## EE 457, Exam 2, Spring 2015, Dr. McCalley, Time: 75 minutes, Calculator allowed; closed book, closed notes, communication devices not allowed

1. ( 25 pts ) Consider the 138 kV transmission system.


Relays are impedance with directionality.
(a) Select the CT ratio so that maximum load current provides 5 amperes on the relay side. Choices are I: 5 where I may be $50,100,150,200,250,300,400,450,500,600,800,900,1000$, or 1200.
(b) Select the VT ratio V:1 so that the rated line-to-neutral voltage provides 67 volts on the on the relay side.
(c) If the primary (line) side of the relay sees an impedance of $\mathrm{V}_{\mathrm{p}} / \mathrm{I}_{\mathrm{p}}=\mathrm{Z}_{\text {line }}$, determine an expression for the impedance seen on the relay side, $\mathrm{Z}_{\text {relay }}$, as a function of $\mathrm{Z}_{\text {line }}$ and the CT and VT ratios.
(d) Identify zone 1, 2, and 3 settings for R12, i.e., the threshold impedance values, on the relay side corresponding to $80 \%$ of line 1-2 (zone 1 ), $120 \%$ of line 1-2 (zone 2 ), $120 \%$ of the longest line beyond bus 2 (zone 3 ) which would be line 2-4 in this case.
(e) Draw zone 1, 2, 3 circles on R-X diagram; plot the point corresponding to maximum cct.1-2 load of $50 \mathrm{MVA}, 0.8 \mathrm{pf}$ lag.

## Solution:

Part A: In this problem we are given both the voltage, 138 kV , and the maximum load on the system, 50 MVA (stated in part e). With both of these factors known we can calculate the maximum load current per phase (each phase requires it's own CT).
$S=\sqrt{3} V_{L} I_{L} \quad$ where L=Load (Equation B3.55: EE 303 Notes)
$I_{L}=\frac{S}{\sqrt{3} V_{L}}=\frac{50 M}{\sqrt{3} * 138 k}=209.2$ Amperes
Therefore we select $\mathrm{I}=200$, which gives us a CT ratio of 200/5. This will produce approximately 5 A in the secondary winding under maximum loading conditions.

Part B: This part is fairly straight forward. First we find the line-neutral voltage
$V_{l-n}=\frac{V}{\sqrt{3}}=\frac{138 k}{\sqrt{3}}=79.67 \mathrm{kV}$
Given that VT ratio is something to 1 , to find that VT ratio that produces 67 volts on the secondary side we simply divide the line-neutral by 67 .
$\frac{79.67 k}{67}=1189.1$ This means the desired VT ratio is $1189.1 / 1$
Part C: This part requires dividing the primary voltage and current by the VT and PT ratios found in parts A and B . By doing this we're transforming the primary voltage/current to the secondary side of the transformer.

On the primary side we have:
$\frac{V_{p}}{I_{p}}=Z_{\text {line }}$
Transforming that to the secondary:

$$
z_{\text {relay }}=\frac{\left(\frac{V_{p}}{(1189.1 / 1)}\right)}{\left(\frac{I_{p}}{(200 / 5)}\right)}=\frac{\left(\frac{V_{p}}{1189.1}\right)}{\left(\frac{I_{p}}{40}\right)}=\underline{.03369 * Z_{\text {line }}}
$$

Part D: Given the formula we derived in part C we can now find the impedances as seen by R12.
Line 1-2 $\quad(3.2+j 32.0) * .033639=(.11+j 1.1) \Omega \quad$ Secondary
Line 2-3 $\quad(3.2+j 32.0) * .033639=(.11+j 1.1) \Omega \quad$ Secondary
Line 2-4 $\quad(4.8+j 48.0) * .033639=(.16+j 1.6) \Omega \quad$ Secondary
The maximum load impedance seen by R12 (using the max current found in part A and the line-neutral voltage) found in part B , assuming a power factor of .8 lagging, will be:
$Z_{\text {load }}=\frac{67}{\left(\frac{209.2}{(200 / 5)}\right)} \angle\left(\cos ^{-1}(.8)\right)=12.8107 \angle 36.87^{\circ}=(10.2+j 7.7) \Omega$
Secondary
The zone-1 setting of R12 must "under reach" the line 1-2, so that the setting should be:
$.8 *(.11+j 1.1)=.088+j .88 \Omega$ Secondary
The zone-2 setting should reach past terminal 2 of the line 1-2. Zone-2 for R12 is therefore set at:

$$
1.2 *(.11+j 1.1)=.13+j 1.32 \Omega \text { Secondary }
$$

The zone- 3 setting should reach beyond the longest line connected to bus 2 . Thus the zone- 3 setting must be:
$(.11+j 1.1)+1.2 *(.16+j 1.6)=.302+j 3.02 \Omega$ Secondary

