Name: $\qquad$

## EE 457, Exam 1, Spring, 2015

## Closed Book, Closed Notes, Closed Computer, No Communication Devices, Calculators allowed

1. ( 35 pts ) Symmetrical Fault Analy sis: Consider a single generator supplying a balanced $\mathrm{R}+\mathrm{jX}$ load as shown in the diagrambelow. The steady-state, pre-fault voltage at bus 2 is 1.02 pu .

a. (7 pts) Draw the per-phase circuit model for this systemunder steady-state, pre-fault, balanced conditions. Label the steady-state generator voltage and all impedances. Solution:

b. (7 pts) Draw the per-phase circuit model for this systemfor the subtransient conditions associated with a fault at bus 2 having impedance of $\mathrm{Z}_{\mathrm{f}}$. Label the subtransient generator voltage and all impedances.

## Solution:


c. (7 pts) Using the impedance values of $Z_{\mathrm{ext}}=\mathrm{j} 0.1 \mathrm{pu}, \mathrm{X}_{\mathrm{d}}=1.0 \mathrm{pu}, \mathrm{X}^{\prime}{ }_{\mathrm{d}}=0.2 \mathrm{pu}, \mathrm{X}^{\prime}{ }_{\mathrm{d}}=0.1 \mathrm{pu}$, $\mathrm{Z}_{\mathrm{L}}=\mathrm{j} 10.0$, compute the Thevenin impedance and voltageof this circuit looking into the network from the fault point at bus 2 .
Solution:
We compute the Thevenin equivalent circuit fromthe following:


The Thevenin impedance is computed fromthe following:


$$
\begin{aligned}
& \mathrm{Z}_{\text {Thev }}=\mathrm{Z}_{\mathrm{L}} / /\left(\mathrm{Z}_{\mathrm{ext}}+\mathrm{j} \mathrm{X}^{\prime}{ }_{\mathrm{d}}\right) \\
& =\mathrm{j} 10^{*}(\mathrm{j} 0.1+\mathrm{j} 0.1) /(\mathrm{j} 10.2) \\
& =\mathrm{j} 0.1961
\end{aligned}
$$

The Thevenin voltage is the voltage at the fault point of the unfaulted network, which is given in the problemstatement as 1.02 pu. And sothe Thevenin equivalent network is

d. (7 pts ) Given a fault at bus 2 through a fault impedance of $Z_{f}=j 0.1 p u$, compute the subtransient current into the fault.

## Solution:

$$
I_{f}^{\prime \prime}=\frac{V_{f}}{Z_{f}+Z_{\text {Thev }}}=\frac{1.02}{j 0.1+j 0.1961}=-j 3.4448 \mathrm{pu}
$$

e. (7 pts ) If you were to purchase a circuit breaker for bus 2 , what is the minimum interruptible current rating you should ensure that it has?

## Solution:

The circuit breaker must be able to interrupt 1.73 times the rms steady-state fault current, which would be (to account for the DC component), $1.73 * 3.4448=5.9595$ pu.
2. (21 pts)Z-bus: A three-bus network is operating sothat all buses have voltage magnitudes equal to 1.0 pu. Each bus is connected to the other two buses via branches having impedance of j0.1 pu. The Z-bus of the network is given as:

$$
Z=\left[\begin{array}{lll}
j 0.073 & j 0.0386 & j 0.0558 \\
j 0.0386 & j 0.0558 & j 0.0472 \\
j 0.0558 & j 0.0472 & j 0.1014
\end{array}\right]
$$

a. (7 pts) For a bolted three-phase fault at bus 1, what is the magnitude of the short-circuit current into the fault?

