The stability of the real polynomial \( F(z) \) in (A.1) may now be represented, using Fact A.1 and the fact that the inverse of an upper triangular PI matrix is also upper triangular (with main diagonal elements inverted) and PI, by the positive otherwise (PI) property of the matrix \( Z^P_m \) defined above, or simply by the positive definiteness (PD) property of the \( m \times m \) matrix
\[
Z^P_m = \begin{pmatrix}
Z^+_1 & Z^+_2 & \cdots & Z^+_m \\
Z^+_m+1 & Z^+_m+2 & \cdots & Z^+_m m \\
\vdots & \vdots & \ddots & \vdots \\
Z^+_{m-m+1} & Z^+_{m-m+1} & \cdots & Z^+_{m+1}
\end{pmatrix}
\] (A.7)
for both \( n \) even (\( n = 2m \)) and odd (\( n = 2m + 1 \)).

**Definition:**

The real polynomial \( F(z) \) has \( 2(m^2 + m) \) Markov parameters \( z^+_1, z^+_2, \ldots, z^+_m, z^+_1, z^+_2, \ldots, z^+_n \) for \( n \) even (\( n = 2m \)), and \( 2(m^2 + m + 1) \) Markov parameters \( z^+_1, z^+_2, \ldots, z^+_m, z^+_1, z^+_2, \ldots, z^+_{m+1} \) for \( n \) odd (\( n = 2m + 1 \)).

**Example A.1:** Consider the Schur polynomial:
\[
F(z) = 5z^4 + z^3 + z^2 + z + 1.
\]

From (A.2)–(A.5) we get the PI matrices \( Z^+_m \)
\[
Z^+_m = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0.2 & 1.12 & -0.128 \\
0 & 0.16 & 0.096 & 1.0176
\end{pmatrix}
\]
and (A.7) the PD matrices \( Z^+_m \)
\[
Z^+_m = \begin{pmatrix}
1.12 & -0.128 \\
0.096 & 1.0176
\end{pmatrix}
\]
and from (A.7) the PD matrices \( Z^+ \)
\[
Z^+ = \begin{pmatrix}
0.16 & 0 \\
-0.16 & 0.8
\end{pmatrix}
\]

Therefore, the Markov parameters of \( F(z) \) are: \( z^+_1 = 0.2, z^+_2 = 1.12, z^+_3 = -0.128, z^+_4 = 0.096, z^+_5 = 1.0176, z^-_1 = -0.2, z^-_2 = -0.16, z^-_3 = 0.8, z^-_4 = 0, z^-_5 = -0.16, \) and \( z^-_6 = 0.8 \).

**References**


**Application of Robust Control to Sustained Oscillations in Power Systems**

Zhihua Qu, John F. Dorsey, John Bond, and James D. McCauley

**Abstract**—Transient control of the sustained oscillations that can occur after a major disturbance to a power system is investigated. A control scheme for an \( n \)-generator system is first developed using a classical machine model, and then extended to a model that includes governor/turbine dynamics. The proposed control strategies are linear and require only local relative angle and velocity measurements for the classical model case, plus the measurement of mechanical power if governor/turbine dynamics are included. Using Lyapunov’s direct method, the control is shown to be robust with respect to parameter and load variations, and topology changes in the power system.

The overall power system is shown to be exponentially stable in the large so that any oscillation, anywhere in the system, can be damped efficiently. The results are obtained without any linearization of the power system model. Simulation results for the 39 Bus New England System demonstrate the effectiveness of the proposed control.

**Keywords**—Exponential stability, Lyapunov stability, power systems, robustness, transient control, uncertainty.

**I. INTRODUCTION**

Over the past decade, Lyapunov’s direct method has been successfully used to design simple, robust control schemes applicable to nonlinear systems [2], [7]. In this paper, we use this approach to design a robust, decentralized, linear control to improve the transient response of power systems, focusing on the problem of damping oscillations that can occur following a major disturbance, such as a fault at a major substation followed by the loss of a heavily loaded transmission line. The analysis is first developed for an \( n \)-generator power system, using classical models for the generators, and then repeated for a generator model that includes governor/turbine dynamics.

Most existing results on transient control of power systems use a simple second-order model linearized about some stable operating point. The validation of these control strategies rests almost exclu-

Manuscript received March 14, 1991; revised March 20, 1992. This paper was recommended by Associate Editor C. C. Liu. Z. Qu is with the Department of Electrical Engineering, University of Central Florida, Orlando, FL 32826.

