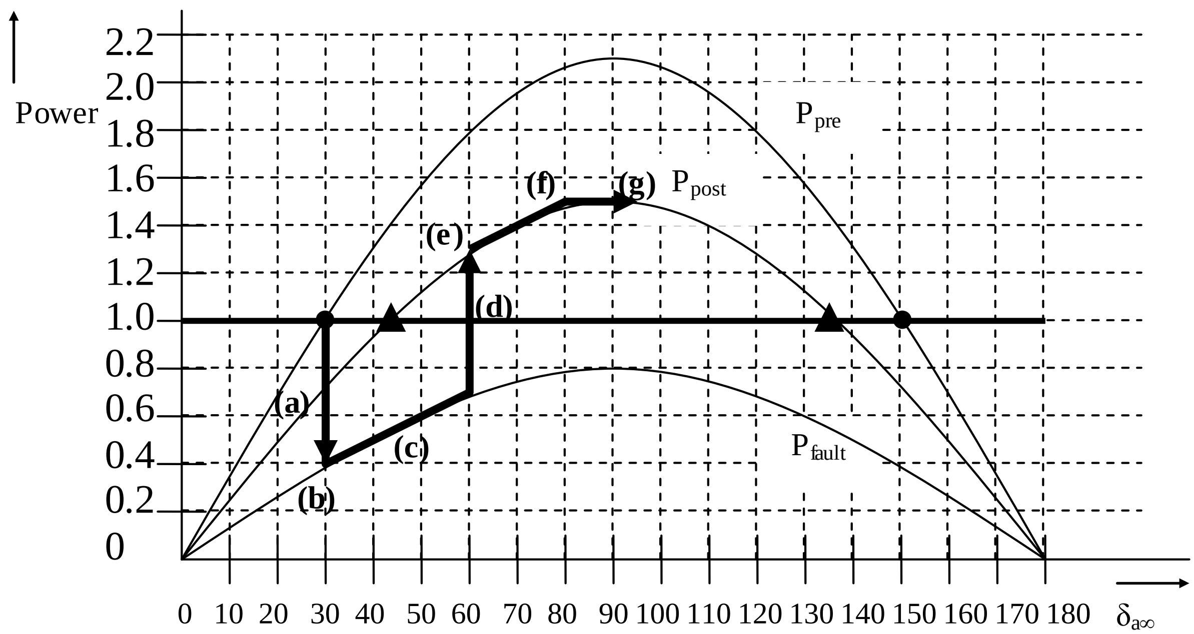

Fig. 1

Fig. 1 shows how the rotor accelerates during the fault-on period (b-c), and then decelerates during the post-fault period (e-f-g), turning around at point g.

Question: Are there ever conditions where the rotor does *not* turn around? That is, are there conditions where the angle continues to increase?

Answer: Unfortunately, yes. We qualify the answer as unfortunate because the system is unstable when the rotor does not turn around, a condition that is highly undesirable.

Our analysis of the last set of notes was for a stable response. Now we want to look at the possibility of an unstable response.

2.0 Unstable – what does it mean?

Let's first consider the ball-bowl analogy. What does it mean for this system to exhibit an unstable response?

Consider the ball is initially at the stable equilibrium, i.e., it is at the bottom of the bowl. Then we perturb the ball by giving it a push.

If the push is not so strong the ball will go up the side of the bowl and then come back, rolling around and finally returning to the stable equilibrium. But if the push is very strong, the ball will go up the side of the bowl and over the edge, leaving the bowl altogether, an unstable response, shown in Fig. 2.

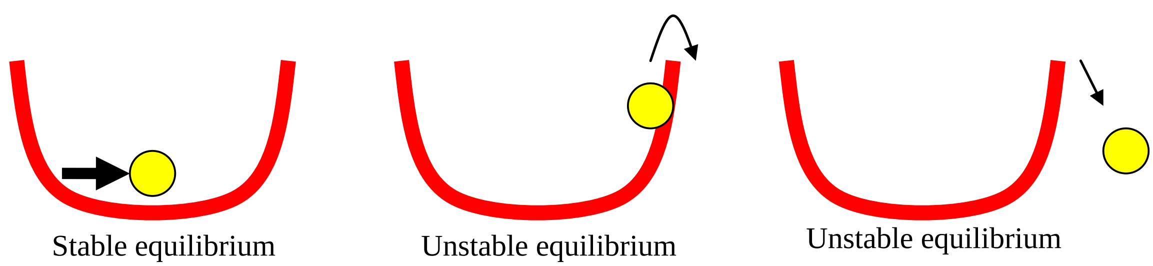

Fig. 2

Key to the whether the outcome is stable (rolls back) or unstable (rolls over the edge) is whether the ball reaches the edge or not. If it does not reach the edge, then clearly the response is stable. If it does reach the edge, then the response is unstable if it has any positive velocity at all when it reaches. The “dividing line” between these two situations is that the ball reaches the edge just as the velocity goes to zero, the so-called marginally stable case.

