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Multi-Round Sensor Deployment for Guaranteed Barrier Coverage

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***Abstract*—Deploying wireless sensor networks to provide guaranteed barrier coverage is critical for many sensor networks applications such as intrusion detection and border surveillance. To reduce the number of sensors needed to provide guaranteed barrier coverage, we propose *multi-round sensor deployment* which splits sensor deployment into multiple rounds and can better deal with placement errors that often accompany sensor deployment. We conduct a comprehensive analytical study on multi-round sensor deployment and identify the tradeoff between the number of sensors deployed in each round of multi-round sensor deployment and the barrier coverage performance. Both numerical and simulation studies show that, by simply splitting sensor deployment into two rounds, guaranteed barrier coverage can be achieved with significantly less sensors comparing to single-round sensor deployment. Moreover, we propose two practical solutions for multi-round sensor deployment when the distribution of a sensor's residence point is not fully known. The effectiveness of the proposed multi-round sensor deployment strategies is demonstrated by numerical and simulation results.**

I. Introduction

Recently, barrier coverage [1] with wireless sensor networks has received great attention. The goal is to deploy a chain of wireless sensors in the barrier, which usually is a long belt region, to prevent mobile objects from crossing the barrier undetected. Applications of barrier coverage include intrusion detection and border surveillance [2]. In this paper, we study the problem of *guaranteed barrier coverage* and the goal is to guarantee that a barrier is covered with probability one using as few sensors as possible.

Various schemes have been proposed to reduce the number of sensors needed to cover a barrier, such as information barrier coverage which exploits the collaboration and information fusion between nearby sensors [3]. We approach this problem from a different angle via investigating sensor deployment strategies, more specifically, line-based sensor deployment strategies. With line-based sensor deployment, sensors are deployed along a line, e.g., sensors are airdropped by an aircraft along a deployment line, and the final residence points of the sensors are distributed along the line with random offsets. Recent work in [4] shows that this is a more realistic sensor placement model and sensor deployment strategies based on this model can achieve more efficient barrier coverage with less sensors than those based on the Poisson point process sensor placement model.

In practice, due to wind and other environmental factors, large placement errors often accompany sensor deployment. For example, when sensors are airdropped into the target region from an aircraft, the final residence points of the sensors may deviate much from the intended deployment points. To provide guaranteed barrier coverage with the conventional single-round sensor deployment, sensors need to be deployed in a conservative manner (i.e., with small deployment interval and hence more sensors are needed) to counter the randomness during the sensor deployment process. In comparison, by splitting sensor deployment into multiple rounds, it is safe to be more aggressive (i.e., with larger deployment intervals) in the earlier rounds and then deploy sensors more conservatively in the final round to fill the coverage gaps generated in the previous rounds. As a result, a significant number of sensors may be saved with multi-round sensor deployment.

Splitting sensor deployment into multiple rounds may incur higher deployment cost, e.g., an aircraft needs to fly along the deployment line multiple times to accomplish the task. From the analytical and simulation studies, we have an interesting discovery that the optimal two-round sensor deployment strategy yields the same barrier coverage performance as other optimal strategies with more than two rounds. This result is particularly encouraging as it implies that the best barrier coverage performance can be achieved with low extra deployment cost by deploying sensors in two rounds.

Furthermore, we propose two practical solutions, the *two-round minimax solution* and the *pilot deployment solution*, to deal with realistic situations when the knowledge about the deviation of sensors' residence points with respect to their intended deployment points is not fully available. The pilot deployment solution performs particularly well and the idea is to introduce an additional pilot round which deploys a small number of sensors to estimate the distribution of sensors' residence points and then use this information to aid the following rounds of sensor deployment.

The rest of the paper is organized as follows. We discuss the related work in Section II. Then, we give the system models and the problem statement in Section III. In Section IV, we analyze multi-round sensor deployment in detail. Section V describes the two-round minimax solution and the pilot deployment solution. Numerical and simulation results are shown in Section VI. Section VII discusses future work and related issues, and Section VIII concludes the paper.

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II. Related Work

The area coverage problem based on uniform and random distribution of sensor deployment has been well-studied in the past [5]–[9]. In these works, a sensor's landing position is assumed to be uniformly and randomly distributed over the monitored area. Controlled sensor deployment for target detection has been studied in [10]–[16]. In [10]–[14], a sensor's position can be arbitrarily controlled, i.e., the placement of each deployed sensor is not subject to any placement error. Based on this assumption, various schemes have been proposed to determine the optimal sensor placement strategy to cover the target region with as few sensors as possible. By comparison, in [15], [16], sensor deployment is partially controlled, i.e., a sensor's position is subject to random offsets with respect to the deployment point.

Sensor mobility has been incorporated into sensor deployment framework [17]–[19], which offers more flexibility for designing more efficient sensor deployment strategies for area coverage. Incremental sensor deployment has been studied in [20], [21], where algorithms are designed to deploy extra sensors after the initial deployment to improve the area coverage performance. The problems of sensor deployment for other purposes, e.g., connectivity, data aggregation, or energy efficiency, have also been investigated [22]–[26]. None of the above works addresses barrier coverage.

Recently, barrier coverage [1], [5] has attracted great attention. [4] studies the line-based sensor deployment strategies and provides both analytical results and interesting observations about how line-based sensor deployment can improve the barrier coverage performance than two-dimensional uniform sensor deployment. Theoretical foundations for weak and strong barrier coverage of a randomly deployed sensor network are studied in [1], [27]. The authors of [28] and [29] study the coverage of a finite-size barrier and provide analytical methods to estimate the required density of deployed sensors for achieving barrier coverage or measuring the quality of barrier coverage. Centralized and distributed algorithms for providing barrier coverage are proposed and evaluated in [30] and [31]–[33], respectively.

III. Models and Problem Statement

A. Sensing and Coverage Models

We consider a network of wireless sensors deployed to monitor a barrier which is a long belt region of length ℓ with two parallel sides: entrance side and destination side (as shown in Fig. 1). An object (or intruder) may cross the barrier via an intruding path starting at the entrance side and ending at the destination side. We assume that sensors are aware of their locations. We also assume that sensors' communication range is reasonably large so that network is connected and sensors can report their location information to the sink. For simplicity, we assume an ideal 0/1 disc sensing model to demonstrate how a well-designed multi-round sensor deployment scheme may help reduce the number of sensors needed to guarantee barrier coverage. Specifically, we assume that each sensor has a sensing disc with a radius of R_s. An object within (outside) a sensor's sensing disc is detected by the sensor with probability one (zero). In Section VII, we will discuss how to extend our work under other sensing models.

