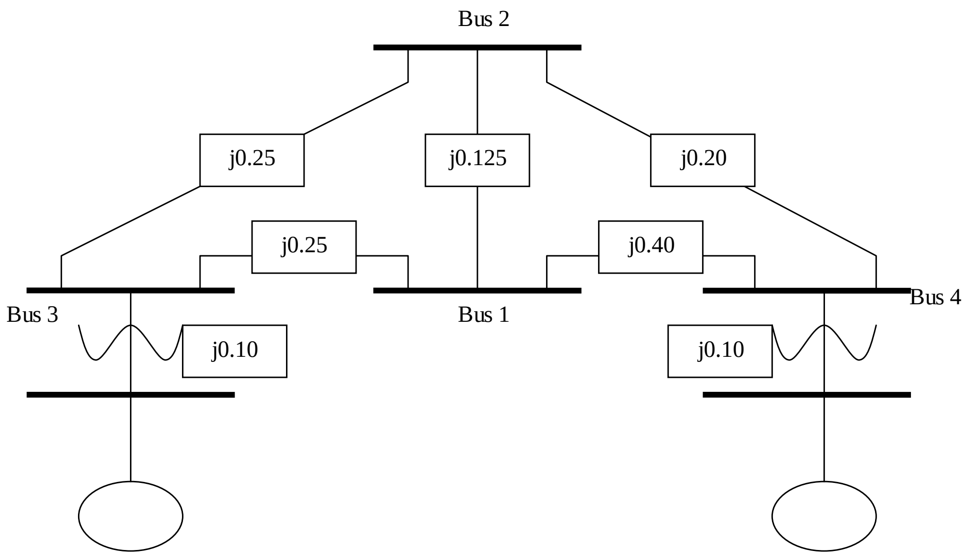
HW3 Solutions

1. Problem 9.8 (see end of this document)
2. Consider the 4-bus system shown below. Both machines have subtransient reactances of 0.20 pu (you can combine the machine subtransient reactance with the transformer impedance to get a single reactance connecting the machine internal voltage with the network).



   a. Construct the Y-bus for this network (should be a 4×4 matrix).
   b. Consider that there is a three-phase (symmetrical) fault at bus 2.
      ii. Use LU decomposition to obtain the 2nd column of the Z-bus.
      iii. Compute the subtransient fault current.
      iv. Use eq. (12) to find the voltages during the fault.
      v. Use eq. (17) to find the subtransient currents in lines 3-2, 1-2, and 4-2.

Solution:

a). Compute the Y-Bus

$$Y_{bus} = \begin{bmatrix} \frac{1}{j.25}+\frac{1}{j.125}+\frac{1}{j.40} & \frac{-1}{j.125} & \frac{-1}{j.25} & \frac{-1}{j.4} \\ \frac{-1}{j.125} & \frac{1}{j.25}+\frac{1}{j.125}+\frac{1}{j.2} & \frac{-1}{j.25} & \frac{-1}{j.2} \\ \frac{-1}{j.25} & \frac{-1}{j.25} & \frac{1}{j.25}+\frac{1}{j.25}+\frac{1}{j.3} & 0 \\ \frac{-1}{j.4} & \frac{-1}{j.25} & 0 & \frac{1}{j.2}+\frac{1}{j.3}+\frac{1}{j.40} \end{bmatrix}$$

$$Y_{bus} = \begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j11.3333 & 0 \\ j2.5 & j5 & 0 & -j10.83333 \end{bmatrix}$$

b).
i). LU Decomposition to obtain Z_2

$$Y_{bus}Z_{bus} = I$$

$$Y_{bus}Z_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Factorization of a matrix $\underline{Y}$ can be done efficiently and easily using the matlab command:
[$\underline{L}$,$\underline{U}$]=lu($\underline{Y}$)
Then it is easy to find $\underline{w}$ by hand using forward substitution from:
$\underline{L}\underline{w} = \underline{I}_k$

➔ $$\underline{L}\underline{w} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

