Voltage Risk Assessment

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Abstract: This paper describes computational techniques for computing risk associated with voltage insecurity, where risk is assessed as the product of probability and consequence of under-voltage and voltage collapse. In contrast to deterministic assessment of voltage security, our approach directly accounts for uncertainties in the analysis. An approach for operational assessment is provided that uses continuation power flow methods. In addition, a planning approach is described which utilizes an interior point optimization method to identify maximum loading conditions over a sequential trajectory of operating conditions. Analysis of the IEEE Reliability Test System illustrates results that are obtained from the approaches.

Keywords: transmission, security, probabilistic risk, voltage collapse, operations, planning.

1 Introduction

The prevailing practice within industry of avoiding voltage insecurity is deterministic. It identifies conditions, in terms of loading, transfers, or voltages, which lead to violation of performance criteria following the most limiting contingency within a specified contingency set. Because of uncertainties, however, a protective "margin" is applied so that the actual conditions remain well away from the danger zone [1]. Yet both the performance criteria as well as the margin are selected quite subjectively, and their use may lead to situations of inadvertent high risk or, more commonly, over-conservative and costly operating conditions. Alternatively, we propose in this paper to use numerical descriptions of the uncertainties in combination with a severity model to compute a voltage-related risk index of an operating condition or of a facility plan. This index can also be used to trend how voltage risk changes as the operating conditions are relieved or stressed, and to better understand the relationship between voltage security levels and economics.

Traditional deterministic performance indices of voltage [2] are used in many on-line or off-line security assessment tools. Other indices, such as sensitivity factors, singular values and eigenvalues, loading margin and closest loadability, and tangent vector index have been proposed [3]. Recently, some researchers have also used first and second order loading margin approximations to estimate the loadability [4]. We pay particular attention to loading margin and margin sensitivity because they closely relate to our approach where these deterministic techniques are used to compute the probability of voltage collapse.

Loading margin to voltage collapse, perhaps the most widely used voltage-related index [4], is the difference between the loading at the operating point and the nose of the P-V curve. Because it requires computation at points away from the operating conditions, it is more computationally expensive than indices that utilize only information at the operating point. On the other hand, once the load margin is computed, it is easy to quickly compute its sensitivity with respect to any power system parameter [5-7]. There are basically two methods to obtain the load margin: continuation power flow [8-10], and optimization [11-15], and there has been considerable research on both methods. However, as mentioned in [16], "there are comparatively few applications of probabilistic analysis to voltage stability problems." References [13-20] covers most of the approaches proposed to date. We build on these approaches in what follows.

We have proposed a method that accounts for uncertainty in event occurrence and consequence and combines them as a product in obtaining a risk index, where deterministic load margin is used in the calculation [21,22]. We provide three tiers of assessment: component, system, and cumulative. The component assessment gives a detailed study of the impact at each load bus under given bus voltages, accounting for uncertainty in load interruption voltage levels. The system assessment, with consideration of uncertainties in loading, dispatch, and outages using margin sensitivities, gives the voltage risk of a given operating condition for a desired region. This risk indicates the expectation of the cost consequence associated with voltage-collapse and voltage-out-of-limit[1]. The cumulative assessment provides a summation of system assessments over a sequential trajectory of operating conditions corresponding to a particular time interval of time.

2 Component Voltage Risk Assessment

In situations where under- or over-voltage protection is used in distribution networks or within the load itself, the distribution components and the loads are protected from over-current or other potential damage due to unacceptable load voltage. Additionally, cascading voltage collapse is avoided by employing load-shedding schemes that automatically trip load. In these situations, when the load voltage exceeds the threshold, end users will experience service interruption [23]. Also, some loads may drop off by themselves without any action of protective relays when voltage is unsustainable.