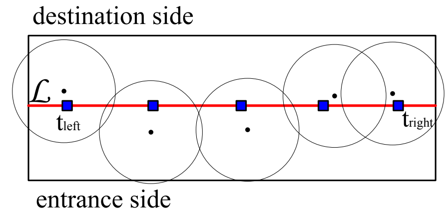


Fig. 1. Illustration of barrier coverage and sensor deployment. Sensors are deployed along the deployment line $\mathcal{L}$ which is parallel to the destination side of the barrier. Deployment points are along $\mathcal{L}$ and shown as small squares. Residence points of the sensors are shown as small dots and they deviate from the deployment points. These sensors form a barrier coverage set because no intruding path can cross the barrier without being sensed by one of the sensors. Two special points, t_{left} and t_{right}, will be explained in Theorem 2.

***Definition* 1 (Coverage of a Path):** *A path across a barrier is covered if there exists a point along the path that is within at least one sensor's sensing disc.*

***Definition* 2 (Barrier Coverage):** *A barrier is covered if all possible paths across the barrier are covered. This is also known as strong barrier coverage.*

***Definition* 3 (Barrier Coverage Set):** *A barrier coverage set is a set of sensors that can together cover the barrier.*

***Definition* 4 (Linkage):** *We say that two sensors a and b have linkage if there exist a series of line segments connecting a and b, and every point on the line segments is within at least one sensor's sensing disc. Particularly, we say that a and b have direct linkage if every point on the line segment connecting them is within the sensing disc of either a or b.*

B. Deployment Model

We assume that sensors are static once they are deployed. We assume that sensors are deployed along a *deployment line* ($\mathcal{L}$) which is parallel to the destination side of the barrier. The assumption of a straight deployment line is due to practical considerations. For example, when using an aircraft to airdrop sensors, it is more plausible that the aircraft flies along a straight line rather than a zigzag route.

We define *deployment point* as the point location where a sensor is to be deployed. In practice, due to environmental factors and terrain characteristics, the deployment point likely is not the location where the sensor finally resides. The sensor may reside at points around the deployment point according to a certain probability density function (pdf). We define *residence point* as the point location where a sensor finally resides. Fig. 1 shows an example of deployment line, deployment points and residence points.

Let (x_i, y_i) denote the coordinates of the deployment point t_i. Assume that the residence point of a sensor (whose deployment point is t_i) follows the pdf $f(x_i', y_i'|t_i) = f_{\text{err}}(x' - x_i, y_i' - y_i)$. We assume that f_{err} has a finite closed support which is a disc with a radius of R_{err} and circular-symmetric with respect to (x_i, y_i). An example of f_{err} is a truncated two-dimensional Gaussian distribution. For the clarity of presentation, we put the deployment line $\mathcal{L}$ along the X-axis of a Cartesian coordinate system with its left end sitting at the origin. Thus, for each deployment point t_i, we have $y_i = 0$.

***Theorem** 1: If R_{err} is smaller than the sensing radius R_s, two sensors are guaranteed (with probability one) to have direct linkage between them when their deployment points are less than $(2R_s - 2R_{err})$ apart. On the other hand, if $R_{err} \geqslant R_s$, direct linkage between two sensors cannot be guaranteed regardless of the distance between their deployment points.*

The proof of the theorem is straightforward and omitted due to space limitation. It is interesting to see that, if f_{err} does not have a finite closed support (i.e., $R_{\text{err}} = \infty$), direct linkage between sensors cannot be guaranteed and, consequently, guaranteed barrier coverage cannot be achieved. Similarly, we have the following theorem.

***Theorem** 2: If $R_{err} < R_s$, a sensor is guaranteed to cover the left (right) boundary of the barrier if its deployment point is t_{left} (t_{right}), which is $(R_s - R_{err})$ away from the left (right) end of the deployment line. t_{left} and t_{right} are shown in Fig. 1. If $R_{err} \geqslant R_s$, such guarantee cannot be achieved.*

C. Problem Statement

Given that sensors can be deployed in multiple ($\geqslant$2) rounds, our goal is to design a proper multi-round sensor deployment scheme to provide guaranteed barrier coverage with as few sensors as possible. More specifically, we aim to strategically decide the deployment points of sensors at each round so that the total number of sensors needed to form a barrier coverage set is minimized. In Section VII, we will discuss how to extend our work to achieve guaranteed k-barrier coverage.

IV. Multi-Round Sensor Deployment

In this section, we first study the two-round sensor deployment problem, where sensors are deployed in two rounds to cover a barrier. Then, we extend our analysis and solution to the general m-round ($m > 2$) sensor deployment problem. We assume that the pdf of a sensor's residence point with respect to its deployment point (f_{err}) is known *a priori*. In Section V, we will loose this assumption and study the problem under practical considerations where f_{err} is only partially known.

A. Overview of Two-Round Sensor Deployment

There are infinite number of ways to deploy sensors in two rounds to cover a barrier. For example, we may choose to deploy different numbers of sensors in each round, and there are infinite number of possible deployment points to choose for each sensor. In general, it is difficult to enumerate all the possibilities. In this paper, we study a specific set of two-round sensor deployment strategies described below.

***Definition** 5 (Gap Distance): Gap distance (denoted as h) between two adjacent sensors is defined as the distance between the leftmost and rightmost deployment points of the sensors deployed to fill the gap between them.*

***Definition** 6 (Two-Round Sensor Deployment Strategy): We consider a special set of two-round sensor deployment strategies that distribute sensors* evenly *in each gap. Specifically, it operates as follows:*

- In the first round, the entire barrier is a single big gap. According to Theorem 2, the leftmost and rightmost deployment points of the sensors deployed to fill the gap must be t_{left} and t_{right}, respectively. We use a special symbol h_ℓ to denote the corresponding gap distance which is $h_\ell = \ell - 2(R_s - R_{\text{err}})$. Let $N(h_\ell, 2, 1)$ denote the number of sensors deployed in the first round. If $N(h_\ell, 2, 1) = 1$, either t_{left} or t_{right} shall be selected as the deployment point. If $N(h_\ell, 2, 1) \geqslant 2$, the deployment points shall be evenly distributed between t_{left} and t_{right} with a distance of $I(h_\ell, 2, 1) = \frac{h_\ell}{N(h_\ell,2,1)-1}$ apart.
- There are three possible outcomes after the first round of deployment:
  - If $N(h_\ell, 2, 1) = 0$, since no sensors are deployed in the first round, the entire barrier needs to be filled in the second round. This in fact is equivalent to single-round sensor deployment.
  - If $N(h_\ell, 2, 1) = 1$, only a single gap is generated to the right (left) of the deployed sensor if the deployment point of the sensor is at t_{left} (t_{right}). This is because the sensor with deployment point at t_{left} (t_{right}) is guaranteed to cover the left (right) boundary of the barrier.
  - If $N(h_\ell, 2, 1) \geqslant 2$, multiple gaps may be generated.
- In the second round, for each gap generated in the first round, deploy a minimum number of evenly distributed sensors to guarantee that it is covered.