J. F. Dorsey, J. Bond, and J. D. McCauley are with the School of Electrical Engineering, Georgia Institute of Technology, Atlanta, GA 30332. IEEE Log Number 9201025.
sively on computer simulation. If the power system is stabilized about some steady-state operating point, it is possible to use a linearized model to study what is often called “steady-state stability.” However, the power system model used to study transient stability (or dynamic stability), that is, the stability of the power system following a major disturbance, such as a fault, is nonlinear. For this latter problem it is preferable to design a control that can be applied directly to the nonlinear model of the power system. Recently, a nonlinear stabilizing control was proposed in [3]. Unfortunately, the control requires the measurement of the derivatives of electrical powers (which in turn requires global information of the power system) and is based on global feedback linearization, which also requires exact knowledge of the parameters of all generators.

The control scheme presented in the present paper is based on Lyapunov’s direct method. The proposed control is linear and local, avoiding the shortcomings of [3]. A judicious choice of the Lyapunov function allows us to show that the control is robust to parameter and load variations, and to changes in the power system topology.

In the next section we give a detailed formulation of the problem discussed in this paper. In Section III, a transient control and corresponding stability result are investigated for a power system using only a classical model of the generators. In Section IV similar results are obtained using a generator model that includes governor/turbine dynamics. Section V provides simulation results that validate the control scheme.

II. PROBLEM FORMULATION

Present day power systems are heavily loaded. As a consequence, a large disturbance to the system, such as a fault or the loss of a major intertie, can result in sustained oscillations, even after the disturbance is cleared. The proposed control, and the assumptions about the power system model to be controlled, can be characterized as follows.

i) Each generator in the power system is represented as a constant voltage behind a transient reactance, with its dynamics represented by a second-order equation.

ii) Governor/turbine dynamics, if included, are represented by a simplified first-order system.

iii) The analysis is done using the post-disturbance electrical network with the disturbance relieved. It is important to note that the proposed control does not depend on the topology of the power system.

iv) The desired or ideal steady-state operating point, predetermined by the system operator, is characterized by \( \delta_i^0, \omega_i^0, P_i^{\text{m}i}, \) and \( v_i^0, \) where the subscript “\( i \)” denotes the \( i \)th machine. This need not be the actual steady state for the post-fault topology, but if it is, it satisfies (1) and (2) with \( \delta_i = 0 \) and \( \omega_i = 0 \) for all \( i \).

v) The control will not require a linearization of the power system nonlinearities. Further, the control will be robust and will not depend on any parameters of the power system, since the exact dynamics of a power system are not known in general.

vi) For ease of implementation, the control will be decentralized or local. Additionally, the control will be linear and use a minimum of feedback information.

vii) In order to achieve good transient performance in damping possible oscillations, the control will guarantee not only asymptotic stability but also a certain speed of convergence, specifically exponential stability.

The control outline above meets essentially all the necessary requirements for practical implementation. The only possible point of controversy is whether the control should be implemented by controlling the mechanical power supplied to the generators, or the output voltage of the generators. In this paper we have elected to control the mechanical power supplied to the generators.

It can readily be argued that the control should be implemented through the excitation system rather than the turbine governor. We intend to extend the results to include excitation control in a future paper. However, it is an open question as to whether excitation control alone will be sufficient to damp transient oscillations in very heavily loaded systems.

Before proceeding to the development of the control strategy, we specify the mathematical model of the power system that we will use. Suppose that the power system consists of \( n \) generators. The nonlinear differential equations describing the dynamics of the \( i \)th machine are

\[
\delta_i = \omega_i, \quad \dot{\omega}_i = \frac{1}{M_i} \left( P_{m_i} - P_{g_i} - D_i \omega_i \right), \quad i = 1, \ldots, n
\]  

(1)

where \( H_i \) is the inertia constant, with \( M_i = H_i / \pi f_0 \), \( D_i \) is the constant damping coefficient of the \( i \)th machine, \( \delta_i \) is the rotor angle, \( \omega_i \) is the rotor speed, \( P_e \) is the electrical output power, \( P_m \) is the mechanical input power, and the subscript “\( i \)” denotes the \( i \)th generator.