In the one-machine-infinite bus case, the unstable equilibrium (represented by the triangle on the right of the Fig. 1 post-disturbance curve), is the "edge." If the initial "push" (the fault) on the rotor angle is so "strong" that the rotor angle increases beyond this point, then $P_a=P_M-P_e$ is positive, and as the angle increases, P_a and the velocity also increases. If the push is so that the velocity is 0 just as the angle reaches the unstable equilibrium, then the case is marginally stable.

<u>3.0 Different problems to solve</u>

Our interest is to be able to predict when an unstable response will occur or the conditions under which an unstable response will occur. To do this, let's first consider what are the conditions that cause the "push" to be "strong."

These are

a. the pre-fault mechanical power into the machine is large;
b. the fault-on power-angle curve has low amplitude;
c. the fault has long duration.

There are a number of different kinds of problems that we could try to solve in relation to predicting, as listed below.

1. For a given pre-fault mechanical power, a given fault type and location, and a given clearing time, determine whether the response is stable or not. This is perhaps the simplest form of the problem.
2. For a given pre-fault mechanical power and a given fault type and location, determine the maximum fault duration for which the system is marginally stable. This duration (a time) is called the critical clearing time, and its corresponding angle is called the critical clearing angle.

3. For a given fault type and location and a given clearing time, determine the pre-fault mechanical power for which the system is marginally stable. When the fault is a "worst-case" fault (a three-phase fault at the machine terminals), this mechanical power is referred to as the operating limit for this machine. Operators will ensure that they never operate the machine above this generation limit.

Let's address the simplest of these three problems – the first one. In doing so, we will establish a criteria for stability.

4.0 Criteria for stability

Let's refer back to eq. (42) of the notes called "Stability 1." This version of the swing equation is:

$$\frac{2H}{\omega_{e0}}\ddot{\delta}(t) = P_{a,pu} \tag{1}$$

where

$$P_{a,pu} = P_M^0 - P_e \tag{2}$$

Define

$$\omega_r = \frac{d\delta}{dt} = \underbrace{\omega}_{\substack{\text{actual}\\\text{rotor}\\\text{speed}}} - \underbrace{\omega_{e0}}_{\substack{\text{synchronous}\\\text{rotor speed}}} \tag{3}$$

Remember that we are working in electrical radians. Also note that $\omega_r=0$ whenever machine is at synchronous speed.

Differentiating eq . (3), we get:

$$\frac{d\omega_r}{dt} = \frac{d^2\delta}{dt^2} \tag{4}$$

Substitute eq. (4) into the swing equation, eq. (1), to get:

$$\frac{2H}{\omega_{e0}}\frac{d\omega_r}{dt} = P_{a,pu} \tag{5}$$

Now multiply the left-hand-side by ω_r and the right-hand-side by $d\delta/dt$ (recall $\omega_r=d\delta/dt$), and also rearrange the left-hand-side, to get:

$$\frac{H}{\omega_{e0}}\left\{2\omega_r \frac{d\omega_r}{dt}\right\} = P_{a,pu}\frac{d\delta}{dt} \qquad (6)$$

The reason we rearranged the left-hand-side is because what is in the brackets is something special, as observed from differentiating $(\omega_r)^2$. Using the chain rule,

$$\frac{d}{dt}\left(\omega_r(t)\right)^2 = 2\omega_r(t)\frac{d\omega_r(t)}{dt} = 2\omega_r(t)\frac{d^2\delta}{dt^2} \qquad (7)$$

Observing the second expression in (7) is what is inside the brackets of (6), then (6) becomes:

$$\frac{H}{\omega_{e0}}\frac{d(\omega_r^2)}{dt} = P_{a,pu}\frac{d\delta}{dt} \qquad (8)$$

Now multiply both sides by dt to obtain:

$$\frac{H}{\omega_{e0}}d(\omega_r^2) = P_{a,pu}d\delta \qquad (9)$$

Consider a change in the state of this system characterized by (9) such that the angle changes from δ_1 to δ_2, and the speed changes from ω_{r1} to ω_{r2}, i.e., condition $(\delta_1, \omega_{r1}) \rightarrow$ condition (δ_2, ω_{r2}).