Note that the key assumption in this type of strategies is that sensors are evenly distributed in each gap while different deployment intervals may be used to deploy sensors in different gaps. We choose this specific strategy because it is quite common in realistic scenarios.

Let $\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$ denote the expected total number of sensors needed to cover the barrier when a two-round sensor deployment strategy described above is used. Clearly, when $N(h_\ell, 2, 1)$ is small, fewer sensors are deployed in the first round but larger coverage gaps may be generated. On the other hand, when $N(h_\ell, 2, 1)$ is large, more sensors are deployed in the first round but smaller gaps are to be covered in the second round. So there is a tradeoff. Our goal is to find the optimal $N(h_\ell, 2, 1)$ so that $\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$ is minimized:

$$N^*(h_\ell, 2, 1) = \arg \min_{N(h_\ell,2,1)} \bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1)) \\ \Longrightarrow \bar{\mathcal{N}}^*_{\text{total}}(h_\ell, 2) = \bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N^*(h_\ell, 2, 1)). \quad (1)$$

B. Theoretical Analysis of Two-Round Sensor Deployment

We now investigate the relation between $N(h_\ell, 2, 1)$ and $\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$. We first study the case when $N(h_\ell, 2, 1) \geqslant 2$. Consider two adjacent sensors a and b deployed in the first round, as shown in Fig. 2. Let s_a and s_b denote their residence points with coordinates of (x'_a, y'_a) and (x'_b, y'_b), respectively, and there may be a coverage gap between them. To fill the gap, additional sensors need to be deployed in the second round. According to Theorem 1, the leftmost and

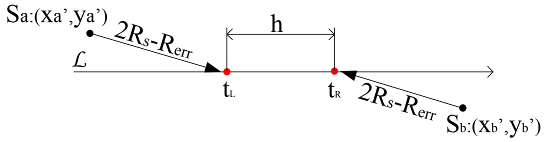


Fig. 2. Two adjacent sensors a and b are deployed in the first round and there may be a coverage gap between them. t_L and t_R are the leftmost and rightmost deployment points of the sensors that are deployed in the second round to fill the gap.

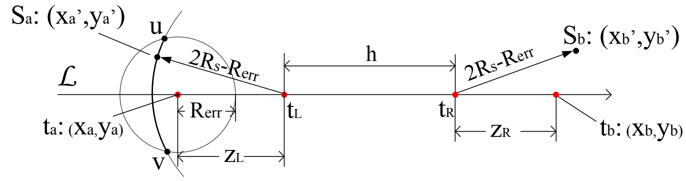


Fig. 4. When the number of sensors deployed in the first round is small, some big coverage gaps (with $h > 0$) may be generated and we need to deploy more than one sensors in the second round to cover each of them.

rightmost deployment points (denoted as t_L and t_R) of these additional sensors must be at a distance of $(2R_s - R_{\text{err}})$ to s_a and s_b, respectively. The distance between t_L and t_R is the corresponding gap distance (denoted as h). Note that, when the left or right boundary of a gap is the left or right boundary of the barrier, $t_L \equiv t_{\text{left}}$ or $t_R \equiv t_{\text{right}}$.

We analyze the following terms: (i) $P_{\text{zero}}(I(h_\ell, 2, 1))$ – the probability that there is no coverage gap between sensors a and b, i.e., there exists a direct linkage between them; (ii) $f_{H(I(h_\ell,2,1))}(h)$ – the pdf of the gap distance between a and b; (iii) $P_{\text{one}}(I(h_\ell, 2, 1))$ – the probability that the gap between a and b can be filled by deploying at most one additional sensor in the second round.

1) $P_{zero}(I(h_\ell, 2, 1))$: Let t_a and t_b denote the deployment points of sensors a and b, and the coordinates of t_a and t_b are $(x_a, 0)$ and $(x_a + I(h_\ell, 2, 1), 0)$, respectively. The residence points s_a and s_b are independently and identically distributed according to the pdf of $f_{\text{err}}(x'_a - x_a, y'_a)$ and $f_{\text{err}}(x'_b - (x_a + I(h_\ell, 2, 1)), y'_b)$, respectively. To guarantee a direct linkage between two sensors, s_b must be within the circle centered at s_a with a radius of $2R_s$, as shown in Fig. 3. Let $\mathcal{A}_1(x'_a, y'_a, I(h_\ell, 2, 1))$ denote the intersection region between the circle centered at s_a with a radius of $2R_s$ and the circle centered at t_b with a radius of R_{err}, which is shown as the shaded area in the figure. Then we have:

$$P_{\text{zero}}(I(h_\ell, 2, 1)) = \iint f_{\text{err}}\left(x'_a - x_a, y'_a\right) \cdot \left[\iint\limits_{\mathcal{A}_1(x'_a, y'_a, I(h_\ell,2,1))} f_{\text{err}}\left(x'_b - (x_a + I(h_\ell, 2, 1)), y'_b\right)\, dx'_b\, dy'_b\right] dx'_a\, dy'_a. \tag{2}$$

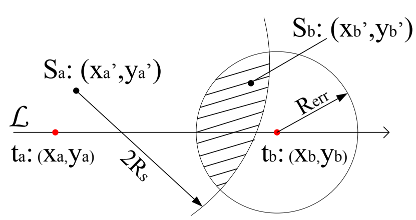


Fig. 3. To guarantee a direct linkage between two adjacent sensors a and b deployed in the first round, s_b must be inside the shaded area. t_a and t_b are the deployment points, and s_a and s_b are the residence points.