And then it is easy to find $\underline{Z}_2$ by hand using backwards substitution from:
$\underline{U}\underline{Z}_2 = \underline{w}$
Alternatively, the manual steps of LU decomposition can be performed per the notes from "LU Decomposition." I did it within Matlab, as follows. Here, y1 is the initial "augmented" matrix (see notes on LU decomposition for the meaning of this term).

```
>> y1=[-14.5i,8i,4i,2.5i,0;8i,-17i,4i,5i,1;4i,4i,-11.33333i,0,0;2.5i,5i,0,-10.833333i,0]

y1 =

   0 -14.5000i   0 + 8.0000i   0 + 4.0000i   0 + 2.5000i        0
   0 + 8.0000i   0 -17.0000i   0 + 4.0000i   0 + 5.0000i   1.0000
   0 + 4.0000i   0 + 4.0000i   0 -11.3333i        0             0
   0 + 2.5000i   0 + 5.0000i        0        0 -10.8333i        0

>> y2=[y1(1,:)/y1(1,1);y1(2,:);y1(3,:);y1(4,:)]

y2 =

   1.0000          -0.5517          -0.2759          -0.1724               0
   0 + 8.0000i   0 -17.0000i   0 + 4.0000i   0 + 5.0000i   1.0000
   0 + 4.0000i   0 + 4.0000i   0 -11.3333i        0             0
   0 + 2.5000i   0 + 5.0000i        0        0 -10.8333i        0

>> y3=[y2(1,:);y2(1,:)*-y1(2,2)+y2(2,:);y2(1,:)*-y1(3,3)+y2(3,:);y2(1,:)*-
y1(4,4)+y2(4,:)]

y3 =

   1.0000          -0.5517          -0.2759          -0.1724               0
   0 +25.0000i   0 -26.3793i   0 - 0.6897i   0 + 2.0690i   1.0000
   0 +15.3333i   0 - 2.2529i   0 -14.4598i   0 - 1.9540i        0
   0 +13.3333i   0 - 0.9770i   0 - 2.9885i   0 -12.7011i        0

>> y3=[y2(1,:);y2(1,:)*-y1(2,1)+y2(2,:);y2(1,:)*-y1(3,1)+y2(3,:);y2(1,:)*-
y1(4,1)+y2(4,:)]

y3 =

   1.0000          -0.5517          -0.2759          -0.1724               0
        0        0 -12.5862i   0 + 6.2069i   0 + 6.3793i   1.0000
        0        0 + 6.2069i   0 -10.2299i   0 + 0.6897i        0
        0        0 + 6.3793i   0 + 0.6897i   0 -10.4023i        0

>> y4=[y3(1,:);y3(2,:)/y3(2,2);y3(3,:);y3(4,:)]

y4 =

   1.0000          -0.5517          -0.2759          -0.1724               0
        0          1.0000          -0.4932          -0.5068          0 + 0.0795i
        0        0 + 6.2069i   0 -10.2299i   0 + 0.6897i        0
        0        0 + 6.3793i   0 + 0.6897i   0 -10.4023i        0
```

```
>> y5=[y4(1,:);y4(2,:);y4(2,:)*-y3(3,2)+y4(3,:);y4(2,:)*-y3(4,2)+y4(4,:)
y5 =

   1.0000            -0.5517            -0.2759            -0.1724                  0
        0             1.0000            -0.4932            -0.5068                  0 + 0.0795i
        0                  0                  0 - 7.1689i        0 + 3.8356i    0.4932
        0                  0                  0 + 3.8356i        0 - 7.1689i    0.5068

>> y6=[y5(1,:);y5(2,:);y5(3,:)/y5(3,3);y5(4,:)]

y6 =

   1.0000            -0.5517            -0.2759            -0.1724                  0
        0             1.0000            -0.4932            -0.5068                  0 + 0.0795i
        0                  0             1.0000            -0.5350                  0 + 0.0688i
        0                  0                  0 + 3.8356i        0 - 7.1689i    0.5068

>> y7=[y6(1,:);y6(2,:);y6(3,:);y6(3,:)*-y5(4,3)+y6(4,:)]

y7 =

   1.0000            -0.5517            -0.2759            -0.1724                  0
        0             1.0000            -0.4932            -0.5068                  0 + 0.0795i
        0                  0             1.0000            -0.5350                  0 + 0.0688i
        0                  0                  0                  0 - 5.1168i    0.7707

>> y8=[y7(1,:);y7(2,:);y7(3,:);y7(4,:)/y7(4,4)]

y8 =

   1.0000            -0.5517            -0.2759            -0.1724                  0
        0             1.0000            -0.4932            -0.5068                  0 + 0.0795i
        0                  0             1.0000            -0.5350                  0 + 0.0688i
        0                  0                  0             1.0000                  0 + 0.1506i


>> z4=y8(4,5)

z4 =

        0 + 0.1506i

>> z3=y8(3,5)-y8(3,4)*z4

z3 =
```

```
        0 + 0.1494i

>> z2=y8(2,5)-y8(2,4)*z4-y8(2,3)

z2 =

  0.4932 + 0.1558i

>> z2=y8(2,5)-y8(2,4)*z4-y8(2,3)*z3

z2 =

        0 + 0.2295i

>> z1=y8(1,5)-y8(1,4)*z4-y8(1,3)*z3-y8(1,2)*z2

z1 =

        0 + 0.1938i

>>
>> Z2=[z1;z2;z3;z4]

Z2 =

        0 + 0.1938i
        0 + 0.2295i
        0 + 0.1494i
        0 + 0.1506i
```