The electrical power outputs \( P_e, i = 1, \ldots, n \), satisfy the power flow equations, namely,

\[
P_{g_i} = P_{m_i} + P_e, \quad \delta_i = \delta_i - \delta_i^0 \]

\[
P_{e_i} = E_i^2 G_{ii} + E_i \sum_{j=1,j \neq i}^n E_j (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij})
\]  

(2)

where \( E_i \) is the magnitude of the constant voltage behind the transient reactance of generator “\( i \),” \( P_{g_i} \) is the electrical bus output power at generator “\( i \),” \( P_{e_i} \) is the local (possibly time-varying) electrical load power at generator “\( i \),” \( G_{ii} \) is the driving point conductance at generator “\( i \),” \( G_{ij} \) is the negative of the mutual conductance between generators “\( i \)” and “\( j \),” and \( B_{ij} \) is the negative of the mutual susceptance between generators “\( i \)” and “\( j \).” The parameters \( G_{ii}, G_{ij}, \) and \( B_{ij} \) are not required to be constant.

In the next section we develop the control strategy characterized above using the classical generator model. This choice of machine model provides a simple setting in which to illustrate an approach that will yield a proof of robustness of the control strategy. The control actually used in a power system will be quite different. However, for the present, using the mechanical input power as the control simplifies the analysis so that the methodology used to develop the control is clearly revealed. Once the methodology is clear, it can be used to develop robust control schemes based on more complex machine models. Later in the paper we take a first step in this direction by extending the results to the case when dynamics of the governor/turbine are included.

III. ROBUST CONTROL OF POWER SYSTEMS: PART I

The proposed robust, local, transient controller for the \( i \)th machine is given by

\[
u_i = \frac{P_{m_i}^d - P_{m_i}}{k_i (\delta_i - \delta_i^0)} - k_i (\omega_i - \omega_i)
\]  

(3)
or

\[ P_m = P_m^f + k_1(\delta_m^f - \delta_i) + k_2(\omega_m^f - \omega_i). \]

The differences \( \delta_m^f - \delta_i \) and \( \omega_m^f - \omega_i \) represent the errors between the states of the power system and the desired steady state. The variable \( \omega_i \) is the transient control input into the \( i \)th machine. It must be emphasized that \( \delta_m^f \) and \( \omega_m^f \) are arbitrary, local references, and that \( \delta_i \) and \( \omega_i \) are local measurements of the machine angle and speed. The control does not require knowledge of the angle of one machine relative to another. Local measurement of \( \delta_i \) and \( \omega_i \) with an optical or magnetic transducer is straightforward.

Reasonable choices for the desired states would be \( \omega_i^f \) equal to the synchronous speed and \( \delta_i^f \) either the prefault steady-state value or some selected contingency value. We show later that the choices of \( \delta_i^f \) and \( \omega_i^f \) will affect the actual post-fault steady state, but do not change the stability result of the whole power system, and therefore can be made arbitrarily.

Let the actual steady-state operating point be characterized by \( \delta_i^f, \omega_i^f, P_m^{f^i}, \) and \( P_e^f \). That is, \( \delta_i, \omega_i, P_m^f, \) and \( P_e^f \) are solutions of the following equations:

\[ \delta_i^f = \omega_i^f = 0, \quad \omega_i^f = 0 = \frac{1}{M_i}(P_m^f - P_{e^f} - D_1 \omega_i^f) \]

\[ P_m^{f^i} = P_m^f + k_1(\delta_i^f - \delta_i) + k_2(\omega_i^f - \omega_i) \]

\[ P_e^f = P_e^f + \delta_i^f - \delta_i \]

\[ P_e^f = E_i^2 G_{ii} + E_i \sum_{j=1, j \neq i}^n E_j (G_{jj} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}). \quad (4) \]

It is important to note here that the actual steady state is never known. Throughout this paper, it is introduced only as a mathematical convenience to clarify and simplify the analysis. It is not assumed to be known and is not used in the proposed control strategy, which is based entirely on local measurements.

To study the stability of power system (1) and (2) under control (3), define the signals representing the error states to be \( x_{1i} = \delta_i^f - \delta_i \) and \( x_{2i} = \omega_i^f - \omega_i \). Then it follows from (1), (2), and (4) that

\[ x_{1i} = x_{2i}, \]

\[ x_{2i} = \frac{1}{M_i}(P_m^f - P_{e^f} - D_1 \omega_i^f). \]