2) $f_{H(I(h_\ell,2,1))}(h)$: As shown in Fig. 4, t_R is the intersection point of the deployment line ($\mathcal{L}$) and the circle centered at s_b with a radius of $(2R_s - R_{\text{err}})$. Let z_L and z_R denote the distance between t_a and t_L, and between t_b and t_R, respectively. Then we have $h = I(h_\ell, 2, 1) - z_L - z_R$. Draw a circle with t_L at the center and a radius of $(2R_s - R_{\text{err}})$. Let u and v denote the intersection points of this circle and the circle centered at t_a with a radius of R_{err}. Note that t_L remains the same as long as s_a falls on the arc $\widehat{uv}$. Therefore, the pdf of z_L is a line integral:

$$f_Z(z_L) = \iint\limits_{(x'_a, y'_a) \in \widehat{uv}(z_L)} f_{\text{err}}\left(x'_a - x_a, y'_a\right)\, dx'_a\, dy'_a. \tag{3}$$

Since the relation between t_b and t_R is the same as that between t_a and t_L, z_R and z_L have the same pdf. Therefore, the pdf of h is

$$\begin{aligned} f_{H(I(h_\ell,2,1))}(h) &= f_{H(I(h_\ell,2,1))}(I(h_\ell, 2, 1) - z_L - z_R) \\ &= \int_{-\infty}^{\infty} f_Z(z) \cdot f_Z(I(h_\ell, 2, 1) - h - z)\, dz. \end{aligned} \tag{4}$$

3) $P_{one}(I(h_\ell, 2, 1))$: Based on the result of $f_{H(I(h_\ell,2,1))}(h)$, we can obtain $P_{\text{one}}(I(h_\ell, 2, 1))$ without much difficulty. We know that, when $h \leqslant 0$, a linkage between them can be guaranteed by deploying at most one additional sensor at either t_L or t_R. On the other hand, such guarantee cannot be achieved if $h > 0$. Therefore, we have:

$$P_{\text{one}}(I(h_\ell, 2, 1)) = \int_{-\infty}^{0} f_{H(I(h_\ell,2,1))}(h)\, dh. \tag{5}$$

Note that $(P_{\text{one}}(I(h_\ell, 2, 1)) - P_{\text{zero}}(I(h_\ell, 2, 1)))$ is the probability that there exists a coverage gap between two adjacent sensors deployed in first round and the gap can be filled by deploying exactly one additional sensor in the second round.

4) Calculation of $\bar{N}_{total}(h_\ell, 2, N(h_\ell, 2, 1)$: Now we are ready to analyze the relation between $\bar{N}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1)$ and $N(h_\ell, 2, 1)$. Let $\bar{N}_2(I(h_\ell, 2, 1))$ denote the average number of additional sensors needed to guarantee a linkage between two adjacent sensors, say a and b, deployed in the first round. We have the following cases:

- When $h \leqslant 0$, we need at most one additional sensor to guarantee a linkage between a and b. There are two subcases:
  - with probability $P_{\text{zero}}(I(h_\ell, 2, 1))$, there already is a direct linkage between a and b and no additional sensor is needed;
  - with probability $P_{\text{one}}(I(h_\ell, 2, 1)) - P_{\text{zero}}(I(h_\ell, 2, 1))$, one additional sensor is needed.
- When $h > 0$, we need at least $\left(1 + \left\lceil \frac{h}{2(R_s - R_{\text{err}})} \right\rceil\right)$ additional sensors to guarantee a linkage between a and b.

Therefore, we have:

$$\begin{aligned}\bar{N}_2(I(h_\ell,2,1)) &= P_{\text{zero}}(I(h_\ell,2,1))\cdot 0\\ &\quad + (P_{\text{one}}(I(h_\ell,2,1)) - P_{\text{zero}}(I(h_\ell,2,1)))\cdot 1\\ &\quad + \int_0^\infty \left(1+\left\lceil \frac{h}{2(R_s-R_{\text{err}})}\right\rceil\right) f_{H(I(h_\ell,2,1))}(h)\,dh\\ &= P_{\text{one}}(I(h_\ell,2,1)) - P_{\text{zero}}(I(h_\ell,2,1)) + (1-P_{\text{one}}(I(h_\ell,2,1)))\\ &\quad + \int_0^\infty \left\lceil \frac{h}{2(R_s-R_{\text{err}})}\right\rceil f_{H(I(h_\ell,2,1))}(h)\,dh\\ &= 1 - P_{\text{zero}}(I(h_\ell,2,1)) + \int_0^\infty \left\lceil \frac{h}{2(R_s-R_{\text{err}})}\right\rceil f_{H(I(h_\ell,2,1))}(h)\,dh. \end{aligned} \tag{6}$$

Finally, $\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell,2,1))$ can be calculated by:

$$\bar{\mathcal{N}}_{\text{total}}\Big(h_\ell,2,N(h_\ell,2,1)\Big) = N(h_\ell,2,1) + (N(h_\ell,2,1)-1)\cdot \bar{N}_2\left(I(h_\ell,2,1)\right). \tag{7}$$

Now let's consider the case when $N(h_\ell,2,1)=1$. Without loss of generality, assume the sensor is deployed at t_{left} and a single coverage gap is generated to the right of the deployed sensor, as shown in Fig. 5. According to Theorem 2, to guarantee this gap is filled, the rightmost deployment point for the additional sensors deployed in the second round must be at t_{right}. So the gap distance $h = h_\ell - z_L$. Since z_L has the same pdf as (3) and all other deployment points are evenly distributed between t_L and t_{right}, we can calculate $\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell,2,1))$ as:

$$\begin{aligned}\bar{\mathcal{N}}_{\text{total}}\Big(h_\ell,2,N(h_\ell,2,1)\Big) &= 1+\int_0^\infty \left(\left\lceil \frac{h}{2(R_s-R_{\text{err}})}\right\rceil +1\right)\cdot f_Z(z_L)\,dz_L\\ &= 1+\int_0^\infty \left(\left\lceil \frac{h_\ell - z_L}{2(R_s-R_{\text{err}})}\right\rceil +1\right)\cdot f_Z(z_L)\,dz_L. \end{aligned} \tag{8}$$

For the case when $N(h_\ell,2,1)=0$, the entire barrier needs to be covered in the second round. Hence, we have

$$\bar{\mathcal{N}}_{\text{total}}\Big(h_\ell,2,N(h_\ell,2,1)\Big) = \left\lceil \frac{h_\ell}{2(R_s-R_{\text{err}})}\right\rceil + 1. \tag{9}$$

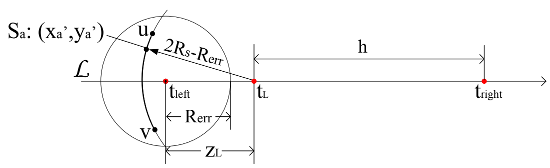


Fig. 5. When $N(h_\ell,2,1)=1$, a single coverage gap is generated after the first round. Without loss of generality, assume the sensor is deployed at t_{left} and a single coverage gap is generated to the right of the deployed sensor. In the second round, the leftmost and rightmost deployment points to fill the gap are at t_L and t_{right}, respectively.