## Solution:

Solutions giving correct magnitude are acceptable.

$$
I_{f}^{\prime \prime}=\frac{V_{f}}{Z_{\text {Thev }}+Z_{f}}=\frac{V_{f}}{Z_{11}}=\frac{1.0}{j 0.073}=-j 13.6986 \mathrm{pu}
$$

b. (7 pts) For a three-phase fault through an impedance of j0.1 at bus 3, what is the magnitude of the short-circuit currentinto the fault?

$$
I_{f}^{\prime \prime}=\frac{V_{f}}{Z_{\text {Thev }}+Z_{f}}=\frac{V_{f}}{Z_{33}+j 0.1}=\frac{1.0}{j 0.1014+j 0.1}=-j 4.9652 p u
$$

c. (7 pts) We developed a formula in class for the voltage at a bus during a fault condition, for a three-phase fault, as

$$
V_{j f}=V_{j}-\frac{Z_{j k}}{Z_{k k}} V_{f}
$$

Use the above formula to obtain the voltage at bus 2 during a three-phase fault at bus 3 .
Solution:

$$
V_{2 f}=V_{2}-\frac{Z_{23}}{Z_{33}} V_{f}=1.0-\frac{.0472}{.1014} \times 1.0=0.5345
$$

3. (24 pts) SLG fault analysis: A three-phase generator is directly connected to a balanced three-phase load having impedance of $j 0.1$ per-unit, per phase. The generator impedances are $Z^{+}=j 1.0, Z^{-}=j 0.1$, and $Z^{0}=j 0.005$, all in per-unit. The generator neutral is not grounded. The load neutral is grounded. A single line-to-ground fault occurs at the terminals of the generator, on phase a. Before the fault occurs, the generator is supplying positivesequence voltages and currents, with the pre-fault voltage at the fault point $\mathrm{V}_{\mathrm{ag}}=1.0<0^{\circ}$. Find the fault current $\mathrm{I}_{\mathrm{f}}$.

## SOLUTION:

Find the sequence networks and corresponding Thevenin equivalents:


Now put the three Thevenin equivalent circuits in series, according to the connections of a single-line-to-ground fault.


The resulting fault current is given as

$$
\begin{aligned}
& I_{f a}^{+}=I_{f a}^{-}=I_{f a}^{0}=\frac{1.0}{j(.0909+.05+.1)}=-j 4.1511 \\
& {\left[\begin{array}{c}
I_{f a} \\
I_{f b} \\
I_{f c}
\end{array}\right]=\underline{A}\left[\begin{array}{c}
I_{f a}^{0} \\
I_{f a}^{+} \\
I_{f a}^{-}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 1 & 1 \\
1 & a^{2} & a \\
1 & a & a^{2}
\end{array}\right]\left[\begin{array}{l}
-j 4.1511 \\
-j 4.1511 \\
-j 4.1511
\end{array}\right]} \\
& =\left[\begin{array}{c}
-j 12.4533 \\
0 \\
0
\end{array}\right]
\end{aligned}
$$

So the fault current is 12.4533 pu .
4. ( 20 pts ) True/false:
_T__The effect of DC offset on fault current depends on when the fault occurs relative to the voltage waveform.
_F_ The subtransient, transient, and synchronous reactances are used to approximate the effect of the DC offset at different times following initiation of a fault.
${ }_{\mathrm{k}}{ }^{\mathrm{F}}$. The Thevenin impedance for a fault at bus $k$ is the inverse of the Y -bus element in row k , colum
_F_ An unsymmetrical set of 3 phas ors may always be decomposed intotwo symmetrical positive sequence sets of phasors.
_F_The zero-sequence set of a set of symmetrical components are not phasors.
_T__Line to line voltages never have zero sequence components.
${ }^{-}$F-_ $1+\alpha+\alpha^{2}=1$
_T_ You can obtain the sequence components of the b and c phase currents from the sequence components of the a-phase currents.
_F_There is no coupling between the positive, negative, and zero-sequence networks unless the applied abc voltages are unbalanced.
_F_If zero-sequence currents flow into the primary of a transformer, then they must als oflow out of the secondary of the transformer.

