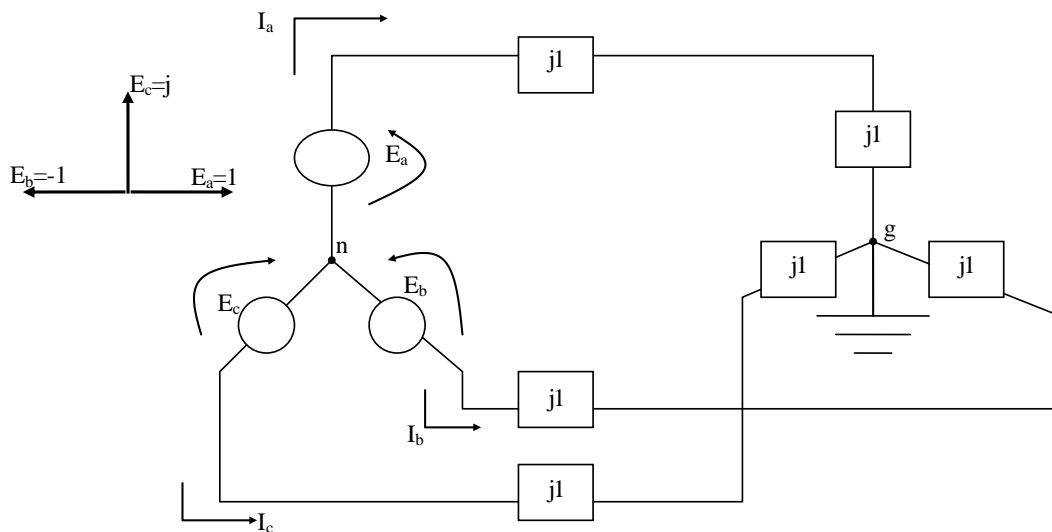


## Examples

I will do 2 of your assigned problems...

### Problem 12.3

The problem requires us to find the three currents  $I_a$ ,  $I_b$ , and  $I_c$ , and also  $V_{ng}$ , given that  $E_a=1$ ,  $E_b=-1$ , and  $E_c=j1$ .



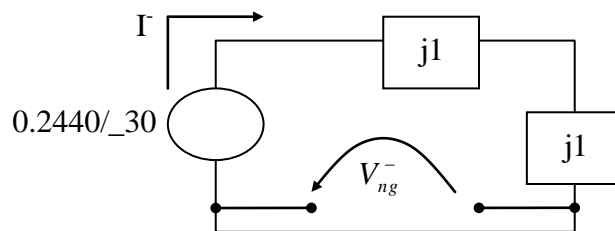
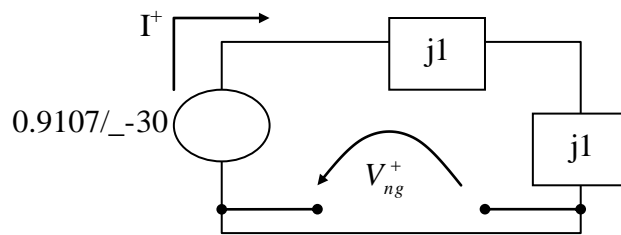
The network is symmetric. Therefore, the sequence circuits will be decoupled and we can analyze them one at a time.

The zero sequence current must be zero since the source is ungrounded and therefore  $I_a + I_b + I_c = 0$ .

But the given voltages are unbalanced. So let's hit them with an  $A^{-1}$  to get the sequence voltages as follows:

$$\begin{bmatrix} E^0 \\ E^+ \\ E^- \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ j1 \end{bmatrix} = \begin{bmatrix} 0.3333 \angle 90^\circ \\ 0.9107 \angle -30^\circ \\ 0.2440 \angle 30^\circ \end{bmatrix}$$

Recall that for loads and transmission lines,  $Z^+ = Z^-$ . So we find the positive and negative sequence currents using the below circuits.



We obtain:

$$I^+ = \frac{0.9107 \angle -30^\circ}{j2} = 0.4554 \angle -120^\circ$$

$$I^- = \frac{0.2440 \angle 30^\circ}{j2} = 0.1220 \angle -60^\circ$$

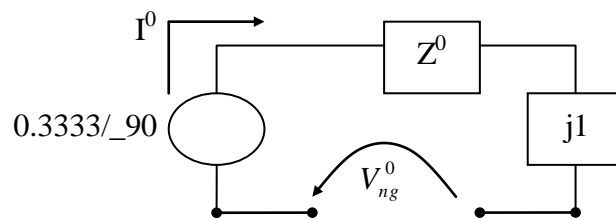
We can then obtain the abc currents as

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.4554 \angle -120^\circ \\ 0.1220 \angle -60^\circ \end{bmatrix} = \begin{bmatrix} 0.5271 \angle -108.44^\circ \\ 0.9107 \angle 108.44^\circ \\ 0.3334 \angle 0^\circ \end{bmatrix}$$

Now what about  $V_{ng}$ ?