## Part E:



Note: As seen by the relay the load impedance $\mathrm{Z}_{\text {load }}$ is more than 3 times the line impedance from bus 1 to bus 4 and lies well beyond the zone- 3 circle of the relay. Therefore there is no danger of tripping the line during any load swings that may occur on the transmission line.
2. $(20 \mathrm{pts})$ For the system below, the values of fault currents are as follows:

- Line-line fault at bus 3: 300 A.
- Three-phase fault at bus 2: 500 A .

Possible CT ratios are:50:5, 100:5. Possible taps are 4, 5, 6, 7, 8 . The coordination time is specified to be at least 0.35 seconds. Provide the CT ratios, the pickups, and the time dial settings for R1 and R2. Use the GE IFC-53 relay. The time-overcurrent characteristic of this relay is given on the last page of the exam.


## Solution:

Set R2: Take safety factor of $3: 300 /=100$. If we choose $50: 5$, we get $(5 / 50) * 100=10$, too high. If we choose $100: 5$, we get $(5 / 100) * 100=5$.
$\rightarrow$ Choose 100:5 as CT Ratio.
$\rightarrow$ Choose tap of 5 .
We will choose a TDS for R2 as fast as possible.
$\rightarrow$ Choose TDS=1/2.
Set R1: R1 must serve as backup for R2 and therefore selection of R2 CT ratio and tap must be based on LL fault at bus 3. Since this is what we used for setting R2, the CT and tap will be the same as R1:
$\rightarrow$ Choose 100:5 as CT Ratio.
$\rightarrow$ Choose tap of 5 .
We must choose a TDS for R1 to give at least 0.35 seconds delay for maximum fault current in the R2 zone. This would be for a three-phase fault at bus 2 , which is specified to be 500 A, giving $500 / 100=5$ times the pickup. From the time-overcurrent curves, we see that R2, with TDS=1/2, will operate in 0.15 seconds. So R1 should operate in $0.15+0.35=0.5$ seconds. From the time-overcurrent curve, we find the TDS that gives a 0.5 second delay at 5 multiples of pickup is 2 .
$\rightarrow$ Choose TDS of 2 .
3. (20 pts) The equal-area criterion is $\underbrace{\int_{\delta_{0}}^{\delta_{\text {clar }}} P_{M}^{0}-P_{\text {fault }} d \delta}_{A_{1}}=\underbrace{\int_{\delta_{\text {clar }}}^{\delta_{\max }} P_{\text {post }}-P_{M}^{0} d \delta}_{A_{2}}$. Consider the very rough approximations of the $\mathrm{P}-\delta$ "power angle curves" for a synchronous machine connected to an infinite bus in the following figure. The curves characterize the $\mathrm{P}-\delta$ variation for pre-fault and postfault conditions. The generator mechanical power is 0.5 pu . Assuming these curves to be an accurate portrayal of the $\mathrm{P}-\delta$ relation, compute the critical clearing angle for a 3-phase fault at the machine terminals. Hint: It is OK to perform calculations in units of degrees for this problem.


## Solution:

The equation of the after-fault clearing curve, for $\delta<90$, is $\mathrm{P}=(0.8 / 90) \delta=0.0089 \delta$. The point of intersection between the $\mathrm{P}=0.5$ line and the after fault-clearing curve is where $0.0089 \delta=0.5 \rightarrow \delta=56.25^{\circ}$. The maximum angle is $180-56.25=123.75^{\circ}$. Let's assume that the critical clearing angle $\theta_{\mathrm{C}}$, is greater than $56.25^{\circ}$. In that case, the figure below illustrates the application of the equal area criterion (EAC) for critical clearing angle:


EAC requires that (area of rectangle)=(area of big triangle)-(area of small triangle), i.e., $\left(\theta_{c}-45\right)(0.5)=1 / 2(123.75-56.25)(0.3)-1 / 2\left(\theta_{c}-56.25\right)\left(0.0089 \theta_{c}-0.5\right) \rightarrow \theta^{2}=4171.3 \rightarrow \underline{\theta}_{c}=64.586^{\circ}$.
4. (20 pts) Consider a lossless two-unit power system where the units have the following cost-rate curves:

$$
\begin{gathered}
C_{I}\left(P_{G 1}\right)=900+45 P_{G 1}+0.01 P_{G 1}^{2} \\
C_{2}\left(P_{G 2}\right)=2500+43 P_{G 2}+0.003 P_{G 2}^{2}
\end{gathered}
$$

and the constraints are

$$
\begin{aligned}
& 50 M W \leq P_{G 1} \leq 200 M W \\
& 50 M W \leq P_{G 2} \leq 600 M W
\end{aligned}
$$

a. Assume the demand is $\mathrm{P}_{\mathrm{D}}$ and express the system incremental cost, $\lambda$, as a function of $\mathrm{P}_{\mathrm{D}}$ under cost-minimum conditions, when both units are regulating.

## Solution:

The Lagrangian is

$$
F=900+45 P_{G 1}+0.01 P_{G 1}^{2}+2500+43 P_{G 2}+0.003 P_{G 2}^{2}-\lambda\left(P_{G 1}+P_{G 2}-P_{D}\right)
$$

KKT conditions result in:

$$
\begin{aligned}
& 0.02 P_{G 1}=\lambda-45 \Rightarrow P_{G 1}=50 \lambda-2250 * \\
& 0.006 \boldsymbol{P}_{G 2}=\lambda-43 \Rightarrow \boldsymbol{P}_{G 2}=166.67 \lambda-7166.7 * *
\end{aligned}
$$

$$
P_{G 1}+P_{G 2}-P_{D}=0 * * *
$$

Substitution of $*$ and ${ }^{* *}$ into ${ }^{* * *}$ results in

$$
50 \lambda-2250+166.67 \lambda-7166-P_{D}=0
$$

Solving the above for $\lambda$ results in

$$
216.67 \lambda-9416-P_{D}=0 \Rightarrow \lambda=\frac{9416+P_{D}}{216.67}=43.46+\frac{P_{D}}{216.67}
$$

b. Assume the demand is $\mathrm{P}_{\mathrm{D}}$ and express the system incremental cost, $\lambda$, as a function of $\mathrm{P}_{\mathrm{D}}$ under cost-minimum conditions, when unit 2 hits its upper limit.

## Solution:

Since unit 2 is at its limit of 600 MW , unit 1 is the only one regulating and must provide $\mathrm{P}_{\mathrm{D}}-600$. Thus, we have that $\mathrm{P}_{\mathrm{G} 1}=\mathrm{P}_{\mathrm{D}}-600$. But the KKT conditions result in

$$
0.02 P_{G 1}=\lambda-45 \Rightarrow P_{G 1}=50 \lambda-2250
$$

Substituting for $\mathrm{P}_{\mathrm{G} 1}$, we have

$$
P_{D}-600=50 \lambda-2250
$$

Solving for $\lambda$, we obtain:

$$
\lambda=\frac{P_{D}}{50}+33
$$

5. (14 pts) True-False:
a. _F_Backup protection is used in order to enhance the overall security of the protection design.
b. _F_Directional relays operate by comparing magnitudes of current phasors entering and exiting the bus.
c. _T_Single loop systems require directional relays in order to perform the necessary coordination.
d. _T_Impedance relays have better discriminatory abilities than overcurrent relays as they depend on I \& V.
e. _F_Impedance relays are set so that impedance associated with normal load conditions is near the trip zone.
f. _F_Impedance relays have a primary reach (zone 1) of about $120 \%$ of the line they are protecting.
g. _T_A reasonable H-constant for a synchronous machine rated at 500 MVA on a 100 MVA base, is 15.0
h. _F_ When $\mathrm{d} \delta / \mathrm{dt}=0$, the rotor is stationary with respect to the stator.
i. _T_If a synchronous machine was operating at its unstable equilibrium, and mechanical power reduced, the rotor would move to its stable equilibrium.
j. $\quad$ _F_A reasonable range for heat rates in units of MBTU/MWhr, is 2.0-7.0.
k. _ T_ The lower the heat rate, the more efficient the unit.
6. _ T_ If the frequency of a 60 Hz power grid is 60.05 , you know it is over-generating.
m. _F_Following loss of a large generator, the action of primary speed control returns frequency to 60 Hz .
n. _F_The North American Reliability Corporation incentives balancing authorities to maintain a large ACE.