Thus, the error system can be described by the following dynamic equations:

\[ x = Ax + BF(x) \]

\[ x = [x_{1i}^T \quad x_{2i}^T \quad \cdots \quad x_n^T]^T, \]

\[ A = \text{diag} \{ A_1 \quad A_2 \quad \cdots \quad A_n \}, \]

\[ A_i = \begin{bmatrix} 0 & \frac{1}{M_i} & -k_1 & -k_2 \end{bmatrix}, \]

\[ B = \begin{bmatrix} B_1 \quad B_2 \quad \cdots \quad B_n \end{bmatrix}, \]

\[ B_i = \begin{bmatrix} 1 \end{bmatrix} \]

\[ F(x) = [f_1(x) \quad f_2(x) \quad \cdots \quad f_n(x)]^T \]

\[ f_i(x) = 2E_i \sum_{j=1, j \neq i}^n E_j \left[ G_{ij} \sin \left( \frac{\delta_{ij} + \delta_{ij}}{2} \right) - B_{ij} \cos \left( \frac{\delta_{ij} + \delta_{ij}}{2} \right) \sin \left( \frac{x_{1j} - x_{1i}}{2} \right) \right]. \]

The submatrices and subvectors with subscript "i" represent the dynamics and the states for the error system of the \( i \)th machine, which constitutes the \( i \)th subsystem. The term \( f_i(x) \) represents the interconnection effects between the \( i \)th subsystem and the other subsystems.

The functions \( f_i \) contain the parameters and steady-state information of the power system and therefore can not be determined. Consequently, they are treated as uncertainties in the following analysis. Although the \( f_i \) are unknown, it is easy to see that

\[ |f_i(x)| \leq \max_{j \neq i} \left\{ 2E_j E_i \left( |G_{ij}| + |B_{ij}| \right) \right\} \]

\[ \sum_{j=1, j \neq i}^n \left( |x_{1j}| + |x_{1j}| \right) \leq C_i \sum_{j=1, j \neq i}^n \left( |x_{1i}| + |x_{1i}| \right) \]

where \( C_i \) is a constant that is independent of the gains \( k_{im}, \ i = 1, 2, \ldots, n \), and depends only on the post-fault topology. In some sense, \( C_i \) is the maximum possible conductance from subsystem "i" provided by the full network. In most system faults, the post-fault topology will have a similar or reduced conductance due to breaker openings. Thus \( C_i \) is a good upper bound for this problem.

We are now in a position to state the main result of this paper.

**Theorem 1**: The error system (5) is guaranteed to be exponentially stable if the gains are chosen such that for all \( i \)

\[ k_{ii} > \frac{3(n - 1)C_i}{2} + \frac{(n - 1)M_i C_i}{D_i} + \max_{j \neq i} \left( \frac{(n - 1)C_j}{2} \right) \]

\[ k_{2i} > \frac{(n - 1)C_i}{2} + \max_{j \neq i} \left( \frac{(n - 1)C_j}{2} \right). \]

More specifically, the states \( x_{1i} \) and \( x_{2i}, \ i = 1, \ldots, n, \) approach zero exponentially, and consequently any possible oscillation in the power system decays to zero exponentially.

**Proof**: The proof uses Lyapunov's direct method. Choose the Lyapunov function to be \( V = (1/2)x^TPx = \sum_{i=1}^n V_i = \sum_{i=1}^n (1/2)x_i^TP_ix_i \), with

\[ P = \text{diag} \{ P_1 \quad P_2 \quad \cdots \quad P_n \}, \]

\[ P_i = \begin{bmatrix} D_i + k_{2i} & M_i k_{ii} \\ M_i & M_i^2 \end{bmatrix}. \]
It is easy to see that $P$ is positive definite. Let $\| \cdot \|$ denote the Euclidean norm and its induced matrix norm. Then,

$$
\dot{V} = x^T P \dot{x} = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} \left( A_{ij} x_i + B_{ij} f_i(x) \right)
$$

$$
= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{s=1}^{n} \left( \frac{M_{is}}{D_s} \frac{M_{js}}{D_s} \right) x_i \left( x_i + \frac{M_{is}}{D_s} x_s \right) f_i(x)
$$

$$
\leq \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{s=1}^{n} \left( \frac{M_{is}}{D_s} \frac{M_{js}}{D_s} \right) \left( |x_i|^2 + \left( \frac{M_{is}}{D_s} \frac{M_{js}}{D_s} \right)^2 |x_s|^2 \right) f_i(x)
$$

$$
\leq - \sum_{i=1}^{n} k_{ii} |x_i|^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij} |x_i||x_j|
$$

$$
\leq -x^T Q x
$$

(7)

where the matrix $Q \in \mathbb{R}^{n \times n}$ is symmetric. Every element $q_{ij}$ of the matrix $Q$ is either 0 or defined by

$$
q_{2i-1,2i-1} = k_{ii} - (n-1)C_i
$$

$$
q_{2i-1,2i} = q_{2i-1,2i-1} = \frac{(n-1)M_i C_i}{2D_i}, q_{2i,2i} = \frac{M_i D_i}{2D_i}
$$

