Homework #4 Solution

Assignment: 12.2, 12.3, 12.9, 12.10, 12.11, 12.12, 12.17 Bergen & Vittal

Solutions:

12.2

$$E_s = A^{-1}E$$

$$E_s = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1\angle 0^\circ \\ 1\angle -90^\circ \\ 2\angle 135^\circ \end{bmatrix}$$

$$E_s = \begin{bmatrix} E_a^0 \\ E_a^+ \\ E_a^- \end{bmatrix} = \begin{bmatrix} .1953\angle 135^\circ \\ 1.311\angle 15^\circ \\ .4941\angle -105^\circ \end{bmatrix}$$

$$Check : E_a^0 + E_a^+ + E_a^- = 0$$

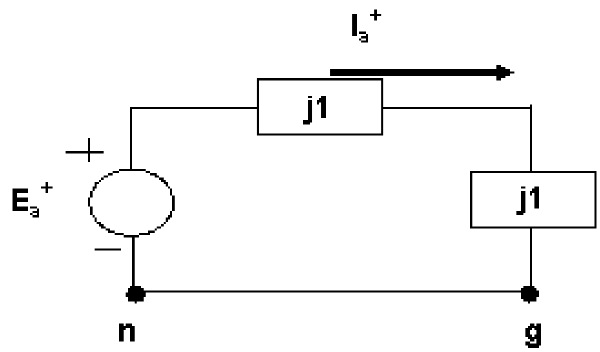
12.3 (Note: A more in depth solution to this problem is in the notes under Examples2)

Part A: In this problem the sources are not balanced. We can replace the unbalanced sources by sums of positive, negative, and zero sequence sets. We find responses to these sets and add.

PartB:

$$\begin{bmatrix} E_a^0 \\ E_a^+ \\ E_a^- \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ j1 \end{bmatrix} = \begin{bmatrix} .3333\angle 90° \\ .9107\angle -30° \\ .2440\angle 30° \end{bmatrix}$$

Redrawing the positive sequence network:



$$I_a^+ = \frac{.9107\angle -30°}{j1 + j1} = .4554\angle -120°$$

$$V_{ng}^+ = 0$$

Similarly we can calculate the negative sequence

$$I_a^- = \frac{.2440\angle 30°}{j1 + j1} = .1220\angle -60°$$

$$V_{ng}^- = 0$$

Finally the zero sequence:

$$I_a^0 = 0$$

$$V_{ng}^0 = -E_a^0 = .3333\angle -90°$$

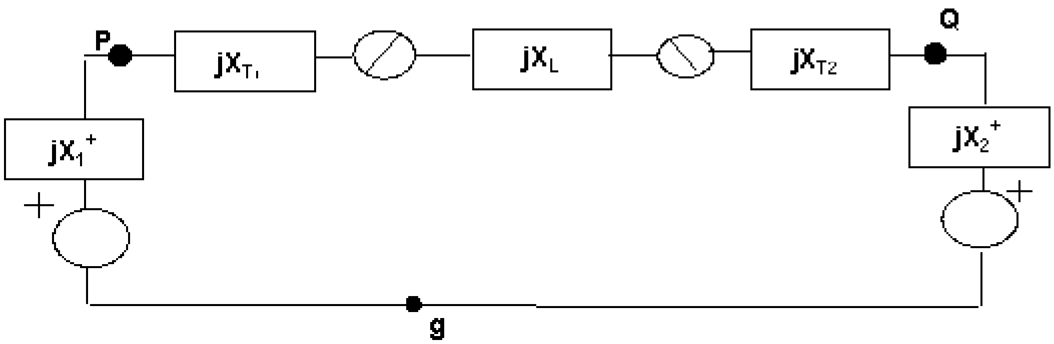
$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \begin{bmatrix} .5271\angle -108.44° \\ .5271\angle 108.44° \\ .3334\angle 0° \end{bmatrix}$$