The component voltage risk is the risk of bus load interruption or the expected impact of load interruption at a load bus, given a specified bus voltage. It indicates the amount of money that one can expect to lose due to load interruption under a given bus voltage. Two measurements

[1] This is not a deterministic voltage limit as treated by other reliability assessment approaches.

are needed to evaluate this bus-oriented risk under given voltage levels: probability of service interruption and expected interruption cost. The risk at the bus is the product of these two measurements.

In transmission models[2], the load represented at a single bus actually represents an aggregation of different load classes, each having its own particular set of statistics regarding its interruption level and its own interruption cost. We assume that there are NCLASS different load classes. Denoting Pj as the load represented at bus j, Kc as the percentage share of load class c in Pj, and Cc as the load interruption cost of load class c, we may compute the risk of load interruption (LI) at bus j, given V_j, as

$$Risk_j(LI \mid V_j) = E(P_j) \times \sum_{c=1}^{N_{CLASS}} E(C_c) \times E(K_c) \times \Pr(V_{L,c} > V_j \text{ or } V_{U,c} < V_j) \quad \text{(eq. 1)}$$

where $V_{L,c}$ and $V_{U,c}$ are random variables representing the lower and upper interruption voltages, respectively, for load class c. The associated probability function may be estimated based on available [24] or gathered statistics. This method of specifying load information at the transmission level in terms of different classes is similar to what has been done before regarding load characterization for dynamic analysis [25]. The notation $E(\bullet)$ is the expectation operation and must be used if one considers uncertainty associated with its argument.

For a simple illustration, we assume an aggregated load at a bus has 100% residential load[3] with mean interruption voltages at 0.85 (lower mean), 1.15 (upper mean), and a 0.02 of standard deviation. Based on the lower mean of the interruption voltage of this residential load class, we expect at least half of the load will be interrupted when the voltage goes below 0.85. On the other hand, we expect more than half of the load will be interrupted if the voltage exceeds 1.15. The expected cost consequence of load interruption at this bus under various voltage levels, computed by eq. 1, is shown in Fig. 1, where an expected cost of $50 per MWhr for an average 6 hour[4] service interruption is assumed for this residential load.

3 System Voltage Risk Assessment

The purpose of system voltage risk assessment is to estimate the voltage risk for near-term operating conditions (e.g., within the next 5 minutes or hour) given knowledge of the present operating conditions. In contrast to the component voltage risk assessment, which evaluates the expected impact of a voltage level at a bus, the system voltage risk assessment uses the probability description of the bus voltage levels throughout the system to compute the risk. This computation combines the system induced probability assessment and the component induced impact assessment.

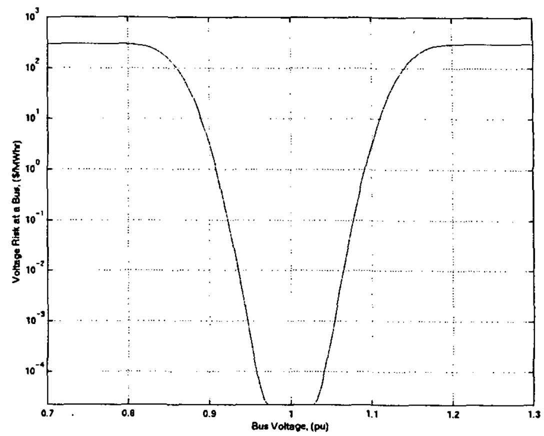

Fig. 1: Load interruption risk as function of voltage

With respect to the entire system, we assume that there are two distinct outcomes for the future performance of system voltages: either they collapse, or they do not. The bifurcation point (the nose) of the P-V curve provides the boundary between these two outcomes. The voltage collapse outcome is commonly known to be a severe event that is generally avoided under all circumstances. Therefore, we assume that the voltage collapse results in complete interruption of all loads[5]. In addition, we recognize that significant impacts may occur due to voltage deviations without voltage collapse, as indicated in the component analysis. Therefore,