To summarize, we have the following result:

$$\bar{\mathcal{N}}_{\text{total}}\Big(h_\ell,2,N(h_\ell,2,1)\Big) = \begin{cases} \left\lceil \frac{h_\ell}{2(R_s-R_{\text{err}})}\right\rceil + 1, & \text{if } N(h_\ell,2,1)=0,\\ 1+\int_0^\infty \left(\left\lceil \frac{h_\ell - z_L}{2(R_s-R_{\text{err}})}\right\rceil +1\right)\cdot f_Z(z_L)\,dz_L, & \text{if } N(h_\ell,2,1)=1,\\ N(h_\ell,2,1) + (N(h_\ell,2,1)-1)\cdot \bar{N}_2(I(h_\ell,2,1)), & \text{if } N(h_\ell,2,1)\geqslant 2. \end{cases} \tag{10}$$

By plugging (10) back into (1), we have completed the definition of the optimization problem for two-round sensor deployment, which can be solved without much difficulty using numeric methods. This is because the search space is limited: $N(h_\ell,2,1)$ is an integer and the number of possible choices for $N(h_\ell,2,1)$ is finite.

C. Multi-Round Sensor Deployment

Now we study the general multi-round sensor deployment problem where sensors are deployed in $m>2$ rounds to cover a barrier. Similar to the two-round sensor deployment strategy described in Section IV, we assume even distribution of sensors in each gap. The goal is to, at each of the m rounds (say, round j where $1\leqslant j\leqslant m$) and for each coverage gap with a gap distance of h, determine the optimal number of sensors to be deployed inside the gap, denoted as $N^*(h,m,j)$, so that the barrier can be covered with fewest sensors. We achieve this goal by using a recursive algorithm which is described below.

Firstly, consider the general case when $1\leqslant i<m$. Suppose that, at round i, $N(h,m,i)$ sensors are deployed inside the gap which may shatter the gap into multiple smaller gaps. Assume that each newly generated smaller gap will be filled with the optimal strategy in the next round, i.e., round $(i+1)$. Then, the expected total number of sensors needed to fill the gap in $(m-i+1)$ rounds, i.e., from round i to round m, can be calculated as:

$$\bar{\mathcal{N}}_{\text{total}}\Big(h,m-i+1,N(h,m,i)\Big) = \begin{cases} \bar{\mathcal{N}}^*_{\text{total}}(h,m-i), & \text{if } N(h,m,i)=0,\\ 1+\int_0^\infty \bar{\mathcal{N}}^*_{\text{total}}(h-z_L,m-i)\cdot f_Z(z_L)\cdot dz_L, & \text{if } N(h,m,i)=1,\\ N(h,m,i)+[N(h,m,i)-1]\cdot\left[P_{\text{zero}}\left(I(h,m,i)\right) - P_{\text{zero}}\left(I(h,m,i)\right)\right] & \\ \quad +[N(h,m,i)-1]\cdot\left[\int_0^\infty \bar{\mathcal{N}}^*_{\text{total}}(h',m-i)\cdot f_{H(I(h,m,i))}(h')\cdot dh'\right], & \text{if } N(h,m,i)\geqslant 2. \end{cases} \tag{11}$$

where

$$I(h,m,i) = \frac{h}{N(h,m,i)-1}. \tag{12}$$

$f_{H(I(h,m,i))}(\cdot)$ and $f_Z(\cdot)$ are given in (4) and (3), respectively. $f_{H(I(h,m,i))}(\cdot)$ is the pdf of the gap distances between adjacent sensors deployed at round i when $N(h,m,i)\geqslant 2$. Moreover, $P_{\text{zero}}(I(h,m,i))$ and $P_{\text{one}}(I(h,m,i))$ are the probability that there is no coverage gap between adjacent sensors deployed at round i and the probability that the coverage gap between adjacent sensors deployed at round i can be filled by deploying at most one additional sensor in the next round, respectively, and they are given in (2) and (5). Then we have:

$$N^*(h,m,i) = \arg\min_{N(h,m,i)} \bar{\mathcal{N}}_{\text{total}}\Big(h,m-i+1,N(h,m,i)\Big), \tag{13}$$

and

$$\bar{\mathcal{N}}^*_{\text{total}}\Big(h,m-i+1\Big) = \bar{\mathcal{N}}_{\text{total}}\Big(h,m-i+1,N^*(h,m,i)\Big). \tag{14}$$

Now consider the special case when $i=m$, i.e., during the final round of sensor deployment. To guarantee that all gaps are filled, according to Theorem 1, we have:

$$N^*(h,m,m) = \left\lceil \frac{h}{2(R_s-R_{\text{err}})}\right\rceil + 1. \tag{15}$$

So, by using this special case as the boundary condition, we have fully specified how to obtain the optimal multi-round sensor deployment strategy that uses fewest sensors to cover a barrier by (13), (14), (11), (12) and (15).

Remarks: As will be shown in Section VI-B, there is an interesting discovery that optimal m-round ($m \geqslant 2$) sensor deployment strategies all yield the same barrier coverage performance regardless of m. In other words, the best barrier coverage performance can be achieved with the optimal two-round sensor deployment. For this reason, we only study two-round sensor deployment in the practical considerations next.

V. Practical Considerations

In practice, the distribution of a sensor's residence point, i.e., the f_{err} function, may not be fully known. For example, if we assume that f_{err} is a truncated two-dimensional Gaussian function with respect to a deployment point at $(x, y = 0)$, as described in (16) below, the parameter σ_{real} may not be known.

$$f_{\text{err}}(x'-x, y') = \frac{1}{A} \cdot I_{(x'-x)^2+y'^2 \leqslant 9\sigma_{\text{real}}^2} \cdot e^{-\left(\frac{(x'-x)^2}{2\sigma_{\text{real}}^2}+\frac{y'^2}{2\sigma_{\text{real}}^2}\right)}, \quad (16)$$

where

$$A = \iint_{(x'-x)^2+y'^2 \leqslant 9\sigma_{\text{real}}^2} e^{-\left(\frac{(x'-x)^2}{2\sigma_{\text{real}}^2}+\frac{y'^2}{2\sigma_{\text{real}}^2}\right)} dx'\, dy', \quad (17)$$

and

$$I_{(x'-x)^2+y'^2 \leqslant 9\sigma_{\text{real}}^2} = \begin{cases} 1, & \text{if } (x'-x)^2 + y'^2 \leqslant 9\sigma_{\text{real}}^2, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Instead, we may only know the range of σ_{real} based on the historical information, i.e., $\sigma_{\text{real}} \in [\sigma_{\min}, \sigma_{\max}]$. We assume that $R_{\text{err}} = 3\sigma_{\max}$ is smaller than R_s; otherwise, barrier coverage cannot be guaranteed according to Theorem 1. In this section, we present two practical solutions to deal with this situation. Note that the proposed solutions can be extended to other f_{err} functions as long as the requirements on f_{err} specified in Section III-B, i.e., circular symmetry and finite closed support, are satisfied.