$$V_{ng} = V_{ng}^+ + V_{ng}^- + V_{ng}^0 = .3333\angle 90°$$

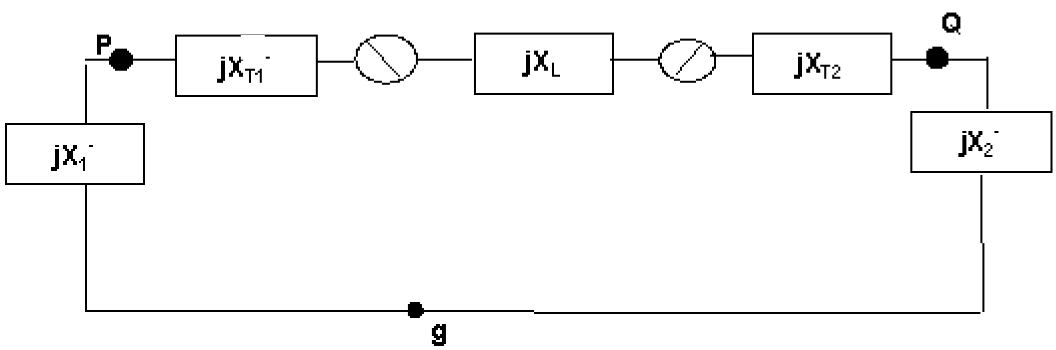
12.9

Part A:

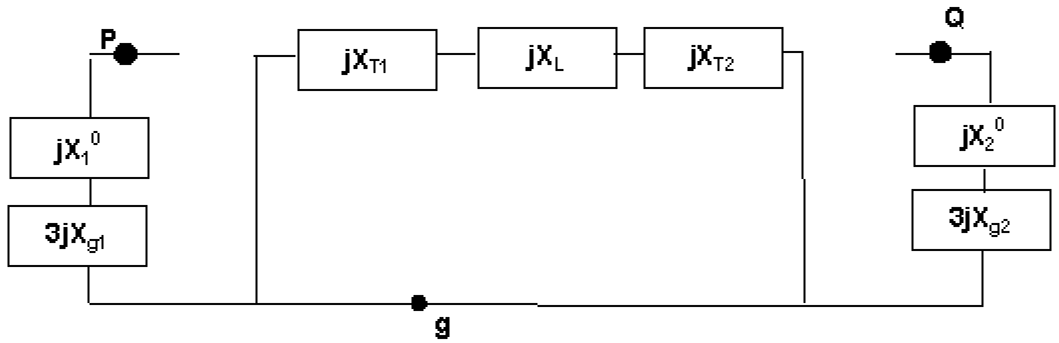
Positive Sequence



Negative Sequence



Zero Sequence



Part B:
We first connect in series at the points P and g. Next, we need to derive the Thevenin Equivalent circuits.

$$Z_{thev}^{+} = jX_1^{+} \parallel (jX_{T1} + jX_L + jX_{T2} + jX_2^{+}) = j.565$$

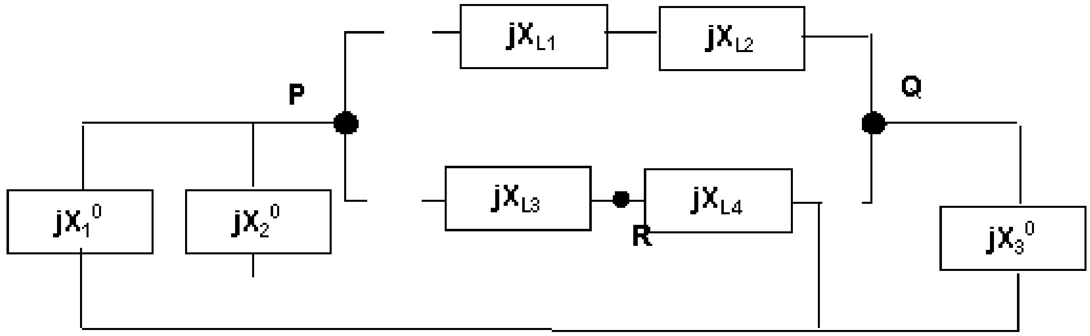
$$Z_{thev}^{-} = jX_1^{-} \parallel (jX_{T1} + jX_L + jX_{T2} + jX_2^{-}) = j.08$$