$$\begin{aligned} Risk(voltage \mid X_0) &= E(\mathrm{Im}(voltage \mid X_0)) \\ &= \Pr(Collapse \mid X_0) \times E\left(\mathrm{Im}(Collapse \mid X_0)\right) + \\ &\quad \left[1.0 - \Pr(Collapse \mid X_0)\right] \times E\left(\mathrm{Im}(NoCollapse \mid X_0)\right) \end{aligned} \quad \text{(eq. 2)}$$

where X_0 stands for the current operating condition. We will drop the notation indicating dependence on X_0, leaving the reader to be cognizant of it in what follows. To compute this voltage risk, eq. 2 indicates we need to obtain the probability of collapse and the impact of collapse and no collapse. In doing so, we will consider three different forms of *system* uncertainty. First, we assume that contingencies E_i can occur, and that their occurrence follows a Poisson distribution such that $\Pr(E_i) \approx 1 - e^{-\lambda t}$ (a companion paper on transient instability risk provides a more rigorous expression for this probability). We generally refer to E_i as a

[2]We assume that transmission models used in this analysis represent the load on the low side of load tap changing transformers.

[3] This load mix assumption is purely for simplicity. Adding more load classes does not change the general shape of Fig. 1.

[4] To obtain $E(C_c)$ in eq. 1, a simple linear regression on interruption duration has been assumed here such that $E(C_c) = \beta_c \times t = 50 * 6$. Generally, it may be any nonlinear function, such as an exponential function, of the duration t where other regression methods on t could be used.

[5] It is possible to mitigate the impact of voltage collapse via corrective or restorative operating actions. It is also possible that partial interruption can mitigate the voltage collapse and prevent full interruption. However, the effectiveness of these actions is very uncertain. Therefore we assume here that voltage collapse results in total system blackout.

contingency state and allow it to include the "normal" condition corresponding to no contingency. Second, although we can know the present load level, some uncertainty does exist in the future load level. Assuming a short-term load forecast provides an expectation of load as μ_L, we model the load L as a normally distributed random variable having standard deviation of σ_L, that is, $L \sim N(\mu_L, \sigma_L^2)$.

The third form of uncertainty is in the maximum system loading, which we call the loadability. This uncertainty arises from short-term uncertainties in parameters which determine it, for example, load sharing factors (percentage of total load at each bus), load power factors, and generation sharing factors (percentage of total load supplied by each generator, as specified by the dispatch policy). These parameters are denoted using the column vector $\underline{K_P}$. For a given contingency state, we consider these uncertainties as deviations away from their expected values: $\underline{K_P} - E(\underline{K_P})$, and assume that future values of these parameters are random. Also, we assume that their deviations follow a multi-variate-normal (MVN) distribution around their expected future values, and that these deviations are small such that linear approximations of parameter-induced variation in loadability is valid. Using the expectation of these parameters, $E(\underline{K_P})$, the continuation power flow (CPF) [8] or other methods provide an expectation of loadability $E(L_{mi})$ and the margin sensitivities $\underline{S_P}$ with respect to these parameters [5-7]. Therefore, we may express loadability in terms of the deviations and sensitivities: $L_{mi} = E(L_{mi}) + \underline{S}_P^T \times (\underline{K}_P - E(\underline{K}_P))$. If we estimate the covariance matrix of uncertain parameters, denoted as V_P, then the loadability, which must follow a normal distribution since it is a linear function of the MVN distributed $\underline{K_P}$, is $L_{mi} \sim N\left(E(L_{mi}),\ \underline{S}_P^T \times V_P \times \underline{S}_P\right)$. For a contingency state E_i, the load margin is the amount by which the loadability exceeds the load, $M_i = L_{mi} - L$. Thus, the load margin distribution is normal, given by $M_i \sim N(\mu_{mi}, \sigma_{mi}^2)$, with mean $\mu_{mi} = E(L_{mi}) - \mu_L$ and the variance is $\sigma_{mi}^2 = \underline{S}_P^T \times V_P \times \underline{S}_P + \sigma_L^2$. Voltage collapse occurs when M_i is negative, therefore, for a particular contingency state E_i, we have $\Pr(Collapse \mid E_i) = \Pr(M_i < 0 \mid E_i)$ which is obtained from the distribution of M_i. The desired probability is then obtained from an application of the law of total probability.