ii). Compute the subtransient fault current.

$$I_f'' = \frac{V_f}{Z_{22}} = \frac{1}{j.2295} = -j4.3573\,pu\ \_\ or\ \_\ 4.3573\angle -90^\circ\,pu$$

iii). Use eq. (12) to find the voltages during the fault.

$$V_{if} = V_j - \frac{Z_{jk}}{Z_{kk}} V_f$$

$$V_{1f} = V_1 - \frac{Z_{12}}{Z_{22}} V_f = 1 - \frac{j.1938}{j.2295} 1 = .15556\,pu$$

$$V_{2f} = 0$$

$$V_{3f} = V_3 - \frac{Z_{32}}{Z_{22}} V_f = .34902\, pu$$

$$V_{4f} = V_4 - \frac{Z_{42}}{Z_{22}} V_f = .343791\, pu$$

iv). Use eq. (17) to find the subtransient currents in lines 3-2, 1-2, and 4-2.

$$I''_{ij} = -V_f \frac{Z_{ik} - Z_{jk}}{Z_b Z_{kk}}$$

$$I''_{32} = -V_f \frac{Z_{32} - Z_{22}}{Z_b Z_{22}} = -1 \frac{j.1494 - j.2295}{(j.25)(j.2295)} = -j1.39608\, pu$$

$$I''_{12} = -V_f \frac{Z_{12} - Z_{22}}{Z_b Z_{22}} == -j1.244\, pu$$

$$I''_{42} = -V_f \frac{Z_{42} - Z_{22}}{Z_b Z_{22}} == -j1.71895\, pu$$

3. A Y-connected load has balanced currents with a-c-b sequence given by

$$I_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 10\angle +120° \\ 10\angle -120° \end{bmatrix}$$

Calculate the sequence currents. How does your answer differ from the answer obtained in Example 1 in these notes?

Solution:

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \underline{A}^{-1} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 10\angle 0° \\ 10\angle 120° \\ 10\angle -120° \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 10\angle 0° \end{bmatrix}$$

In this case the quantity that is non-zero is negative sequence component.