A. The Two-Round Minimax Solution

The two-round minimax solution works as follows:

- Let $\bar{N}^{\sigma}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$ denote the expected total number of sensors needed to cover the barrier when (i) $N(h_\ell, 2, 1)$ sensors are deployed in the first round; and (ii) sensors are deployed in the second round by setting $R_{\text{err}} = 3\sigma_{\max}$ to guarantee filling of all coverage gaps.
- Recall that $\bar{N}^{*}_{\text{total}}(h_\ell, 2) = \bar{N}_{\text{total}}(h_\ell, 2, N^*(h_\ell, 2, 1))$ is the minimum expected total number of sensors needed to cover the barrier when σ_{real} is known. The difference between $\bar{N}^{\sigma}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$ and $\bar{N}^{*}_{\text{total}}(h_\ell, 2)$ is then the number of extra sensors deployed due to the lack of knowledge about σ_{real}, and it varies with $N(h_\ell, 2, 1)$.
- With the two-round minimax solution, the following number of sensors are deployed in the first round so that the maximum number of extra sensors can be minimized:

$$N^*_{\text{minimax}}(h_\ell, 2, 1) = \arg \min_{N(h_\ell,2,1)} \max_{\sigma_{\text{real}}} \left[\bar{N}^{\sigma}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1)) - \bar{N}^{*}_{\text{total}}(h_\ell, 2)\right], \quad (19)$$

which can be obtained using numerical methods because the number of possible choices for $N(h_\ell, 2, 1)$ is finite.

B. The Pilot Deployment Solution

Now we present a more efficient solution by introducing an additional pilot round prior to the two rounds of sensor deployment. The basic idea is to use the residence points of the sensors deployed in the pilot round to estimate σ_{real}, which is then used to guide the next two rounds of sensor deployment.

Let N_{pilot} and I_{pilot} denote the number of sensors deployed in the pilot round and the deployment interval. The deployment could start from either t_{left} or t_{right}. Recall that, in order to guarantee coverage of the left (right) boundary of the barrier, t_{left} (t_{right}) should be at a distance of $(R_s - 3\sigma_{\max})$ from the left (right) end of the deployment line. After the pilot round, the residence points of deployed sensors are collected and used to estimate σ_{real} as follows. The sample variance of the deviation of the residence points along X-axis and Y-axis with respect to the corresponding deployment points is calculated as follows:

$$S = \frac{\sum_{i=1}^{N_{\text{pilot}}} (x_i' - x_i)^2 + \sum_{i=1}^{N_{\text{pilot}}} (y_i')^2}{2N_{\text{pilot}} - 1}, \quad (20)$$

based on which we propose the following estimator for σ_{real}:

$$\hat{\sigma}_{\text{real}} = \begin{cases} \sigma_{\min}, & \text{if } S \leqslant \sigma_{\min}^2, \\ \sqrt{S}, & \text{if } \sigma_{\min}^2 < S < \sigma_{\max}^2, \\ \sigma_{\max}, & \text{if } S \geqslant \sigma_{\max}^2. \end{cases} \quad (21)$$

This estimator makes sense because the truncated two-dimensional Gaussian distribution (symmetrically at 3σ) is very similar to the non-truncated version. Therefore, the deviations along X-axis and Y-axis, i.e., $(x_i' - x_i)$ and y_i', can be treated as two sets of independent samples to collectively contribute to the estimation of σ_{real}. The conditional pdf of $\hat{\sigma}_{\text{real}}$ is approximately:

$$f_{\hat{\sigma}_{\text{real}}}(\hat{\sigma}|\sigma_{\text{real}}) = B_1 \cdot \delta(\hat{\sigma} - \sigma_{\min}) + B_2 \cdot \delta(\hat{\sigma} - \sigma_{\max}) + I_{\hat{\sigma}} \cdot f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}, \quad (22)$$

where

$$B_1 = \int_0^{\sigma_{\min}} f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}\, d\hat{\sigma},$$
$$B_2 = \int_{\sigma_{\max}}^{\infty} f_{\Gamma}\left(\hat{\sigma}^2; \frac{2N_{\text{pilot}} - 1}{2}, \frac{2\sigma_{\text{real}}^2}{2N_{\text{pilot}} - 1}\right) \cdot 2\hat{\sigma}\, d\hat{\sigma}, \quad (23)$$
$$I_{\hat{\sigma}} = \begin{cases} 1, & \text{if } \sigma_{\min} \leqslant \hat{\sigma} \leqslant \sigma_{\max}, \\ 0, & \text{otherwise,} \end{cases}$$

and $f_{\Gamma}(\cdot)$ is the Gamma distribution. The derivation of Eq. (22) is omitted due to space limitation and details can be found in [34]. Then, the next two rounds of sensor deployment are planned as follows:

- For each coverage gap (with a gap distance of h) generated after the pilot round, deploy $N^*_{\hat{\sigma}_{\text{real}}}(h, 3, 2)$ sensors in the first round, where $N^*_{\hat{\sigma}_{\text{real}}}(h, 3, 2)$ is obtained based on the assumption that $\sigma_{\text{real}} = \hat{\sigma}_{\text{real}}$;

- For each coverage gap of size h generated after the first round, deploy $\left(\left\lceil \frac{h}{2(R_s-3\sigma_{\max})}\right\rceil + 1\right)$ sensors in the second round to guarantee coverage of the gap.

Finally, we can find the optimal $<N^*_{\text{pilot}}, I^*_{\text{pilot}}>$ that minimizes the maximum number of extra sensors deployed when the pilot deployment solution is used:

$$< N^*_{\text{pilot}}, I^*_{\text{pilot}} >= \arg \min_{< N_{\text{pilot}}, I_{\text{pilot}} >} \max_{\sigma_{\text{real}}} \left[\int_0^\infty \bar{\mathcal{N}}^p_{\text{total}}(h_\ell, N_{\text{pilot}}, I_{\text{pilot}}, \sigma_{\text{real}}, \hat{\sigma}) \cdot f_{\hat{\sigma}_{\text{real}}}(\hat{\sigma}|\sigma_{\text{real}})\, d\hat{\sigma} - \bar{\mathcal{N}}^*_{\text{total}}(h_\ell, 2) \right], \tag{24}$$

where $\bar{\mathcal{N}}^p_{\text{total}}(h_\ell, N_{\text{pilot}}, I_{\text{pilot}}, \sigma_{\text{real}}, \hat{\sigma})$ is the expected total number of sensors needed to cover the barrier by following the above pilot deployment solution, whose analysis is similar to that of $\bar{\mathcal{N}}_{\text{total}}$ in Section IV and omitted due to space limitation.