$$\Pr(Collapse) = \sum_{E_i} \Pr(Collapse \mid E_i) \times \Pr(E_i) \quad \text{(eq 3)}$$

Turning to the impact term in eq. 2, $\mathrm{Im}(NoCollapse|X_0)$, we see that eq. 1 provides the risk at a particular bus when the buses' voltage is specified. For a given loading and contingency state, but under the parametric system uncertainties described earlier, the bus voltages will also be uncertain. With $\frac{\partial \underline{V}}{\partial \underline{K}_p}$ as the sensitivity matrix of bus voltages with respect to parametric uncertainties, the bus voltage vector is

$$\underline{V} = E(\underline{V} \mid E_i, L) + \left(\frac{\partial \underline{V}}{\partial \underline{K}_p}\right) \times \left(\underline{K}_p - E(\underline{K}_p)\right)$$

from which we obtain a MVN distribution of bus voltages:

$$\underline{V} \sim MVN\left(E(\underline{V} \mid E_i, L),\ \left(\frac{\partial \underline{V}}{\partial \underline{K}_p}\right) V_p \left(\frac{\partial \underline{V}}{\partial \underline{K}_p}\right)^T\right) \quad \text{(eq. 4)}$$

where we again employ the covariance matrix V_P. The expectation of bus voltages $E(\underline{V}|E_i,L)$ is obtained by solving the power flow based on the specified system conditions and the contingency state. Eq. 4 provides a normal probability density for each bus voltage $\Pr(V_j|E_i,L)$, from which we may compute the impact[6] of no-collapse given the loading and contingency state as:

$$\mathrm{Im}(NoCollapse \mid E_i, L) = \sum_{j=1}^{N_{BUSES}} \int_{V_j} Risk_j(LI \mid V_j) \Pr(V_j \mid E_i, L) dV_j \quad \text{(eq. 5)}$$

where $Risk_j(LI \mid V_j)$ is given by eq. 1 and NBUSES is the total number of buses in the region or system being analyzed. Accounting for the exposure to uncertain contingencies and loading levels, we have:

$$\mathrm{Im}(NoCollapse) = \sum_{i=1}^{N_T} \Pr(E_i) \int_L \mathrm{Im}(NoCollapse \mid L, E_i) \Pr(L) dL \quad \text{(eq. 6)}$$

where we use eq. 5 together with the previously described distributions on contingencies and loading.

The impact $\mathrm{Im}(Collapse|X_0)$ is the interruption cost of the entire system's load (see footnote 5, previous page). This, together with eqs. 3 & 6, provides the information necessary to compute the operating point voltage risk per eq. 2.

We provide an illustration of a modified IEEE Reliability Test System (RTS). We have chosen a scenario where 3 contingencies, each one a transmission line outage, result in low voltages. An illustration of the region is shown in Fig. 3 of the companion overview paper and a one-line of the entire system in Fig. 1 of the companion paper on transient instability risk. Fig. 2 shows the P-V curves for each contingency state. The solid line corresponds to the "normal" state, without any contingency. It shows the loadability as 4070 MW if no contingency occurs. With the possibility of contingencies, the P-V curve is typically more restrictive. The most constraining contingency results in a loadability of 3689 MW. Using deterministic procedures, we would employ a safety margin, e.g., 3%, to establish the loading capability, as indicated by the boundary between the shaded and non-shaded areas of Fig. 2.

Under a 1 hour time horizon, we assume that the forecasted expectation of future loading is the same as the current loading, with a standard deviation of 2%. We further assume load sharing factors to have expected values equal to

[6] The impacts expressed by eqs. 5 and 6 are actually expectations on the impact since they include some uncertainty.