Notice from the positive and negative sequence circuits that  $V_{ng}^+ = 0$  and  $V_{ng}^- = 0$ .

But consider  $V_{ng}^0$ . The zero sequence circuit is



From the zero-sequence circuit, a KVL equation results in

$$0.3333 \angle 90^\circ + V_{ng}^0 = 0$$

$$\Rightarrow V_{ng}^0 = -0.3333 \angle 90^\circ = 0.3333 \angle -90^\circ$$

Now we have all of the symmetrical components of  $V_{ng}$ , and we can therefore obtain  $V_{ng}$  as their sum:

$$V_{ng} = V_{ng}^+ + V_{ng}^- + V_{ng}^0 = 0 + 0 + 0.3333 \angle -90^\circ = 0.3333 \angle -90^\circ$$

Clarification: When getting abc quantities from 0+- quantities, we hit the 0+- quantities with A (which contains terms in  $\alpha$  for the b- and c-phase quantities). But that is to obtain ALL abc quantities from the *a-phase* 0+- quantities, as indicated below.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^+ \\ V_a^- \end{bmatrix}$$

Notice that  $V_a = V_a^0 + V_a^+ + V_a^-$ .

If we were to have the b-phase 0+- quantities, then the b-phase quantity would just be the sum of those b-phase 0+- quantities, i.e., it would be  $V_b = V_b^0 + V_b^+ + V_b^-$ . And if we were to have the c-phase 0+- quantities, then the c-phase quantity would just be the sum of those c-phase 0+- quantities, i.e., it would be  $V_c = V_c^0 + V_c^+ + V_c^-$ . And so a phase quantity is just the sum of the sequence quantities in that phase.

## Problem 12.17

From book: Repeat Ex 12.5 if there is a DLG fault (between phases b and c) at the terminals of G1. Assume that the prefault (phase a) voltage at the fault point is  $1/\_0^\circ$ . Find the fault currents  $I_{bf}$  and  $I_{cf}$ .

Ex12.5 is a good one (review it carefully), below. In P.12.17, the fault location is left of T1, not right of T1 as below for Ex12.5.

### Example 12.5 Fault Calculation

The one-line diagram is given in Figure E12.5(a) with a single line-to-ground (SLG) fault shown at the point  $a'$  at the left end of the transmission line. Assume that the prefault voltage  $V_{a'n} = 1/\_0^\circ$ . Calculate  $I', I_a, I_b, I_c, I_{a'}, I_{b'}, I_{c'}, V_{b'g},$  and  $V_{c'g}$ , using generator transient reactances. This information is useful in designing a protective relaying scheme to protect the equipment in the event of an SLG fault at point  $a'$ .