If the right-hand-side and the left-hand-side of (9) are indeed equal functions, with their variables $(\omega_r)^2$ and δ related by (7), then the integration of the functions in (9) with respect to their variables should also be equal. Therefore:

$$\int_{\omega_{r1}^2}^{\omega_{r2}^2} \frac{H}{\omega_{e0}} d(\omega_r^2) = \int_{\delta_1}^{\delta_2} P_{a,pu} d\delta \qquad (10)$$

Bring the constant in the left-hand integral out front:

$$\frac{H}{\omega_{e0}} \int_{\omega_{r1}^2}^{\omega_{r2}^2} d(\omega_r^2) = \int_{\delta_1}^{\delta_2} P_{a,pu} d\delta \qquad (11)$$

Noting that the variable of integration in the left-hand integral is $(\omega_r)^2$, we see that the left-hand integral is simple to evaluate.

$$\frac{H}{\omega_{e0}} (\omega_r^2)\Big|_{\omega_{r1}^2}^{\omega_{r2}^2} = \int_{\delta_1}^{\delta_2} P_{a,pu} d\delta \qquad (12)$$

or

$$\frac{H}{\omega_{e0}}(\omega_{r2}^2 - \omega_{r1}^2) = \int_{\delta_1}^{\delta_2} P_{a,pu} d\delta \tag{13}$$

We have so far not specified anything about the two states characterized by (δ_1, ω_{r1}), (δ_2, ω_{r2}). It is useful to do so now.

Let's let both of these states be zero-velocity states such that $\omega_{r1}= \omega_{r2}=0$. Then the left-hand-side of eq. (13) is 0, and we have:

$$\int_{\delta_1}^{\delta_2} P_{a,pu} d\delta = 0 \tag{14}$$

Now return to Fig. 1, repeated below for convenience, except I have also labeled the initial, clearing, and maximum angles as δ_0, δ_{clear}, and δ_{max}, respectively.

<u>Question</u>: Where are zero-velocity points?

