

Fault Analysis Using Z-bus

1.0 Introduction

The previous notes on Z-bus resulted in some useful knowledge: diagonal element Z_{kk} of the Z-bus is the Thevenin impedance seen looking into the network from bus k.

Combining this knowledge with equation (4) from notes called “Symmetrical Faults 2”, which was

$$I_f'' = \frac{V_f}{Z_f + Z_{Thev}} \quad (1)$$

enables us to efficiently obtain the fault current for any bus in the network.

This is easy if we have Z-bus and V_f .

However, one other thing that we will frequently need is the line currents, because the circuit breakers are going to be in series with the lines, not with the faults.

2.0 Fault calculations using Zbus

Recall that

$$\underline{V} = \underline{Z}\underline{I} \quad (2)$$

Since (2) represents a set of linear equations, superposition holds, and we may write:

$$\underline{\Delta V} = \underline{Z}\underline{\Delta I} \quad (3)$$

This says that the change in voltage at all buses $\underline{\Delta V}$ may be computed if the change in injections at all buses $\underline{\Delta I}$ are known. We can write eq. (3) in expanded form as:

$$\begin{bmatrix} \Delta V_1 \\ \vdots \\ \Delta V_k \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kN} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & \cdots & Z_{Nk} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} \Delta I_1 \\ \vdots \\ \Delta I_k \\ \vdots \\ \Delta I_N \end{bmatrix} \quad (4)$$

Now consider a fault at bus k, where the pre-fault voltage at bus k is V_f . Let the fault current be I'_f , and assume that the fault impedance $Z_f=0$ (this is typically worst-case scenario).

Since a fault is a short circuit, then $\Delta V_k = -V_f$. Also, since the fault current is out of bus k, then $\Delta I_k = -I''_f$. Substituting these into eq. (4) results in

$$\xrightarrow{\text{row } k} \begin{bmatrix} \Delta V_1 \\ \vdots \\ -V_f \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & \cdots & Z_{1k} & \cdots & Z_{1N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{k1} & \cdots & Z_{kk} & \cdots & Z_{kN} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & \cdots & Z_{Nk} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ -I''_f \\ \vdots \\ 0 \end{bmatrix} \quad (5)$$

Note that the right-hand-side results in, for each row j, only the Z_{jk} being multiplied by a non-zero current. Therefore:

$$\xrightarrow{\text{row } k} \begin{bmatrix} \Delta V_1 \\ \vdots \\ -V_f \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} -Z_{1k} I''_f \\ \vdots \\ -Z_{kk} I''_f \\ \vdots \\ -Z_{Nk} I''_f \end{bmatrix} \quad (6)$$

We observe from row k that:

$$-V_f = -Z_{kk} I''_f \quad (7)$$

Solving (7) for I''_f results in

$$I''_f = \frac{V_f}{Z_{kk}} \quad (8)$$

Notice that eq. (8) is consistent with eq. (1) when $Z_f=0$.

Now substitute eq. (8) into eq. (6) to get:

$$\xrightarrow{\text{row } k} \begin{bmatrix} \Delta V_1 \\ \vdots \\ -V_f \\ \vdots \\ \Delta V_N \end{bmatrix} = \begin{bmatrix} (-Z_{1k} / Z_{kk})V_f \\ \vdots \\ -V_f \\ \vdots \\ (-Z_{Nk} / Z_{kk})V_f \end{bmatrix} \quad (9)$$

Now eq. (9) provides the change in the bus voltages due to the fault. Change from what?

It is the change from the voltage without the fault, i.e., it is the pre-fault voltage.

Consider any bus, let's say bus j , with a pre-fault voltage of V_j . Then we can compute the bus j voltage under the faulted condition as

$$V_{jf} = V_j + \Delta V_j \quad (10)$$

From eq. (9), we know that

$$\Delta V_j = -\frac{Z_{jk}}{Z_{kk}} V_f \quad (11)$$

Substitution of (11) into (10) results in

$$V_{jf} = V_j - \frac{Z_{jk}}{Z_{kk}} V_f \quad (12)$$

Now eq. (12) is useful for computing fault currents in the circuits. Consider Fig. 1.