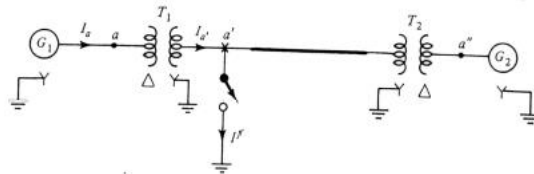


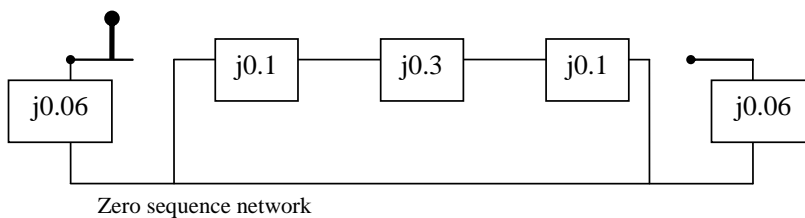
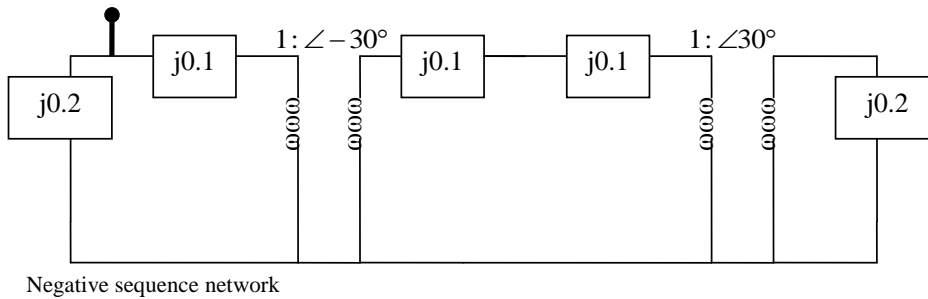
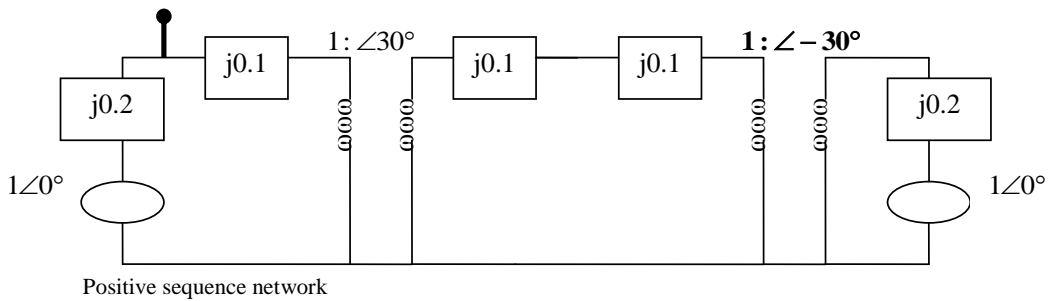
Figure E12.5(a)

Recall that a LLG fault requires a parallel combination of all three sequence networks. So we must find all 3 sequence networks. Below are some comments about doing this.

- We get positive sequence phase shift of  $30^\circ$  going from low to high; negative sequence of  $-30^\circ$  going from low to high.

- The problem requires pre-fault, phase a voltage at the fault point be  $1.0/_0^\circ$ .
  - This  $1.0/_0^\circ$  is a positive sequence voltage and therefore lies in the positive sequence network.
  - Pre-fault implies no fault current, and since we are ignoring load currents, there is no current flow in the pre-fault network.
  - In the text, Fig. E 12.5(b), because the fault was on the line (in the middle of the diagram), and therefore on the opposite side, from G1 and G2, of the transformer phase shifts, the source voltages had to be  $1/_{-30}^\circ$  in order to impose the fault point to have a voltage of  $1/_0^\circ$ .
  - Here, because the fault is at the generator terminals, and therefore on the same side of the transformer phase shift as G1, G1 must be  $1/_0^\circ$  in order to impose the fault point to have a voltage of  $1/_0^\circ$ . Because there are equal and opposite phase shifts between the fault point and G2, G2 must also be  $1/_0^\circ$ .

- The zero sequence network is
  - to ground at each gen since both gens are solidly grounded.
  - open between each gen and the network because of low side delta connection in the xfmrs.

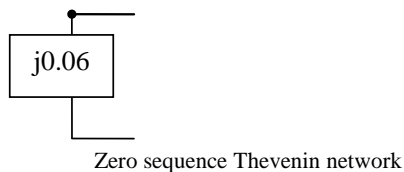


The fault point, at the terminals of G1 (on the left), is indicated in the sequence networks using the bold, short vertical lines.

We must compute the Thevenin impedances and voltages for each sequence network, as seen from the fault point.

The Thevenin voltage for each network is easy – it will be zero for zero and negative sequence, and it will be the pre-fault voltage at the fault point,  $1/\_0^\circ$ , for the positive sequence network.

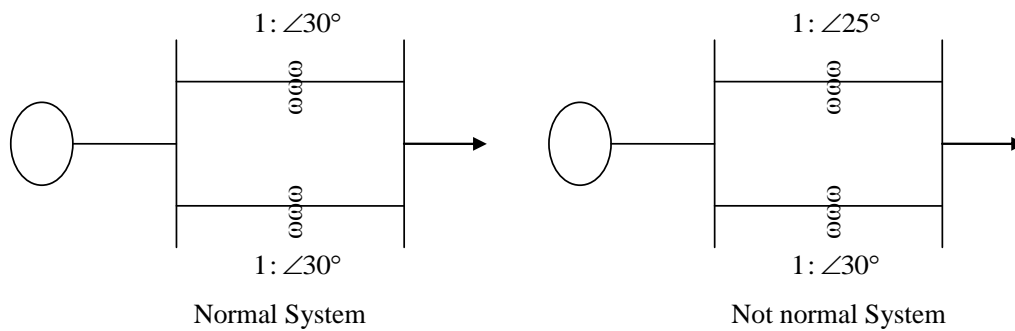
Obtaining the zero-sequence Thevenin network is also easy because of the open circuit to the right of the fault point; it is shown below.