$$Z_{thev}^{0} = jX_1^{0} + 3jX_{g1} = j.305$$

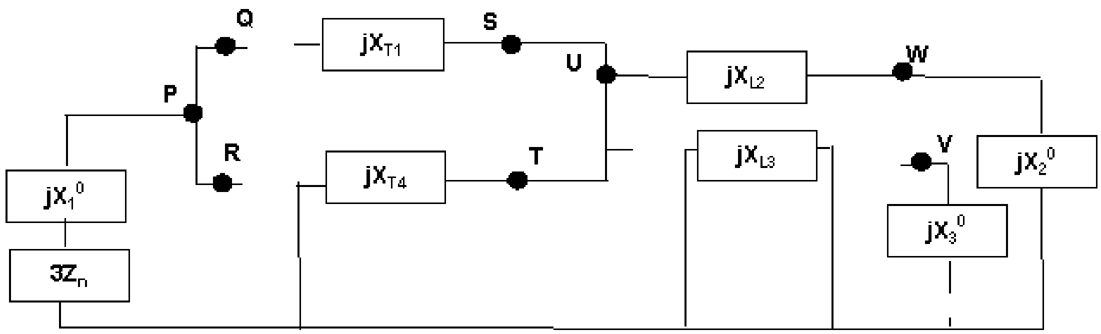
$$\frac{I^f}{3} = \frac{1\angle 0^\circ}{Z_{thev}^{+} + Z_{thev}^{-} + Z_{thev}^{0}} = 1.05263\angle -90^\circ$$

$$I^f = 3.15789\angle -90^\circ$$

12.10



12.11



12.12

Reference to Figs. 12.9 and 12.10 in the text indicates that the given connection of positive and negative sequence networks for a line-to-line fault assumes the fault is between the b and c-phases, with the voltage source in the sequence connection (Fig. 12.10) having the same angle as the E_{an} in the abc connection (assumed 0°). So let's re-label as follows:

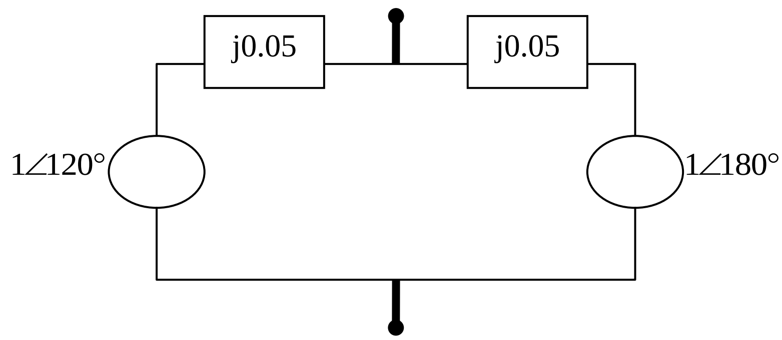
- a becomes bnew
- b becomes cnew
- c becomes anew

So the fault between a to b is now between bnew and cnew. But we now need to get the phase angle correct for the sources in the positive sequence network.

The phase angle for the positive sequence voltage source of the source on the left should be the same as the phase angle of the voltage source in the anew phase, which is the voltage source in the c phase. This would be 120° (c-phase is 240° behind a-phase or 120° ahead) assuming that the a-phase voltage is the reference as indicated in the figure of Problem 12.12.

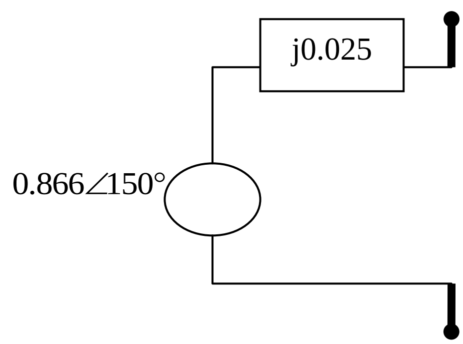
Similar reasoning results in the positive sequence voltage source of the source on the right should be 120° ahead of the a-phase voltage source, and since the a-phase voltage source on the right is indicated in the figure of Problem 12.12 to be 60°, the corresponding positive sequence voltage source should have an angle of 60+120=180°.

So the positive sequence circuit appears as below, with the dark lines indicating the fault point.