$$
q_{2i-1,2i-1} = q_{2i-1,2i-1} = \frac{C_i}{2} - \frac{C_j}{2}
$$

$$
q_{2i,2i} = \frac{M_i C_i}{2D_i} - \frac{M_j C_j}{2D_j}
$$

where $1 \leq i < j \leq n$. By the Gershgorin Theorem in [10], we know that the matrix is positive definite if the two inequalities in (6) hold for all $i$. By choosing the gains $k_{ii}$ and $k_{ij}$ to satisfy the two inequalities in (6), the matrix $Q$ is positive definite. Thus, we have $\lambda_{\min}(Q) > 0$ and

$$
\dot{V} \leq -\lambda_{\min}(Q) |x|^2 \leq -\lambda_{\min}(Q) \frac{\lambda_{\max}(Q)}{\lambda_{\max}(P)} V = -\lambda \cdot V
$$

where $\lambda > 0$. Let $\dot{V} = -\lambda \cdot V + s(t)$. It follows that $s(t) \leq 0$ for all $t$ and that

$$
V(t) = e^{-\lambda t} V(t_0) + \int_{t_0}^{t} e^{-\lambda (t-\tau)} s(\tau) d\tau \leq e^{-\lambda t} V(t_0)
$$

which shows that $V(x)$ decreases to zero exponentially. Then, since $V(x) \geq \lambda_{\min}(P) \| x \|^2$, it follows that $\| x \|$ decreases to zero exponentially. Therefore, (5) is exponentially stable in the large under control (3) with finite gains.

It is important to note the following. One, the control guarantees that the system approaches the post-disturbance steady state at an exponential rate, even though the steady state is assumed to be unknown. Two, although the lower bounds of the gains that guarantee stability are given by the inequality (6), it is not necessary to know the parameters $M_i, D_i$, and $C_i$ only the range of values of these parameters. Further, these bounds provide only a very conservative estimate for the magnitude of the gains. It is unavoidable that this inequality is not a necessary condition, since Lyapunov's second method always yields very conservative results. The magnitude of the gains really needed to control the power system can and should be determined by computer simulation, as done in Section V.

IV. ROBUST CONTROL OF POWER SYSTEMS: PART II

We now extend the results obtained in the previous section by including the governor and turbine dynamics in the system. The analysis proceeds as before, but the control and the Lyapunov function are chosen to accommodate the increased number of state variables. The following analysis assumes that the governor/turbine dynamics can be approximated by a first-order system. The case that the dynamics are modeled as a higher order system and that loads are not represented by constant impedance loads can be similarly treated, but finding a proper Lyapunov function will be more tedious.

Suppose that the dynamics of turbine and governor can be modeled as a simple lag transfer function. That is,

$$
\dot{P}_{mi} = -\alpha_i \dot{P}_{mi} + v_i
$$

(8)

where $1/\alpha_i$ is the governor/turbine time constant of the $i$th subsystem and $v_i$ is the corresponding input. In this case, the proposed robust, local, transient controller for the $i$th machine is given by

$$
\Delta u_i = \omega_i^* - \omega_i = -k_{ii} (\delta_i^d - \delta_i) - k_{2i} (\omega_i^d - \omega_i) - k_{3i} (P_{mi} - P_{m_i})
$$

(9)

The control has properties similar to those given in the previous section.

We again assume there exists an unknown post-disturbance steady-state operating point of the power system, characterized by $\delta_i^d, \omega_i^d, P_{m_i}^d, P_{m_i}'$, and $\omega_i^d'$. That is, $\delta_i', \omega_i', P_{m_i}'$, $P_{m_i}^d$, and $\omega_i^d$ are solutions of the following equations:

$$
\delta_i' = \omega_i' = 0, \omega_i^d' = 0 = \frac{1}{M_i} \left( P_{m_i}' - P_{m_i}^d - D_i \omega_i^d \right)
$$

$$
\dot{P}_{m_i}' = 0 = v_i' - \alpha_i P_{m_i}'
$$

$$
\omega_i' = \omega_i^d' + k_{ii} (\delta_i^d - \delta_i) + k_{2i} (\omega_i^d - \omega_i') + k_{3i} (P_{m_i} - P_{m_i}' - P_{m_i}^d)
$$

$$
P_{m_i}' = P_{m_i}^d + P_{m_i}, \delta_i' = \delta_i - \delta_i^d
$$

$$
P_{m_i}' = \sum_{j=1}^{n} E_{ij} (G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}).
$$

