In this technical report, we present a simple model to calculate the maximum service rate (MSR) of a station.

**A. Calculation of MSR with Two Contending Stations**

We first consider the basic scenario when two and only two stations (s and t) are associated with the same interface and contend for channel access. The transmission rate, the traffic rate and the data payload length of a station are denoted as $r$, $\lambda$ and $L_{data}$, respectively. By definition, MSR of station $s$ is the maximum throughput that $s$ may obtain when it increases $\lambda_s$ to $\infty$ (i.e., becomes saturated), given that the traffic rate of station $t$ ($\lambda_t$) remains unchanged.

Similar to [1]–[3], we use virtual slot to characterize the airtime usage, which is defined as a full transmission cycle including backoff, data transmission time and ACK transmission time, or a single slot time if no station is contending for the channel. Let $q$ denote the probability that a station contends for the channel at the end of a virtual slot. Clearly, when calculating the MSR of station $s$, $q_s$ is always one as $s$ is assumed to be saturated. Therefore, we have approximately the following:

$$
P_s = q_s (1 - q_t) + \frac{1}{2} q_t q_t = 1 - \frac{1}{2} q_t,
$$

$$
P_t = q_t (1 - q_s) + \frac{1}{2} q_s q_t = \frac{1}{2} q_t,
$$

where $P_s$ and $P_t$ are the probability that $s$ and $t$ complete a transmission successfully in a virtual slot. Eq. (1) is based on the assumption that two stations have an equal probability of 0.5 to seize the channel successfully when they both contend in the same virtual slot. In the following, we derive $q_t$ based on a Markov chain model.

As shown in Fig. 1, we define a Markov chain with $N + 1$ states for station $t$, where $N$ is the maximum queue length allowed at $t$. State $i$ ($0 \leq i \leq N$) of the Markov chain represents that $t$’s queue length is $Q_t = i$ at the end of a virtual slot. In this Markov chain, state $i$ can only transit to state $j$ where $j \geq i - 1$, as the station can transmit at most one packet in a virtual slot. In particular, state $i$ transits to state $i - 1$ if $t$ transmits successfully in the virtual slot and there are no new packet arrivals at $t$ during the transmission. The state transition probabilities are:

$$
P_{j|i} = P\{Q_t = j|Q_t = i\}
= \begin{cases} 
\delta(T_s, j), & 0 = i \leq j < N, \\
0, & 1 \leq j + 1 \leq i \leq N, \\
\frac{1}{2} \delta(T_t, i), & 1 \leq j + 1 = i \leq N, \\
\frac{1}{2} \delta(T_t, j - i) + \frac{1}{2} \delta(T_t, j + i), & 1 \leq i \leq j < N, \\
\frac{1}{2} \sum_{k=N-i}^{\infty} \delta(T_s, k) + \frac{1}{2} \sum_{k=N-i+1}^{\infty} \delta(T_t, k), & 0 \leq i \leq j = N,
\end{cases}
$$

where $T$ is the effective transmission duration of a station, including backoff, data transmission time, and ACK transmission time. $\delta(T, n)$ is the probability that $n$ packets arrive at station $t$ during the time interval $T$. For example, in the case of Poisson arrivals, $\delta(T, n) = (\lambda_T)^n e^{-\lambda_T T}$.

The meanings of the transition probabilities are explained next. The first equation in Eq. (2) states that at state 0, $t$ has no packet in its queue. Therefore, it will transit to state $j$ if there are $j$ packet arrivals during station $s$’s effective transmission time $T_s$ since $s$ is saturated and always ready to transmit. The second equation states that state $i$ can never transit to state $j$ if $j < i - 1$. This is because the station can transmit at most one packet in a virtual slot. The third equation means that, in order to transit from state $i$ to state $i - 1$, the following conditions need to be satisfied: (i) $t$ competes with $s$ and seizes the channel.
\[ \text{Pr}(Q = j | Q = i) = P_{ij} \]

0 \leq i \leq N

0 \leq j \leq N

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{markov_chain}
\caption{The Markov chain model for station \( t \), which is competing with a saturated station \( s \).}
\end{figure}

Fig. 1. The Markov chain model for station \( t \), which is competing with a saturated station \( s \).