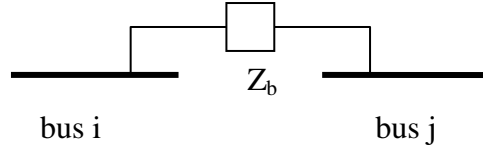


Fig. 1

We can use eq. (12) to write down the voltages under the faulted condition for buses i and j, as

$$V_{if} = V_i - \frac{Z_{ik}}{Z_{kk}} V_f \quad (13)$$

$$V_{jf} = V_j - \frac{Z_{jk}}{Z_{kk}} V_f \quad (14)$$

Now we can compute the subtransient current flowing from bus i to bus j under the fault condition as

$$I''_{ij} = \frac{V_{if} - V_{jf}}{Z_b} \quad (15)$$

Substituting eqs. (13) and (14) into (15) results in

$$\begin{aligned} I''_{ij} &= \frac{V_i - \frac{Z_{ik}}{Z_{kk}} V_f - V_j - \frac{Z_{jk}}{Z_{kk}} V_f}{Z_b} \\ &= \frac{V_i - V_j}{Z_b} - V_f \frac{Z_{ik} - Z_{jk}}{Z_b Z_{kk}} \end{aligned} \quad (16)$$

We can use eq. (16) to get the fault current in the circuits. These values provide us with the appropriate information for selecting the circuit breakers in the lines.

3.0 Some important comments

- Zbus should be developed using subtransient reactances in generator/motor models.
- Because fault currents are typically much larger than load currents, it may be assumed that there are no loads.
 - All pre-fault currents are 0.
 - All buses have voltage (pre-fault) equal to V_f .
 - Equation (16) becomes:

$$I''_{ij} = -V_f \frac{Z_{ik} - Z_{jk}}{Z_b Z_{kk}} \quad (17)$$

- From (16) and (17), we see that only the k^{th} column of the Z-bus is required to analyze a fault at bus k.

The last observation can be utilized in an effective fashion when performing fault analysis. Let's assume that we want to compute the short circuit currents for a fault

at only one bus k . So we just want to get the k^{th} column of Z-bus, but we do not need the entire Z-bus.

There is an efficient way to get the k^{th} column of Z-bus. Let's study it.

Consider that the Z-bus and the Y-bus are inverses of each other, i.e.,

$$\underline{Z} = \underline{Y}^{-1} \quad (18)$$

This means that their product gives the identity matrix.

$$\underline{Y}\underline{Z} = \underline{I} \quad (19)$$

where \underline{I} is given by a matrix of zeros except the diagonal which contains all ones, i.e.,

$$\underline{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \quad (20)$$

Our approach will depend on two ideas:

1. Column of \underline{Z} : We can just consider a single column of \underline{Z} , instead of the entire matrix. Call it \underline{Z}_k ; it is a column vector. The right hand side of (19) will just be a column of \underline{I} . Call it \underline{I}_k . It will also be just a vector and will contain zeros in every row except for row k . The resulting relation is:

$$\underline{Y}\underline{Z}_k = \underline{I}_k \quad (21)$$

2. LU-Decomposition: If you have taken EE 456 or a linear algebra course in math, then you are familiar with LU decomposition. LU decomposition provides a way to solve for the vector \underline{x} in the matrix relation

$$\underline{A}\underline{x} = \underline{b} \quad (22)$$

where \underline{A} is a $n \times n$ square matrix, \underline{x} is an unknown $n \times 1$ vector, and \underline{b} is a known $n \times 1$ vector. The advantage to LU decomposition is it does not require

inverting the matrix \underline{A} . The basis of LU-decomposition is that we may factor \underline{A} into a matrix product \underline{LU} , i.e.,

$$\underline{A} = \underline{L}\underline{U} \quad (23)$$

where \underline{L} is a lower diagonal $n \times n$ matrix of the form

$$\underline{L} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad (24)$$

and \underline{U} is an upper diagonal $n \times n$ matrix of the form

$$\underline{U} = \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} \quad (25)$$

Substitution of eq. (23) in (22) yields

$$\underline{L}\underline{U}\underline{x} = \underline{b} \quad (26)$$

Defining

$$\underline{U}\underline{x} = \underline{w} \quad (27)$$

provides that eq. (25) becomes:

$$\underline{\underline{L}}\underline{\underline{w}} = \underline{\underline{b}} \quad (28)$$

If we have $\underline{\underline{L}}$ and $\underline{\underline{U}}$, then (28) is easily solved for $\underline{\underline{w}}$ (without inverting $\underline{\underline{L}}$) using forward substitution, and then (27) can be easily solved for $\underline{\underline{x}}$ (without inverting $\underline{\underline{U}}$) using backwards substitution. More details on LU decomposition may be found in the notes called “LUDecomposition.doc.”