(10)

Then, proceeding as in the last section, we obtain the error system described by the following dynamic equations.

$$
\dot{x} = Ax + BF(x)
$$

(11)

where $x_i = \delta_i - \delta_i^d, x_{2i} = \omega_i - \omega_i^d, x_{3i} = P_{m_i} - P_{m_i}', k_{3i} = k_{3i} + \alpha_i$

$$
x = \begin{bmatrix} x_1 \ x_2 \ \vdots \ x_n \end{bmatrix}, \dot{x} = \begin{bmatrix} x_{1i} \ x_{2i} \ \vdots \ x_{ni} \end{bmatrix}^T,
$$

$$
A = \text{diag} \begin{bmatrix} A_1 \ A_2 \ \vdots \ A_n \end{bmatrix}
$$

$$
A_i = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{D_i}{M_i} & 1 & 0 & 0 \\ -k_{ii} & -k_{2i} & -k_{3i} & 0 \end{bmatrix}
$$

$$
B = \text{diag} \begin{bmatrix} B_1 \ B_2 \ \vdots \ B_n \end{bmatrix}, B_i = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T
$$

and $F(x)$ is the same as that defined in (5).
We now state the following theorem, which guarantees the stability
of the power system (1), (2), and (8) under the control (9).

**Theorem 2**: Given constants $\beta_i > 0$, $\rho_i > 0$, and $\gamma > 1$ there exist constants $k_{3i}^* > 0$ such that, whenever $k_{3i} \geq k_{3i}^*$, $k_{3j} = (1/\rho_i) k_{3i}^{1/2}$, and $k_{3j} = \beta_i k_{3i}$ for $i = 1, \ldots, n$, and $\gamma \geq k_{3i}/k_{3j}$ for $1 \leq i, j \leq n$, the error system (11) is exponentially stable.

**Proof**: The proof is conceptually the same as that of Theorem 1, and therefore only the major steps are presented for brevity.

The main issue is to determine a lower bound $k_{3i}^*$ for $k_{3i}$ such that whenever $k_{3i} > k_{3i}^*$, the error system (11) is exponentially stable. Choose the Lyapunov function to be $V = (1/2) x^T P x = \sum_{i=1}^{n} V_i$

\[
P = \text{diag} \{ P_1, P_2, \ldots, P_n \},
\]

\[
P_i = \begin{bmatrix}
k_{3i} + \beta_i \kappa_{3i} + 2 D_i \beta_i \kappa_{3i} & 2 M_i \beta_i \kappa_{3i} & \beta_i \\
2 M_i \beta_i \kappa_{3i} & 2 M_i \kappa_{3i} & 1 \\
\beta_i & 1 & \frac{\rho_i}{\sqrt{k_{3i}}} 
\end{bmatrix}
\]

The necessary and sufficient condition for the matrix $P$ to be positive definite is that all determinants of its successive principal minors and itself are positive. It is then easy to verify that $P_i$ is positive definite if

\[
k_{3i} > \max \left\{ \rho_i^2 (D_i - M_i \beta_i)^2, \frac{1}{M_i}, 4M_i^2 \beta_i^2 \rho_i^2 \right\}.
\]  

(12)

Moreover, it is easy to verify that

\[
W_i = -\frac{1}{2} (P_i A_i + A_i^T P_i)
\]

\[
= \begin{bmatrix}
\beta_i \kappa_{3i} & 0 & 0 \\
0 & -2(\beta_i M_i - D_i) \kappa_{3i} + \kappa_{3i} & 1 \\
0 & 1 & -\frac{1}{M_i} + \rho_i \sqrt{k_{3i}} 
\end{bmatrix}
\]

Then, differentiating the Lyapunov function, we have

\[
\dot{V} = \sum_{i=1}^{n} \frac{1}{2} x_i^T (P_i A_i + A_i^T P_i) x_i + x_i^T P_i B_i f_i(x)
\]

\[
\leq \sum_{i=1}^{n} x_i^T W_i x_i
\]

\[
= \sum_{i=1}^{n} \left( 2 \beta_i \kappa_{3i} |x_{i1}| + 2 \kappa_{3i} |x_{2i}| + \frac{1}{M_i} |x_{3i}| \right)
\]

\[
\cdot C_i \sum_{j=1, j \neq i}^{n} (|x_{ij}| + |x_{ji}|)
\]

\[
\leq -x^T Q x
\]

where the matrix $Q \in R^{3n \times 3n}$ is symmetric. Every element $q_{ij}$ of the matrix $Q$ is either 0 or defined by