B. Calculation of MSR with Multiple Contending Stations

Algorithm 2 estimates the MSR of station \( s \) given that a set of stations \( \vec{t} \) contend with \( s \) for the same interface. The basic idea is to pair \( s \) with the first station in \( \vec{t} \), say \( x \), and treat \( s \) and \( x \) as a virtual node with an updated effective transmission duration. Then the virtual node is paired with the second station in \( \vec{t} \). The process goes on until all stations in \( \vec{t} \) have been considered. As we will show next, this simple approximation performs reasonably well in various scenarios.

\begin{algorithm}
\caption{Calculation of MSR of \( s \) given that only a single station \( t \) contends with \( s \).}
\begin{algorithmic}[1]
\State \textbf{INPUT:} \( s \), \( t \) and their statistics;
\State \textbf{OUTPUT:} MSR\((s, t)\);
\State Calculate \( T_s \) and \( T_t \) based on \( r_s, \ L_{data}(s), r_t, L_{data}(s) \);
\State Calculate \( P_s \) and \( P_t \) based on the Markov chain model;
\State \( T_{avg} = P_s T_s + P_t T_t \);
\State MSR\((s, t) = P_s L_{data}(s) / T_{avg} \);
\end{algorithmic}
\end{algorithm}

\begin{algorithm}
\caption{Calculation of MSR, given that a set of stations \( \vec{t} \) contend with \( s \).}
\begin{algorithmic}[1]
\State \textbf{INPUT:} \( s, \vec{t} \) and their statistics;
\State \textbf{OUTPUT:} MSR\((s, \vec{t})\);
\For {each \( x \) in \( \vec{t} \) do}
\State Run Algorithm 1 to calculate MSR\(_{imp}\) = MSR\((s, x)\);
\State // Treat \( s \) and \( x \) as a virtual node with an updated \( T_s \)
\State Update \( T_s = L_{data}(s) / MSR(s, x) \);
\State Update \( \vec{t} = \vec{t} \setminus \{s\} \);
\EndFor
\State MSR\((s, \vec{t}) = MSR_{imp} \);
\end{algorithmic}
\end{algorithm}

C. Validation of the Model and Estimation of \( G \)

We use simulation results to validate the Markov chain model and the algorithms for calculating the MSR. In the first simulation, there are two stations in the network. STA\(_1\) has a PHY rate of 54 Mbps and STA\(_2\) has a PHY rate of 6 Mbps. We fix the traffic rate of STA\(_2\) to be 200 packets/s, where each packet carries 1500 bytes of payload. We increase the traffic rate of STA\(_1\) gradually. The throughput of two stations are plotted in Fig. 2(a). In this figure, we observe that the throughput of STA\(_1\) first increases almost linearly with the traffic rate, and then keeps almost constant around 14 Mbps when the traffic successfully in the virtual slot, with a probability of 0.5; and (ii) no new packets arrive during \( T_t \). The fourth equation states that a transition from state \( i \) to \( j \) (\( j \geq i \)) occurs when either (i) \( s \) seizes the channel and there are \( j - i \) packet arrivals during \( T_s \); or (ii) \( t \) seizes the channel and transmits one packet, and there are \( j - i + 1 \) packet arrivals during \( T_t \). The fifth equation describes a special case caused by the buffer size of \( N \). That is, if \( Q_t > N \) at the end of a virtual slot, \( t \) simply drops the extra packets and the Markov chain stays at state \( N \).

With the transition probabilities of the Markov chain defined, we can calculate the steady state probability of state \( 0 \), i.e., \( \pi_0 \). Then, by definition, we have \( q_t = P(\{Q_t \neq 0\}) = 1 - \pi_0 \); plugging it back to Eq. (1), \( P_s \) and \( P_t \) can then be derived. With \( P_s \) and \( P_t \), MSR of station \( s \) can be calculated with Algorithm 1.

Algorithm 1 Calculation of MSR of \( s \) given that only a single station \( t \) contends with \( s \).

Algorithm 2 Calculation of MSR, given that a set of stations \( \vec{t} \) contend with \( s \).

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rate is high. In other words, the MSR of STA_1 is 14 Mbps. Based on this observation, we approximate the throughput of a station as follows:

$$G = \min\{\text{MSR}, \lambda\}. \quad (3)$$

In comparison, our model gives an MSR of 13.83 Mbps, which is very close to the simulation result.

In the second simulation, we compare the simulated and analytical MSR results in a random setup. In this setup, we vary the number of competing stations, and each station chooses its PHY rate as well as traffic rate randomly. We plot the difference between the simulated and analyzed MSR results in Fig. 2(b). It shows that with a single competing station, our modeling scheme has an estimation error of about 2%. With 16 competing stations in the network, our model yields an estimation error of around 10% which is an acceptable tradeoff considering the simplicity of our model and low computational complexity of the algorithms.

REFERENCES