VI. Performance Evaluation

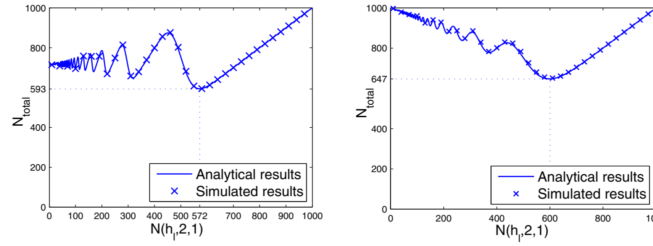
We develop a custom simulator based on MATLAB and use it to evaluate the performance of our proposed multi-round sensor deployment schemes for guaranteed barrier coverage. In the numerical and simulation studies, we consider a barrier with a length of ℓ = 1000 units. Sensing radius (R_s) is 1 unit. A sensor's residence point follows a truncated two-dimensional Gaussian distribution with respect to its deployment point, as described in (16), and the parameter σ_{real} is within the range of [1/30, 3/10] units, which corresponds to $R_{\text{err}} \in$ [0.1, 0.9] units. Results with other σ_{real} values or other distributions of sensors' residence points yield similar trends and are not included here.

A. Two-Round Sensor Deployment when R_{err} is Known

We first study the proposed two-round sensor deployment strategy when R_{err} is known. Fig. 6(a) shows the results when R_{err} = 0.3 units; it plots the expected total number of sensors needed to cover the barrier ($\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$) when the number of sensors deployed in the first round ($N(h_\ell, 2, 1)$) varies from 0 to 1000. We also plot the simulation results as "×" marks, where each point is averaged over 1000 simulation runs. As shown in the figure, simulation results match analytical results closely. We have the following observations.

In general, as $N(h_\ell, 2, 1)$ increases starting from zero, $\bar{\mathcal{N}}_{\text{total}}$ firstly decreases (with fluctuations) till reaching the minimum, and then increases almost linearly as $N(h_\ell, 2, 1)$ increases further. As shown in Fig. 6(a), $\bar{\mathcal{N}}_{\text{total}}$ reaches the minimum of 593 when $N(h_\ell, 2, 1)$ is 572, meaning that the numbers of sensors deployed in the two rounds is 572 and 21, respectively. On the other hand, when $N(h_\ell, 2, 1)$ is zero, the corresponding $\bar{\mathcal{N}}_{\text{total}}$ is 700, meaning that if sensors are deployed in one round, 700 sensors are needed to cover the barrier. This shows that, with two-round sensor deployment, 15.3% of the sensors are saved.

Now let's take a look at the optimal two-round deployment strategy in more detail. One salient feature of the strategy is that a large number (572) of sensors are deployed in the first round, while a very small number (21) of sensors are deployed in the second round to fill the gaps. To provide guaranteed barrier coverage with single-round sensor deployment, sensors need to be deployed in a conservative manner (i.e., with small deployment interval and hence more sensors are needed) to counter the deviation of the sensors' residence points with respect to their deployment points. In comparison, with two-round sensor deployment, it is safe to be more aggressive (i.e., with larger deployment interval) in the first round and then deploy sensors more conservatively in the second round to fill the limited number of gaps generated in the first round. As a result, a significant number of sensors are saved by simply splitting sensor deployment into two rounds.



(a) R_{err} = 0.3 units (b) R_{err} = 0.5 units

Fig. 6. Expected total number of sensors needed to cover the barrier ($\bar{\mathcal{N}}_{\text{total}}(h_\ell, 2, N(h_\ell, 2, 1))$) vs. the number of sensors deployed in the first round ($N(h_\ell, 2, 1)$) with the proposed two-round sensor deployment strategy. Note that $\bar{\mathcal{N}}_{\text{total}}$ fluctuates in both figures. This is because, when $N(h_\ell, 2, 1)$ increases from small, the number of sensors to fill each gap generated in the first round decreases approximately according to a staircase function. So $\bar{\mathcal{N}}_{\text{total}}$ fluctuates with respect to $N(h_\ell, 2, 1)$ until $N(h_\ell, 2, 1)$ gets sufficiently large.

In the optimal two-round sensor deployment strategy, the deployment interval in the first round is $I^*(h_\ell, 2, 1) = \frac{h_\ell}{N^*(h_\ell,2,1)-1} = \frac{1000-2(1-0.3)}{572-1} \approx 1.75$ units. Besides, we find that (i) the number of gaps generated in the first round is small; and (ii) all the gaps are small gaps which can be filled by deploying one extra sensor in the second round. The latter one is an important observation as it shows that, as long as the sensors' communication range is reasonably large (e.g., $\geqslant 4R_s$), when the optimal two-round sensor deployment strategy is used, connectivity among sensors deployed in the first round can be guaranteed, which is a critical assumption (discussed in Section III) for the strategy to function properly.

In Fig. 6(b), R_{err} is increased from 0.3 to 0.5 units and all the above observations still hold. Particularly, with the proposed two-round sensor deployment strategy, $\bar{\mathcal{N}}_{\text{total}}$ reaches the minimum of 647 when $N(h_\ell, 2, 1)$ is 600. In comparison, when $N(h_\ell, 2, 1)$ is zero, the corresponding $\bar{\mathcal{N}}_{\text{total}}$ becomes 1000, meaning that 1000 sensors are needed to cover the barrier if deployed in a single round. This shows that, a larger portion of sensors (35.3%) can be saved when R_{err} is increased to 0.5 units. In Fig. 7, we vary R_{err} from 0.1 to 0.9 units and show the benefit of two-round sensor deployment in terms of the percentage of sensors saved. It can be seen clearly from the figure that although more sensors are needed to cover the barrier as R_{err} goes up, it saves more sensors (percentage wise) by increasing the number of sensor deployment rounds from one to two. This is because a larger R_{err} requires a more conservative sensor deployment round (i.e., the only round in single-round sensor deployment vs. the second round in two-round sensor deployment) to deal with.