Now observe that eq. (21) and (22) are in the same form. Therefore we want to solve the following equations in the order they are given:

$$\underline{\underline{L}}\underline{\underline{w}} = \underline{\underline{I}}_k \quad (29)$$

$$\underline{\underline{U}}\underline{\underline{Z}}_k = \underline{\underline{w}} \quad (30)$$

Factorization of a matrix $\underline{\underline{Y}}$ can be done efficiently and easily using the matlab command:

$[\underline{\underline{L}},\underline{\underline{U}}]=\text{lu}(\underline{\underline{Y}})$

Then it is easy to find $\underline{\underline{w}}$ by hand using forward substitution from:

$$\underline{\underline{L}}\underline{\underline{w}} = \underline{\underline{I}}_k$$

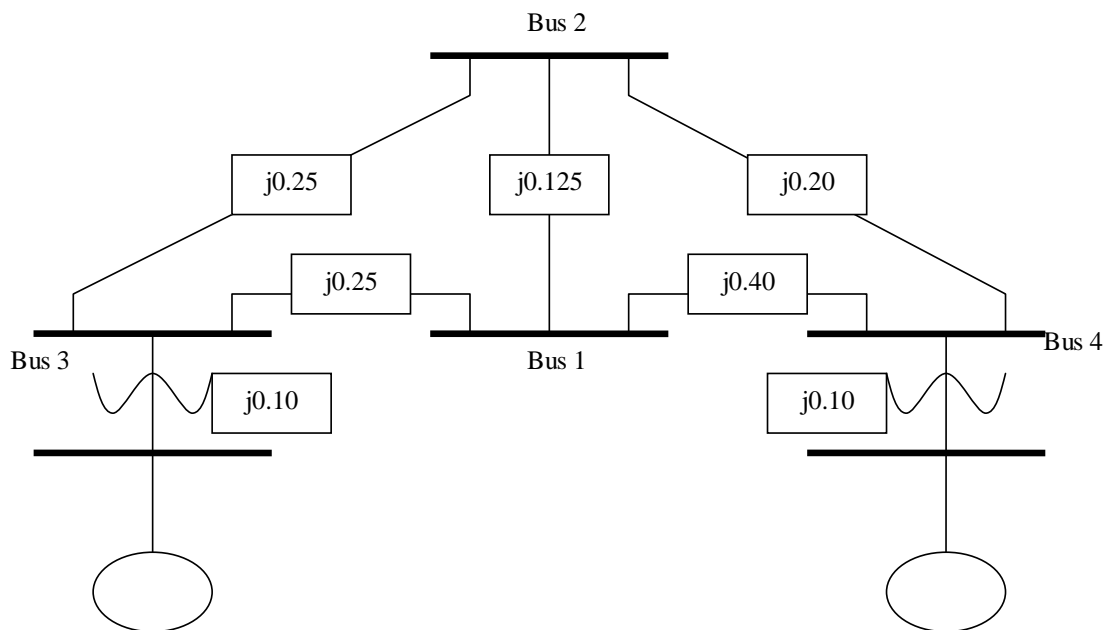
And then it is easy to find $\underline{\underline{Z}}_2$ by hand using backwards substitution from:

$$\underline{\underline{U}}\underline{\underline{Z}}_2 = \underline{\underline{w}}$$

Alternatively, the manual steps of LU decomposition can be performed per the notes from “LU Decomposition.”

Homework #3: Due Tuesday, Feb 3.

1. Work problem 9.8 in book.
2. Consider the 4-bus system shown below. Both machines have subtransient reactances of 0.20 pu (you can combine the machine subtransient reactance with the transformer impedance to get a single reactance connecting the machine internal voltage with the network).



- a. Construct the Y-bus for this network (should be a 4×4 matrix).
- b. Consider that there is a three-phase (symmetrical) fault at bus 2.

- i. Use LU decomposition to obtain the 2nd column of the Z-bus.
 - ii. Compute the subtransient fault current.
 - iii. Use eq. (12) to find the voltages during the fault.
 - iv. Use eq. (17) to find the subtransient currents in lines 3-2, 1-2, and 4-2.
3. See notes called “Examples.”
 4. See notes called “Examples.”