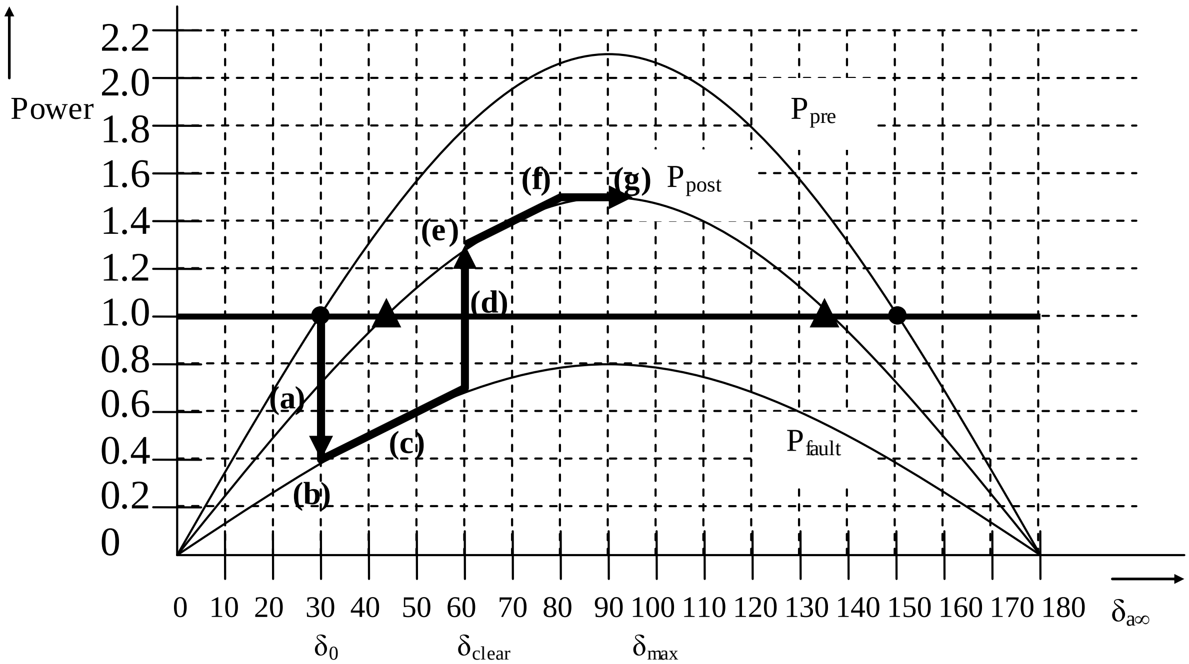

Fig. 3

Clearly, one zero-velocity point is the initial condition, characterized by the intersection of the pre-fault power-angle curve and the mechanical power (dark) line, the initial point for stage (a), which has an angle of δ_0. Also, since velocity cannot change instantaneously, the point (b) is also a zero-velocity point.

Another zero-velocity point is point (g), since it is at this point that the rotor "turns around."

Therefore, we can write eq. (14) as:

$$\int_{\delta_0}^{\delta_{\max}} P_{a,pu} d\delta = 0 \tag{15}$$

Let's replace P_a in eq. (15) using eq. (2).

$$\int_{\delta_0}^{\delta_{\max}} P_M^0 - P_e d\delta = 0 \tag{16}$$

Equation (16) is a wonderful relation. It says that in order for there to be two zero-velocity points (and therefore be a stable response since the second zero-velocity point will not occur if the response is unstable), the total area under the curve of P_M-P_e from δ_0 to δ_{max} must be zero!

Now, what is the total area under the curve of P_M-P_e from δ_0 to δ_{max} in Fig. 3? It is just the area *between* the P_M and P_e curves. This area is appropriately colored in Fig. 4.

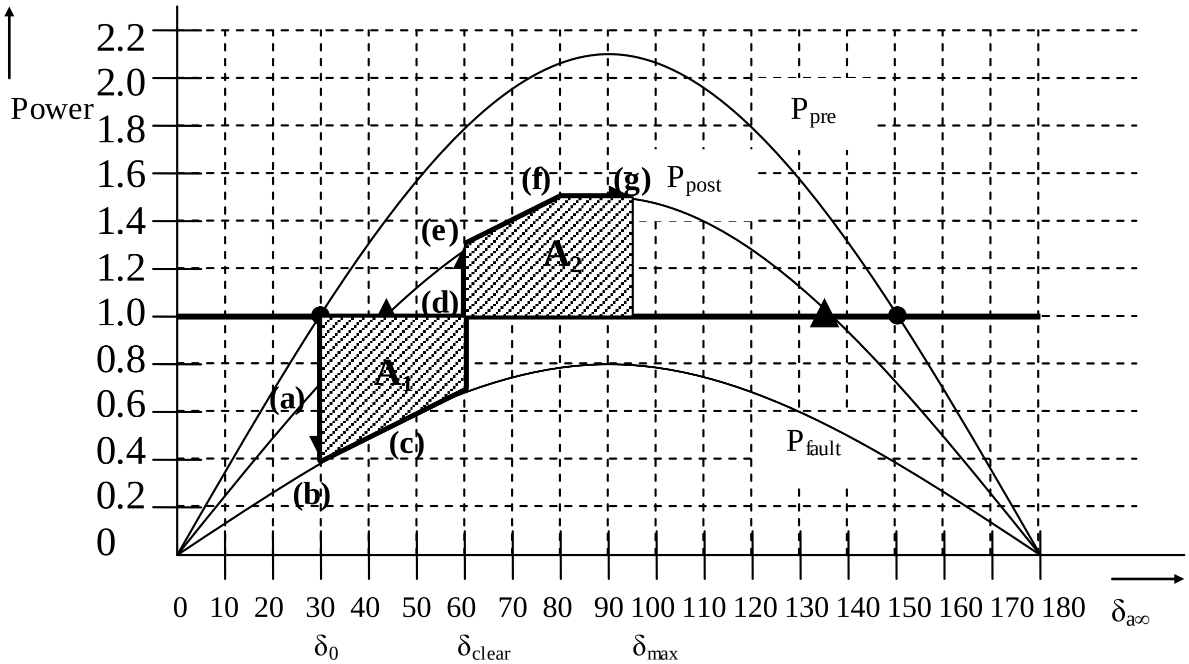

Fig. 4

Note that the lower area, A_1, uses the fault-on power-angle curve, whereas the upper area, A_2, uses the post-fault power-angle curve. So the P_e curve is discontinuous. Therefore, in analytically evaluating eq. (16), we need to account for this discontinuity, as follows:

$$\int_{\delta_0}^{\delta_{\max}} P_M^0 - P_e d\delta$$

$$= \int_{\delta_0}^{\delta_{clear}} P_M^0 - P_{fault} d\delta + \int_{\delta_{clear}}^{\delta_{\max}} P_M^0 - P_{post} d\delta = 0 \tag{17}$$