4. A feeder provides service to a delta-connected load having the following phase currents:

$$I_{ab} = 208.3\angle -18.19°$$

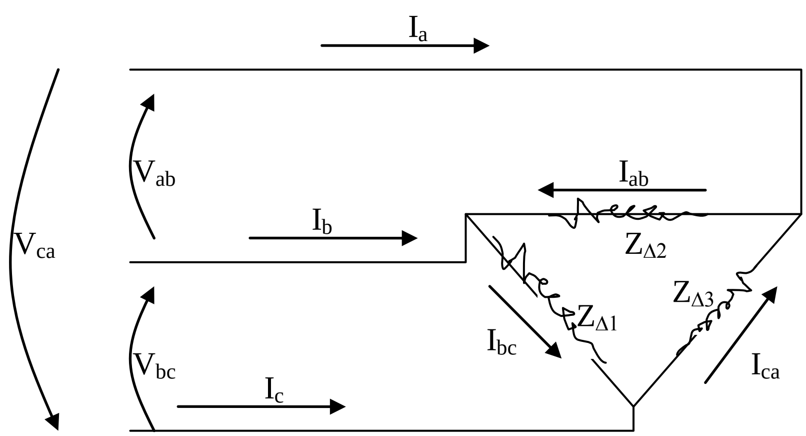
$$I_{bc} = 138.89\angle -151.788°$$

$$I_{ca} = 131.94°\angle 145.84$$

a. For the phase currents:
   i. Are they balanced or unbalanced?
   ii. What is their sum?
   iii. Obtain their sequence quantities.
   iv. What is the 0-sequence quantity?
b. Obtain the line currents. For these currents:
   i. Are they balanced or unbalanced?
   ii. What is their sum?
   iii. Obtain their sequence quantities.
   iv. What is the 0-sequence quantity?
c. Use what you have learned in the parts (a) and (b) to answer the three questions (ii, iv) from part (b) for the following a-b-c quantities:
   i. Unbalanced currents into a grounded-Y.
   ii. Unbalanced currents into an ungrounded-Y.
   iii. Unbalanced line-to-line voltages.

Solution:

The situation is shown in the figure below.



Part a:

i. Phase currents are unbalanced.

ii. Their sum is….

$$I_{ab} + I_{bc} + I_{ca} =$$

$$208.3\angle -18.19° + 138.89\angle -151.788° + 131.94°\angle 145.84$$

$$= 65.9252\angle -121.01°$$

iii. Their sequence quantities are:

$$\begin{bmatrix} I_{ab}^{0} \\ I_{ab}^{+} \\ I_{ab}^{-} \end{bmatrix} = \underline{A}^{-1}\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^{2} \\ 1 & \alpha^{2} & \alpha \end{bmatrix}\begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^{2} \\ 1 & \alpha^{2} & \alpha \end{bmatrix}\begin{bmatrix} 208.3\angle -18.19° \\ 138.89\angle -151.788° \\ 131.94°\angle 145.84 \end{bmatrix}$$

$$= \begin{bmatrix} 21.9531\angle -120.753° \\ 147.373\angle -10.5147° \\ 67.0451\angle -16.699° \end{bmatrix}$$

iv. The zero-sequence quantity is (21.9531 /_ -120.753º) (it's non-zero).

Part b: Consider the figure above and note that we may relate the line currents to the phase currents using KCL:

$I_a=I_{ab}-I_{ca}=(1)I_{ab}+(0)I_{bc}+(-1)I_{ca}$

$I_b=-I_{ab}+I_{bc}=(-1)I_{ab}+(1)I_{bc}+(0)I_{ca}$

$I_c=-I_{bc}+I_{ca}=(0)I_{ab}+(-1)I_{bc}+(1)I_{ca}$

Writing in matrix form, we have:

$$I_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

Denote the matrix as:

$$\underline{K} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

Now let's use it to obtain the line currents:

$$I_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 208.3\angle -18.19° \\ 138.89\angle -151.788° \\ 131.94°\angle 145.84 \end{bmatrix}$$

$$= \begin{bmatrix} 208.3\angle -18.19° - 131.94°\angle 145.84 \\ -208.3\angle -18.19° + 138.89\angle -151.788° \\ -138.89\angle -151.788° + 131.94°\angle 145.84 \end{bmatrix}$$

$$= \begin{bmatrix} 337.108\angle -24.3718° \\ 320.282\angle -179.887° \\ 140.367\angle 84.5983° \end{bmatrix}$$

i. They are unbalanced.

ii. Their sum is

$$337.108\angle-24.3718° + 320.282\angle-179.887° + 140.367\angle 84.598° = 0$$

iii. Obtain their sequence quantities.

$$\begin{bmatrix} I_a^0 \\ I_a^+ \\ I_a^- \end{bmatrix} = \underline{A}^{-1}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 337.1\angle-24.37° \\ 320.3\angle-168.81° \\ 140.4\angle 84.56° \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 255.258\angle-40.5147° \\ 116.125\angle 13.301° \end{bmatrix}$$

iv. The 0-sequence quantity is zero.