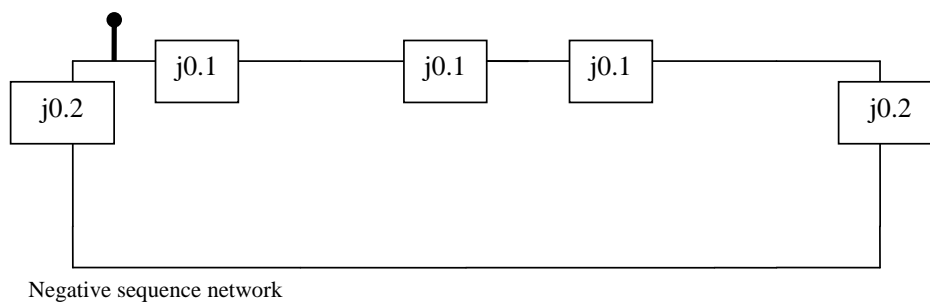
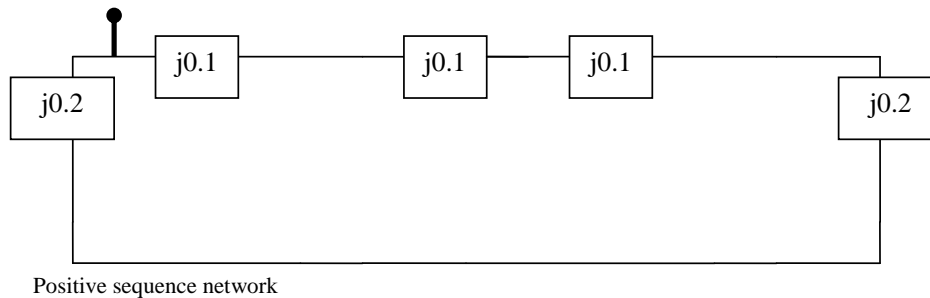
Obtaining positive, negative sequence Thev. impedances requires analyzing networks with phase shifts. However, because the phase shifts occur to voltages *and* currents, Thevenin impedances may be computed without consideration of phase shifts.



This is true for what your text calls *normal systems* (see pp 151-157), which are systems for which the product of the voltage transformations around any closed loop is  $1.0/_0^\circ$ . Below are examples.



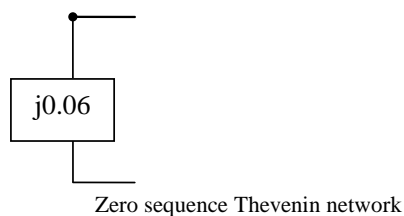
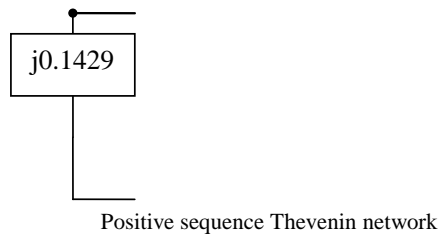
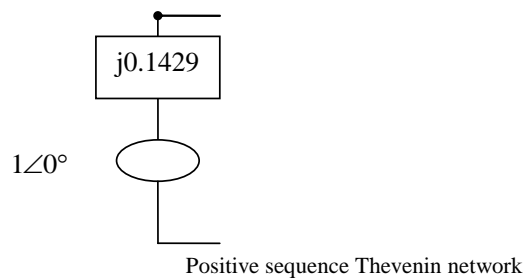
So the positive and negative sequence networks to use in obtaining the Thevenin impedances are below. Note that the positive sequence sources were idled.



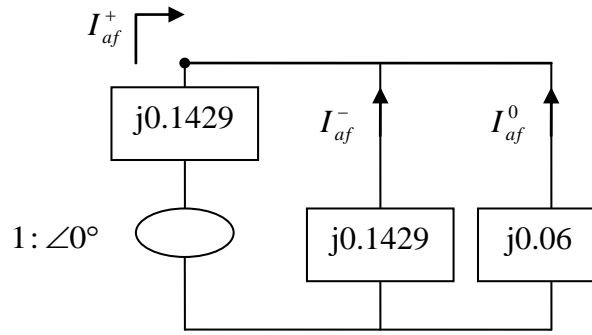
These are identical networks. Therefore the positive and negative sequence Thevenin reactances are the same:

$$X^+ = X^- = \frac{(0.2)(0.1+0.1+0.1+0.2)}{(0.2) + (0.1+0.1+0.1+0.2)} = 0.1429$$

Now we may draw the three Thevenin equivalent networks.