Therefore,

$$\int_{\delta_0}^{\delta_{clear}} P_M^0 - P_{fault} d\delta = -\int_{\delta_{clear}}^{\delta_{\max}} P_M^0 - P_{post} d\delta \tag{18}$$

Taking the negative sign on the right inside the integral, we have:

$$\underbrace{\int_{\delta_0}^{\delta_{clear}} P_M^0 - P_{fault} d\delta}_{A_1} = \underbrace{\int_{\delta_{clear}}^{\delta_{\max}} P_{post} - P_M^0 d\delta}_{A_2} \tag{19}$$

Here, we have the so-called equal-area criterion:

For stability, $A_1=A_2$, which means the decelerating energy (A_2) must equal the accelerating energy (A_1) in order for the system response to be stable.

When will instability occur?
To answer this question, note that the maximum value of decelerating energy is bounded by the maximum angle δ_{max}, i.e., once the angle exceeds δ_{max}, then the energy becomes accelerating energy.

If the maximum amount of available decelerating energy (A_2) is not enough to counteract the accelerating energy, then the system response will be unstable. Such a case is illustrated in Fig. 5.

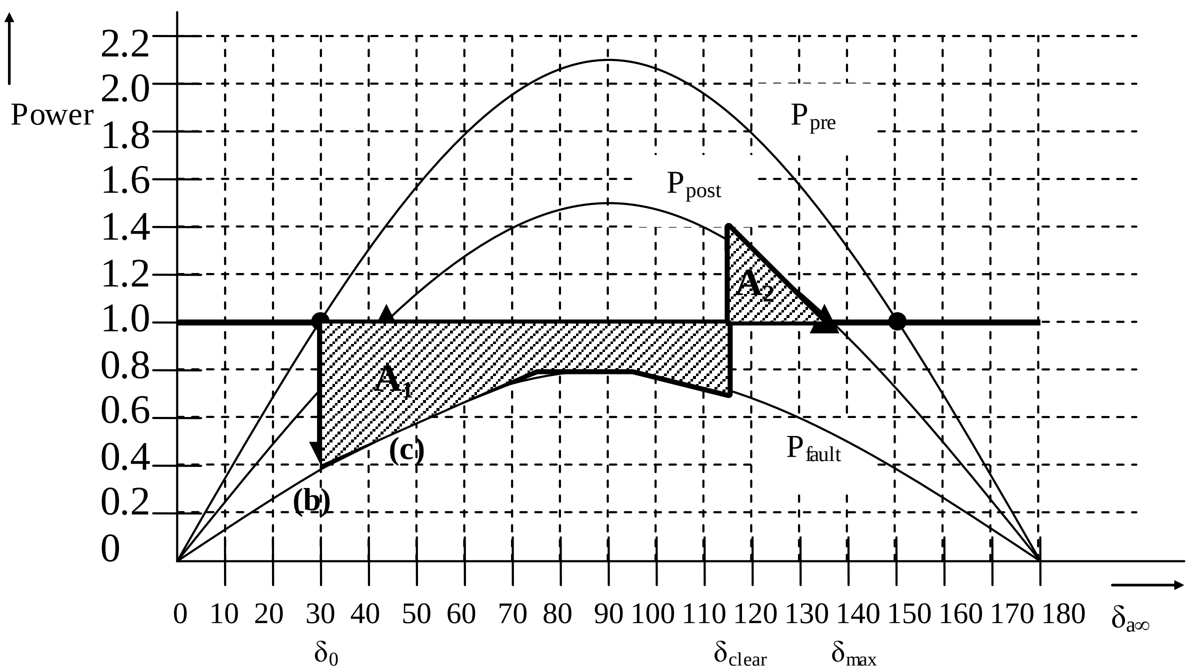

Fig. 5