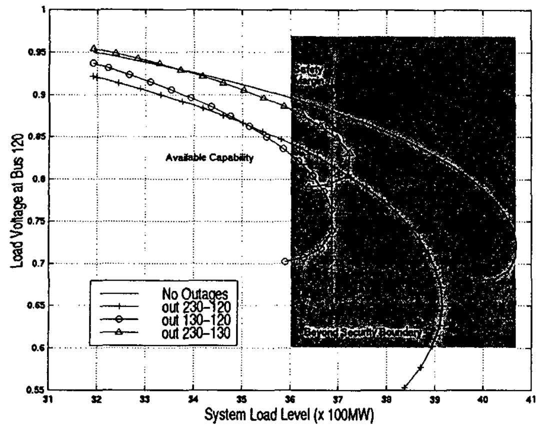

Fig. 2: P-V curves at a load bus

the current ones and standard deviations of 5%, and the three contingencies to have equal probabilities of 4.58E-5 in the next hour[7]. We also use the load interruption risk curve of Fig. 1 for each bus in the system. Fig. 3 shows the expected impact of no collapse for each contingency state, corresponding to eq. 5, and the total, corresponding to eq. 6. Because of the low contingency probability, the expected impact of no collapse is dominated by the no-contingency case, and the two curves are indistinguishable.

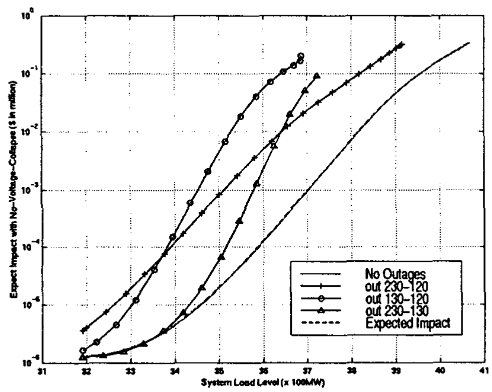

Fig. 3: Expected impact of no collapse

Figure 4 shows the probability of collapse for each contingency state, which indicates Pr(collapse|*Ei*), and also the total probability corresponding to eq. 3. Fig. 5 shows the risk with no collapse, the risk of collapse, and the total risk, corresponding to eq. 2. The shaded region denotes loading between the deterministic boundary of about 3600 MW and a value of 4070 MW, beyond which the voltage will collapse (the no contingency limit). Identifying an acceptable operating condition depends on selecting an acceptable risk [26]. One way to do this is to identify risks that were acceptable in the past. Another way is to use optimization to identify the best tradeoff between economics security. Solution to such a problem would require computation of the marginal risk, as indicated in Fig. 6.