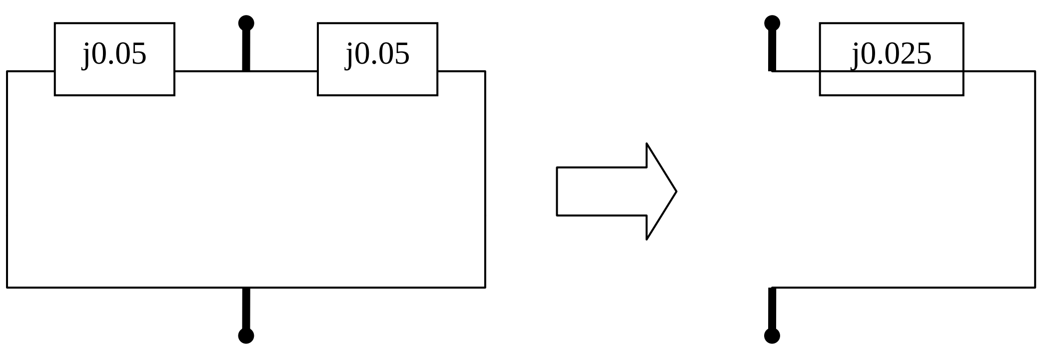


The Thevenin impedance is j.05/2=j.025. The Thevenin voltage is found from KVL: 1<120°-jI(0.1)=1<180°➔ I=(1<120°-1<180°)/j0.1=10<-30°.

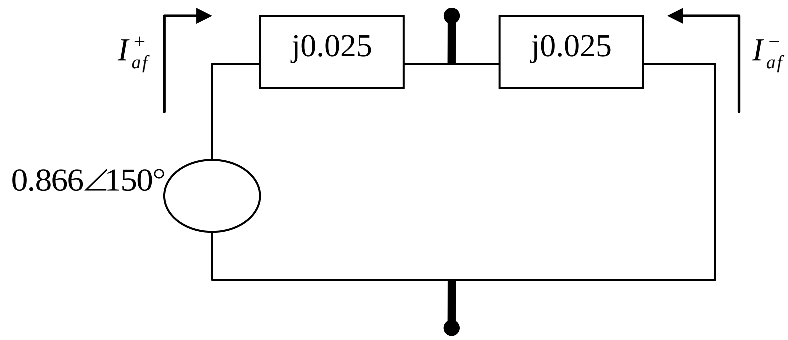
Therefore V_{thev}=1<120°-j0.05(10<-30°)=0.866<150°. So the Thevenin equivalent of the positive sequence network as seen from the fault point is



The negative sequence network appears as below, with the dark lines indicating the fault point, and the Thevenin equivalent is shown to the right.



Since this is a line-to-line fault, we do not need the zero-sequence network, but need only to connect the positive and negative sequence networks as shown below.



The fault positive sequence current is computed as

$$I_{af}^{+} = \frac{0.866\angle 150°}{j0.05} = 17.32\angle 60°$$

And the negative sequence current is clearly the negative of the positive sequence current, so that

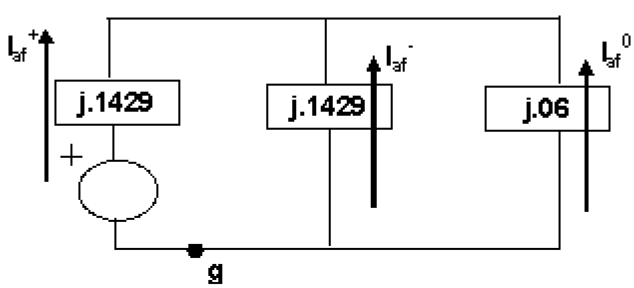
$$I_{af}^{-} = 17.32\angle -120°$$

With 0 zero sequence current (zero sequence network is dead for line-to-line fault), the abc currents are:

$$\begin{bmatrix} I_{anew} \\ I_{bnew} \\ I_{cnew} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 17.32\angle 60° \\ 17.32\angle -120 \end{bmatrix} = \begin{bmatrix} 0 \\ 30\angle -30° \\ 30\angle 150° \end{bmatrix}$$

12.17

We connect the sequence networks shown in Figure E12.5 (b) at the generator terminals (points a and g) and in parallel (as suggested by Figure 12.8). After reduction (the phase shifters can be ignored) we have the following equivalent circuit.



$$j.1429 = j.2 \,\|\, (j.1 + j.1 + j.1 + j.2)$$

Now we just have a simple circuit to solve for (use KVL or current divider).

$$I_{af}^{+} = -j.53996$$

$$I_{af}^{-} = j1.5967$$

$$I_{af}^{0} = j3.8029$$

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{af}^{0} \\ I_{af}^{+} \\ I_{af}^{-} \end{bmatrix} = \begin{bmatrix} 0 \\ -8.324\angle -43.27^\circ \\ 8.324\angle 43.27^\circ \end{bmatrix}$$