Part c:

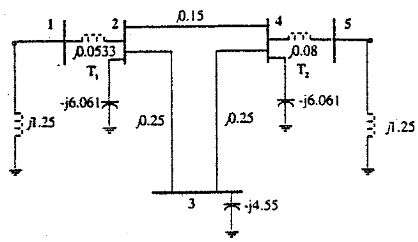
i. Unbalanced currents into a grounded-Y will not sum to zero and therefore will have a non-zero zero-sequence component .

ii. Unbalanced currents into an ungrounded-Y will sum to zero and therefore will have a zero zero-sequence component .

iii. Unbalanced line-to-line voltages must sum to zero (do a KVL!) and therefore will never have a zero-sequence component.

Problem 9.8

The network considered in Example 9.5 together with the appropriate branch impedances is shown in the figure below



We apply the Z_{bus} building algorithm described in Section 9.5.

Step 1

Add node 1 to the reference node. The branch has a reactance of j 1.25. This falls under the category Modification 1.

$$Z_{bus} = [j1.25]$$

Step 2

Add node 2 to node 1. The branch has a reactance of j 0.0533. This falls under the category Modification2.

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.3033 \end{bmatrix}$$

Step 3

Add node 3 to node 2. The branch has a reactance of j 0.25. This falls under the category Modification 2.

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 & j1.25 \\ j1.25 & j1.3033 & j1.3033 \\ j1.25 & j1.3033 & j1.5533 \end{bmatrix}$$

Step 4

Add node 4 to node 3. The branch has a reactance of j 0.25. This falls under category Modification 2.

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.3033 & j1.3033 & j1.3033 \\ j1.25 & j1.3033 & j1.5533 & j1.5533 \\ j1.25 & j1.3033 & j1.5533 & j1.8033 \end{bmatrix}$$

Step 5

Add branch between node 2 and node 4. The branch has a reactance of j 0.15. This falls under category Modification 4.

$$\mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ -j0.25 \\ -j0.50 \end{bmatrix} \qquad \gamma = (j0.15 + j1.3033 + j1.8033 - (2 \times j1.3033))^{-1} = (j0.65)^{-1} = -j1.5385$$

$$Z^{n}_{bus} = Z_{bus} - \gamma \mathbf{b}\mathbf{b}^{T}$$

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.3033 & j1.3033 & j1.3033 \\ j1.25 & j1.3033 & j1.4571 & j1.3610 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 \end{bmatrix}$$

Step 6

Add node 5 to node 4. The branch has a reactance of j 0.08. This falls under the category Modification 2.

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.3033 & j1.3033 & j1.3033 & j1.3033 \\ j1.25 & j1.3033 & j1.4571 & j1.3610 & j1.3610 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 & j1.4187 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 & j1.4987 \end{bmatrix}$$

Step 7

Add the branch between node 5 and the reference node. The branch has a reactance of j 1.25. This falls under the category Modification 3.

We first augment the Z_{bus} with an additional row and column as shown below

$$Z_{bus} = \begin{bmatrix} j1.25 & j1.25 & j1.25 & j1.25 & j1.25 & j1.25 \\ j1.25 & j1.3033 & j1.3033 & j1.3033 & j1.3033 & j1.3033 \\ j1.25 & j1.3033 & j1.4571 & j1.3610 & j1.3610 & j1.3610 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 & j1.4187 & j1.4187 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 & j1.4987 & j1.4987 \\ j1.25 & j1.3033 & j1.3610 & j1.4187 & j1.4987 & j2.7487 \end{bmatrix}$$

Kron reduce the last row and column to obtain

$$Z_{bus} = \begin{bmatrix} j0.6815 & j0.6573 & j0.6311 & j0.6048 & j0.5685 \\ j0.6573 & j0.6853 & j0.6580 & j0.6306 & j0.5927 \\ j0.6311 & j0.6580 & j0.7832 & j0.6585 & j0.6189 \\ j0.6048 & j0.6306 & j0.6585 & j0.6865 & j0.6452 \\ j0.5685 & j0.5927 & j0.6189 & j0.6452 & j0.6815 \end{bmatrix}$$

We will now add the capacitances due to the half line charging at nodes 2, 3, and 4. This falls under the category Modification 3.