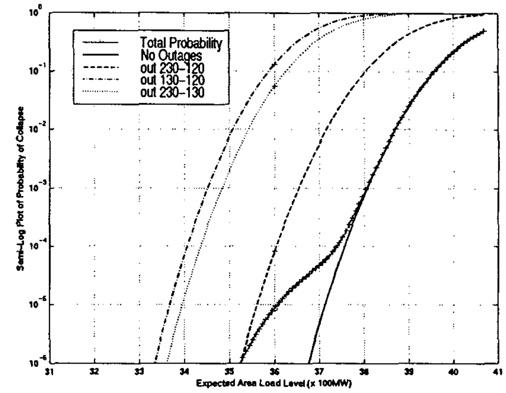

Fig. 4: Probability of Collapse

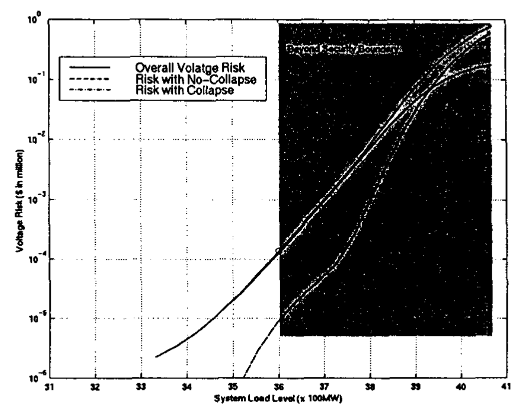

Fig. 5: Final risk plots

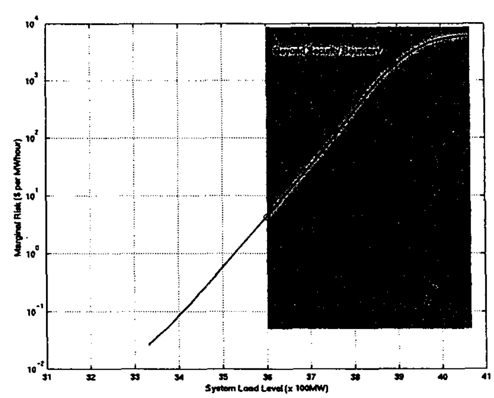

Fig. 6: Marginal risk

4 Cumulative Voltage Risk Assessment

This paper so far has addressed computation of near-term voltage risk given a particular operating condition, by eq. 2, which is useful for operational or operational planning. We now turn attention to facility planning, where we desire to assess the risk of a planning alternative for a certain time period. Most existing planning software for assessing

[7]This probability changes over time depending on weather conditions.

reliability does so by comparing the influence of different facility plans for a limited number of operating conditions, sometimes just one. We call this the "snapshot" approach. Eq. 2 could be used to implement this approach, but we choose to implement a sequential trajectory approach instead [27], whereby we simulate the power system operation hour by hour for the time frame of interest, e.g., 1 year[8]. There are two reasons for our choice. First, planning decisions based on associated results tend to optimize over the entire study period rather than just a few conditions. This is important because decisions made based on results for a few conditions may not be best for other conditions. Second, the sequential trajectory approach is able to capture chronological influences such as load cycles, weather conditions, maintenance strategies, and unit commitment plans, and consequently reflect the dependence of each state on previous states. This is not possible with the snapshot approach.

Denoting the dependence of operating condition on time as Xt and a trajectory over an interval T as Ω, the cumulative voltage risk is

$$
\begin{aligned}
Risk(voltage|\,\Omega) &= \sum_{t=1}^{T} E(\mathrm{Im}(voltage|\,X_t)), \qquad X_t \subset \Omega \quad \forall\, t \\
&= \sum_{t=1}^{T} \Pr(Collapse|\,X_t) \times E\Big(\mathrm{Im}(Collapse|\,X_t)\Big) + \\
&\quad [1.0 - \Pr(Collapse|\,X_t)] \times E\Big(\mathrm{Im}(NoCollapse|\,X_t)\Big) \\
&\quad X_t \subset \Omega \quad \forall\, t
\end{aligned}
$$

Conceptually, this calculation is a simple extension of the system risk calculation in that we merely perform the system risk calculations for each time t and sum them. Practically, there are two significant problems to solve. One is the development of the trajectory Ω so that it reflects actual operating conditions and policies, and the other is performing the calculations within an acceptably short amount of computation time. Reference [27] partially addresses the first problem using a Monte Carlo scheme whereby entire trajectories are repetitively chosen at random for reliability assessment. However, the computational expense of this scheme is excessive. In contrast, we assess only the expected trajectory formed by (1) developing an hour by hour load forecast, (2) identifying and modeling the load forecast error, (3) forming a maintenance schedule for all generation units, and (4) developing a unit commitment plan based on the load forecast and maintenance schedule. Descriptions of each of these steps can be found in [22]. In addition, we assume that Im(*nocollapse*), given by eq. 6, is zero and therefore assess only the impact of collapse.[9] Because collapse only requires identification of loadability, we replace CPF with a zone-based optimization problem to enhance the assessment speed. Define *Ptot,i* as total zone i load and *wi* as the corresponding weight, and solve:

$$\max \sum w_i P_{tot,i}$$

subject to real and reactive power balance equations at each bus, and real and reactive limits at each PV bus, and limits on bus voltages. The weights are used to weight certain zones regarding load increase and to identify how load should be curtailed in case of non-convergence. We have used an interior point algorithm to solve this optimization problem [28]. One attractive feature of the solution is that it is possible to decompose the overall risk according to cause.

Modeling the normal distribution on load at each hour, with forecasted value as mean and a specified standard deviation (2%), identification of the loadability Lmax enables calculation of the expected load curtailment as

$$E(L_c) = \int_{L\max}^{\infty} L \Pr(L)\, dL$$

from which we may obtain a dollar-based risk value.

A 1 year calculation of the IEEE RTS is used to illustrate the approach. Although voltage limits were enforced in the calculation, at 0.85 pu, voltage collapse did occur, and the total 1 year voltage collapse risk was $4,494. Figure 7 shows the hour by hour variation in voltage collapse risk.

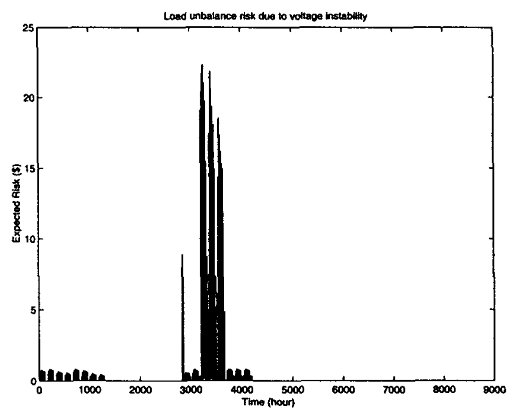

Fig. 7: Hourly voltage collapse risk for 1 year

5 Conclusions

We have developed a unified framework for computing probabilistic risk for voltage problems in power systems. This framework provides assessment of the risk of voltage collapse as well as the risk of voltage deviations. In operations, it is useful in a monitoring role as a leading indicator of the system voltage security. It also serves as a

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[8] We have used a 1 hour time step as it is the longest interval for which the load may be assumed reasonably constant. We have used a 1 year study period vecause of our perception that new facility construction requiring 5+ years to plan and build is no longer common in the industry. Rather, there is tendency to utilize existing transmission where feasible and reject large-scale transmission requests when not. As a result, transmission reinforcements for small-scale transmission requests are most often considered. Typically, they have relatively short completion times, so that a 1 year interval can be appropriate in many cases. When longer simulation time is needed, one can string together multiple 1 year simulations.

[9] We can enforce constraints on bus voltage magnitudes in the optimization and attribute load interruption to them when they are active. However, this assumes a single interruption voltage for all loads and consequently ignores the uncertainty modeled in our component analysis described in Section 2. Nonetheless, we believe there are ways to include the load interruption risk of eq. 1 into cumulative assessment, and we are working on this.

means for assessing security in economic terms, which may be very useful for providing security-related price signals. In planning, we think that use of sequential trajectories is essential in assessing the long term effect of a facility alternative. Continuation power flow (CPF) and optimization are both effective methods to obtaining loadability, each with drawbacks and benefits. Although optimization does not readily yield itself to assessing the risk of voltage deviation, it does offer considerable flexibility in modeling, it is generally faster than CPF, and the shadow prices that result from its solution offer intriguing insight into the effect of different network constraints on voltage risk [22]. In operations and planning, a very attractive feature of the risk assessment, not illustrated in this paper, is that it is possible to obtain a composite index that reflects the security level for overload, voltage, and dynamic security, and multiple contingency states [22]. A paper to illustrate the significance of composite security assessment is in preparation.

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