\[
q_{3j - 2, 3j - 2} = \frac{\beta_i^2}{\rho_i} k_{3i}^{1/2} - 2(n - 1) C_i \beta_i \kappa_{3j - 1},
\]

\[
q_{3j - 2, 3j - 1} = -\frac{n - 1}{M_i} C_i,
\]

\[
q_{3j, 3j - 1} = \rho_i \sqrt{k_{3j}} - \frac{1}{M_i}, q_{3j - 2, 3j - 1} = -2(n - 1) k_{3j} C_i,
\]

\[
q_{3j - 1, 3j - 1} = -\frac{1}{M_i}, q_{3j - 1, 3j - 2} = -\frac{2}{M_i} - D_i k_{3j},
\]

\[
q_{3j - 1, 3j - 2} = -C_i k_{3j}, q_{3j - 1, 3j - 2} = -C_j k_{3j} C_i,
\]

\[
q_{3j - 1, 3j - 2} = -C_j k_{3j} C_i, q_{3j - 1, 3j - 2} = -\frac{C_i}{2 M_i} - \frac{C_j}{2 M_j}
\]

(13)

where $1 \leq i, j \leq n$ and $i \neq j$. By the Gerschgorin Theorem in [10], we know that the matrix $Q$ is positive definite if

\[
k_{3j}^{1/2} > \max \{ a_{ij}, a_{2i}, a_{3i} \}, \forall i \in \{1, \ldots, n\}
\]

(13)

where $\gamma \geq \gamma_{ij} = k_{3j}/k_{3i}$ and

\[
a_{ii} = \frac{(n - 1) \rho_i}{\beta_i^2} \left[ 3C_i \left( \beta_i + 1 + \frac{1}{2M_i k_{3j}} \right) \right]
\]

\[
+ \max_{j \neq i} \left[ C_j \left( \beta_j \gamma_{ij} + \gamma_{ij} + \frac{1}{2M_j k_{3j}} \right) \right]
\]

\[
a_{ij} = \rho_i \left[ 2 |\beta_i M_i - D_i| + \frac{M_i}{2 k_{3j}} - \beta_i \right]
\]

\[
+ 3(n - 1) C_i + (n - 1) \max_{j \neq i} C_j \gamma_{ij}
\]

\[
a_{ij} = \frac{1}{\rho_i} \left[ 1 + \frac{1}{M_i} \left( 2 \beta_i \kappa_{3j} + \frac{M_i}{2 M_j} \right) \right]
\]

\[
+ (n - 1) C_i + (n - 1) \max_{j \neq i} C_j \kappa_{3j}
\]

It is not entirely evident that inequality (13) can be satisfied by choice of $k_{3j}$ large since $k_{3i}$ and $k_{3j}$ are involved in the computation of $a_{ii}$ and $a_{ij}$. However, by keeping the ratio, $k_{3j}/k_{3i}$, bounded above by $\gamma$, as stated in the theorem, the effect of increasing $k_{3j}$ is a reduction in the magnitude of $a_{ij}$ and $a_{3i}$. By choosing the gain $k_{3j}$, to satisfy the inequalities (12) and (13), the matrices $P$ and $Q$ are positive definite. Therefore, system (11) is exponentially stable under control (9) with finite gains.

It follows from the above discussion that the magnitude of the control gain $k_{3j}$ is crucial for the asymptotic stability of the whole power system. Recall that $k_{3j} = k_{3j}^* + a_{ji}$ is the reciprocal of the time constant of the closed-loop governor/turbine under local linear feedback control. The control gain $k_{3j}^*$ is used to move the dominant pole of the governor/turbine further away from the imaginary axis.
and consequently reduce the time constant of the response of the governor/turbine dynamics. For Theorem 2 to be useful, we must be able to reduce the time constant to a reasonably small magnitude in order to stabilize the power system for any possible initial conditions under the proposed linear control law. Also, in the case that $\alpha_i$ is large enough, we can let $k_i = 0$. This means that the control (9) requires only relative rotor angles and velocities if the governor/turbine dynamics are fast enough to be neglected. However, in most cases, feedback of the governor/turbine state will be required.

Again, the inequalities (12) and (13) provide sufficient but very conservative estimates for the gains. The actual magnitude of the gains and the actual upper bound of the magnitude of the time constant under which the governor/turbine can respond quickly enough to stabilize the whole power system can be obtained by computer simulation, as is done in Section V.