Step 8

Add capacitive reactance between node 2 and the reference node. This branch has a reactance of $(j0.11 + j0.055)^{-1} = -j6.061$

As in step 7 we first augment the original Z_{bus} with an additional row and column and Kron reduce. The augmented matrix and the Kron reduced matrix are given below.

$$Z_{bus} = \begin{bmatrix} j0.6815 & j0.6573 & j0.6311 & j0.6048 & j0.5685 & j0.6573 \\ j0.6573 & j0.6853 & j0.6580 & j0.6306 & j0.5927 & j0.6853 \\ j0.6311 & j0.6580 & j0.7832 & j0.6585 & j0.6189 & j0.6580 \\ j0.6048 & j0.6306 & j0.6585 & j0.6865 & j0.6452 & j0.6306 \\ j0.5685 & j0.5927 & j0.6189 & j0.6452 & j0.6815 & j0.5927 \\ j0.6573 & j0.6853 & j0.6580 & j0.6306 & j0.5927 - j5.3757 & \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.7619 & j0.7411 & j0.7115 & j0.6819 & j0.6409 \\ j0.7411 & j0.7727 & j0.7419 & j0.711 & j0.6683 \\ j0.7115 & j0.7419 & j0.8637 & j0.7357 & j0.6915 \\ j0.6819 & j0.711 & j0.7357 & j0.7604 & j0.7147 \\ j0.6409 & j0.6683 & j0.6915 & j0.7147 & j0.7469 \end{bmatrix}$$

Step 9

Add capacitive reactance between node 3 and the reference node. This branch has a reactance of $(j0.11 + j0.11)^{-1} = -j4.55$

Once again we first augment the original Z_{bus} and Kron reduce.

$$Z_{bus} = \begin{bmatrix} j0.7619 & j0.7411 & j0.7115 & j0.6819 & j0.6409 & j0.7115 \\ j0.7411 & j0.7727 & j0.7419 & j0.711 & j0.6683 & j0.7419 \\ j0.7115 & j0.7419 & j0.8637 & j0.7357 & j0.6915 & j0.8637 \\ j0.6819 & j0.711 & j0.7357 & j0.7604 & j0.7147 & j0.7357 \\ j0.6409 & j0.6683 & j0.6915 & j0.7147 & j0.7469 & j0.6915 \\ j0.7115 & j0.7419 & j0.8637 & j0.7357 & j0.6915 - j3.6817 & \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j0.8944 & j0.8845 & j0.8785 & j0.821 & j0.7746 \\ j0.8845 & j0.9222 & j0.9159 & j0.8593 & j0.8076 \\ j0.8785 & j0.9159 & j1.0665 & j0.9083 & j0.8537 \\ j0.821 & j0.8593 & j0.9083 & j0.9075 & j0.8529 \\ j0.7746 & j0.8076 & j0.8537 & j0.8529 & j0.8768 \end{bmatrix}$$

Step 10

Add capacitive reactance between node 4 and the reference node. This branch has a reactance of

$(j0.11 + j0.055)^{-1} = -j6.061$

We again augment the matrix and Kron reduce to obtain

$$Z_{bus} = \begin{bmatrix} j0.8944 & j0.8845 & j0.8785 & j0.821 & j0.7746 & j0.821 \\ j0.8845 & j0.9222 & j0.9159 & j0.8593 & j0.8076 & j0.8593 \\ j0.8785 & j0.9159 & j1.0665 & j0.9083 & j0.8537 & j0.9083 \\ j0.821 & j0.8593 & j0.9083 & j0.9075 & j0.8529 & j0.9075 \\ j0.7746 & j0.8076 & j0.8537 & j0.8529 & j0.8768 & j0.8529 \\ j0.821 & j0.8593 & j0.9083 & j0.9075 & j0.8529 - j5.1351 & \end{bmatrix}$$

$$Z_{bus} = \begin{bmatrix} j1.0312 & j1.0219 & j1.0237 & j0.9693 & j0.911 \\ j1.0219 & j1.0655 & j1.0674 & j1.0106 & j0.9498 \\ j1.0237 & j1.0674 & j1.2266 & j1.0683 & j1.004 \\ j0.9693 & j1.0106 & j1.0683 & j1.0673 & j1.003 \\ j0.911 & j0.9498 & j1.004 & j1.003 & j1.0179 \end{bmatrix}$$