As mentioned at the beginning of this problem, a LLG fault requires a parallel combination of all three sequence networks. The connected circuits for analysis of a LLG at the G1 terminals is shown below.



From this network, we may compute the three currents shown, as

$$I_{af}^+ = \frac{1}{j(0.1429 + 0.1429 // 0.06)} = \frac{1}{0.1852} = -j5.4$$

Then using current division, we obtain

$$I_{af}^- = -(-j5.4) \frac{0.06}{0.1429 + 0.06} = j5.4 \frac{0.06}{0.2029} = j1.597$$

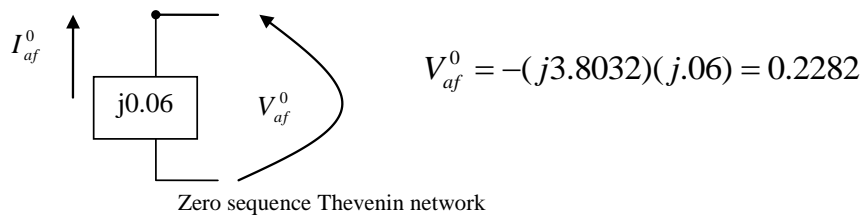
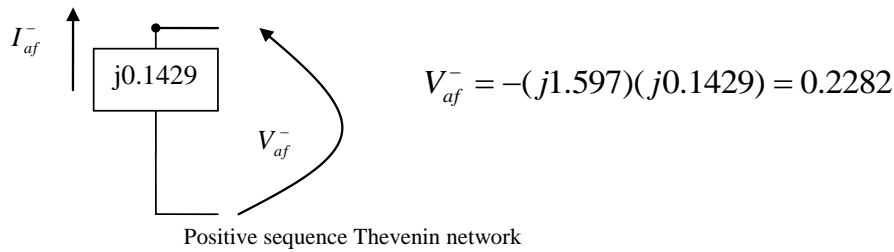
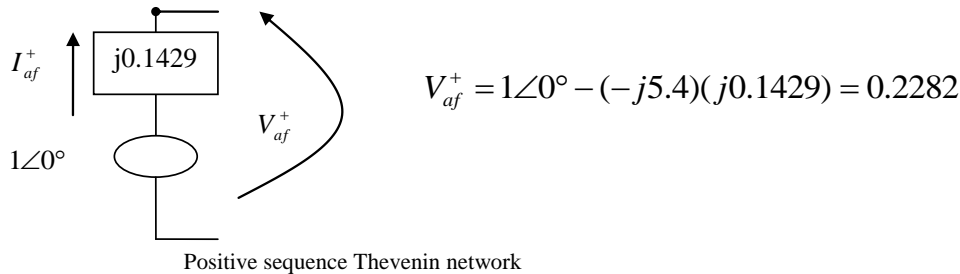
$$I_{af}^0 = -(-j5.4) \frac{0.1429}{0.1429 + 0.06} = j5.4 \frac{0.1429}{0.2029} = j3.8032$$

Now we can compute the abc currents:

$$\begin{aligned} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} &= \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} j3.8032 \\ -j5.4 \\ j1.597 \end{bmatrix} = \begin{bmatrix} 0 \\ -6.0596 + j5.7047 \\ 6.0596 + j5.7047 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 8.3224 \angle 136.73 \\ 8.3224 \angle 42.27 \end{bmatrix} \end{aligned}$$

What if we wanted to obtain abc currents in other parts of the system?

The first thing to do is to compute the fault-on sequence voltages at the fault point. This is easily done from the Thevenin circuits and knowledge of the fault currents, as seen below.



As a check on this work, let's take a look at the abc voltages at the fault point.

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0.2282 \\ 0.2282 \\ 0.2282 \end{bmatrix} = \begin{bmatrix} 0.6846 \\ 0 \\ 0 \end{bmatrix}$$

Then we would use the sequence voltages at the fault point, in each sequence network, to compute the currents in the various elements. To do this, we would need to go back to circuits with the phase shifts (see page 6 above). Your book, pg. 481, shows an example of this. Key point here is that in these calculations, the phase shifts matter!!!!