It is also worth noting that the parameters, $G_{ij}$, $G_{ji}$, $B_{ij}$, $\alpha_i$, in (1), (2), and (8) are not required to be time invariant. This means that the above analysis is robust for unmodeled dynamics, namely, it accommodates the case of a time-invariant linear system approximated by a time-varying linear system of lower order and/or when a linearization of nonlinearities is performed.

V. SIMULATION EXAMPLES

The proposed control algorithms were tested using the loading of the 39 Bus New England System given in [12]. The topology of the system and the load data is readily available. The disturbance chosen was a four cycle fault at bus 23, which is midway between generators 6 and 7. These two machines are the most coherent in the system, but Fig. 1(a) shows that there is a persistent intermachine oscillation once the fault is removed, where the label "machine angle" in Fig. 1 represents the angle difference between machine 6 and machine 7. As might be expected, this is the largest intermachine oscillation in the system. Fig. 1(b) shows the response with the proposed control using classical generator models for all machines. Fig. 1(c) shows the results when the governor/turbine dynamics are included. The big change in response between Fig. 1(b) and 1(c) is due to the 8-s time constant of each governor. Fig. 1(d) shows the results when the governor input is limited to 1.02 of the initial steady state operating power. The feedback gains were also increased from Fig. 1(c) to 1(d) by a factor of three. As can be seen, the control is still very effective.

Fig. 1(e) and (f) illustrate the increased time required for the oscillation to disappear as a result of changing the gain ratios away from the selected gain ratios while holding their squared sum constant. It was found by simulation that the gain ratio is more important than gain magnitude, beyond a certain threshold magnitude, in reducing oscillation time when the power available for control is limited. Although there is not room to discuss it at length in this paper, the oscillation duration can be used to tune the gains.

VI. CONCLUSIONS

In this paper we have developed a general framework based on Lyapunov's direct method for analyzing and developing robust control schemes to improve the transient response of power systems. A general approach to finding the appropriate Lyapunov function is demonstrated for the case of classical machine models and for machines with simple governor/turbine control. The proposed control is feasible since it uses only local feedback information. Only the local speed and angle of each machine, relative to an arbitrary local reference is required. Further, the control is robust to uncertainties in the system parameters and topology, as well as loading. The principles and procedures of designing such controls
A Bound for the Zeros of Polynomials

M. S. Žilović, L. M. Roytman, P. L. Combettes, and M. N. S. Swamy

Abstract—In this paper we introduce a new bound for the zeros of polynomials. Our result is obtained by applying Cauchy's theorem to a polynomial derived from the original polynomial through an exponential transform.

I. INTRODUCTION

The problem of determining bounds on the magnitude of the zeros of a polynomial has a long history that dates back to the work of Cauchy [2], who gave a very simple expression for the bound in terms of the coefficients of the polynomial. An account of several developments on this topic can be found in the comprehensive book by Marden [5]. Recently, Zeheb [6] gave new extensions of Cauchy's result, by introducing polynomial transformations, treating separately cases when the original polynomial is real and complex. Minimizing Cauchy's bound for a transformed pair first, Zeheb defined two circular bounds for the original polynomial. His result demonstrated improvement over known results.

Starting from the idea of determining a bound on the zeros of polynomial, determining Cauchy's bound on zeros of its transformed pair first, we will define a new circular bound. Our transformation is based on a nonlinear transformation of the variable, which conceptually should give us a better upper bound. The advantage of such a transformation will be illustrated through several examples, where our results are compared to the existing ones. This paper will show the improvement over the existing bounds, when they are applied to the transformed polynomial. It will also examine convergence of the bound after iterative transformations of the original polynomial.

A. Bound for the Zeros of a Polynomial

Let

\[ P(z) = z^n + \sum_{k=0}^{n-1} a_k z^k, \quad a_k \in \mathbb{C} \]  

be a complex polynomial of degree \( n \).

Theorem: All zeros of \( P \) lie in the disk

\[ \{ z \in \mathbb{C} \mid |z| < \sqrt{1 + A} \} \]  

with

\[ A = \max_{0 \leq k \leq n-1} \left\{|a_k^2 + 2(-1)^{k}(B - C)|\right\} \]  

where

\[ B = \sum_{0 \leq k \leq n/2} a_{2k}a_{2k} \]  

(4)

\[ C = \sum_{l+j=k \leq n/2} a_{2l}a_{2l+1} \]  

(5)

\( a_0 = 1 \), and \( [l] \) denotes the integer part of \( l \).

REFERENCES


Manuscript received May 6, 1991. This paper was recommended by Associate Editor P. Agathoklis.