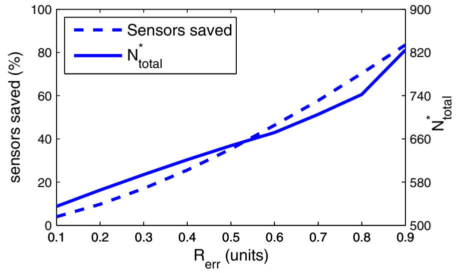


Fig. 7. Percentage of sensors saved and $\bar{\mathcal{N}}^*_{\text{total}}(h_\ell, 2)$ vs. R_{err}.

B. Two-Round vs. M-Round ($M \geqslant 3$) Sensor Deployment

In Section VI-A, we observe that significant performance improvement may be achieved by increasing the number of sensor deployment rounds from one to two. In this section, we study how or whether the performance may be improved further by increasing the number of deployment rounds more.

Fig. 8 compares the performances of two-round and three-round sensor deployment strategies when R_{err} is 0.3 units. Results reveal a rather surprising discovery that the optimal three-round sensor deployment strategy produces the same $\bar{\mathcal{N}}^*_{\text{total}} = 593$ as the optimal two-round strategy. As shown in the figure, with the three-round sensor deployment strategy, $\bar{\mathcal{N}}^*_{\text{total}} = 593$ is reached when $N(h_\ell, 3, 1) = 0$ or 572. When $N(h_\ell, 3, 1) = 0$, the number of sensors deployed in the three rounds is 0, 572 and 21, respectively, while when $N(h_\ell, 3, 1) = 572$, the numbers are 572, 21 and 0. We can see that, the optimal three-round strategy essentially only deploys sensors in two out of three rounds. We have also done simulations with m-round ($m \geqslant 4$) sensor deployment and $\bar{\mathcal{N}}^*_{\text{total}}$ remains the same. This means that the minimum number of sensors needed to cover the barrier is the same regardless of the number of sensor deployment rounds, as long as the sensors are deployed in multiple rounds. This discovery has high practical significance because it implies that the best barrier coverage performance can be achieved with low extra deployment cost by deploying sensors in two rounds.

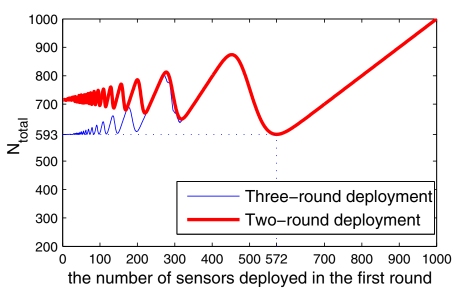


Fig. 8. Comparison of the expected total number of deployed sensors with two-round and three-round sensor deployment strategies.

C. Two-Round Sensor Deployment with R_{err} Partially Known

Finally, we study the scenario when R_{err} is only partially know, i.e., the range of R_{err} is known ($0.1 \leqslant R_{\text{err}} \leqslant 0.9$) but the actual value of R_{err} is unknown. Using the proposed two-round minimax solution, N^*_{minimax} is 587, while with the pilot deployment solution, N^*_{pilot} is 51 and I^*_{pilot} is 1.69 units. Fig. 9 compares the performances of these two solutions against the ideal solution when R_{err} is known, with the actual value of R_{err} varying from 0.1 to 0.9 units. As shown in the figure, the two-round minimax solution works well in most scenarios except when the actual value of R_{err} is close to 0.1 or 0.9 units. In these scenarios, as many as 50 extra sensors may be needed to guarantee barrier coverage. In comparison, with the pilot deployment solution, the number of extra sensors is reduced significantly to no more than 9. This is due mainly to the extra pilot round of sensor deployment which collects some preliminary statistics about R_{err} and then uses them to aid the following two rounds of sensor deployment. We have also simulated the pilot deployment solution. The results are averaged over 1000 simulation runs and a close match between analytical and simulation results can be observed.

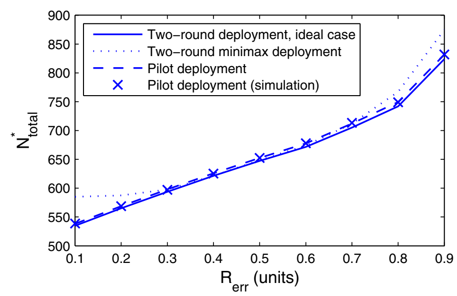


Fig. 9. Expected total number of deployed sensors vs. the actual value of R_{err} (unknown) with the proposed practical solutions.

D. Summary

We summarize the key findings from our numerical and simulation studies as follows:

- By splitting sensor deployment into multiple ($\geqslant$2) rounds, the number of sensors needed to provide guaranteed barrier coverage can be reduced significantly.
- The performance gain of multi-round sensor deployment over single-round sensor deployment becomes more significant as the deviation of a sensor's residence point with respect to its deployment point gets larger, i.e., when more randomness is present during sensor deployment.
- With the optimal two-round sensor deployment strategy, all the coverage gaps generated in the first round are small gaps. This means that, as long as the communication range of sensors is reasonably large, the sensors deployed in the first round are connected, which supports the practical feasibility of the strategy.
- Optimal m-round ($m > 2$) sensor deployment strategies yield the same performance as the optimal two-round sensor deployment strategy. Thus, the best barrier coverage performance can be achieved with low extra deployment cost by deploying sensors in two rounds.
- When the information about the deviation of a sensor's residence point with respect to its deployment point is not fully available, the pilot deployment solution performs particularly well and is comparable to the ideal solution.

VII. Discussions and Future Work

In this section, we discuss how our proposed multi-round sensor deployment may be extended and some related issues.

1) Different Sensing Models. We assume the disc sensing model in this paper. In practice, the shape of a sensing area could be irregular. In general, it is difficult to have a rigorous analysis of the coverage performance in this situation. One simple way is to approximate the sensor area with the largest disc centered at a sensor and contained in its sensing area, which would result in conservative sensor deployment strategies. We also assume the 0/1 sensing model in this paper. A potential extension to the work is to incorporate the more realistic probabilistic sensing model and the concept of information coverage (to exploit the collaboration between sensors) into multi-round sensor deployment, which could save even more sensors when covering a barrier.

2) k-Barrier Coverage. The barrier coverage studied in this paper is the simple 1-barrier coverage. The work can be extended to k-barrier coverage by making the following modifications in the analysis. With k-barrier coverage, a coverage gap between two adjacent sensors deployed in the previous rounds exists if there are less than k disjoint linkages between them. Therefore, coverage gaps can be classified into multiple categories according to the number of additional disjoint linkages that are needed to fill the gap. Different types of coverage gaps need to be analyzed differently.

3) Other Issues. Recall that according to our numerical and simulation studies, with the optimal two-round sensor deployment strategy, as long as the sensors' communication range is at least four times the sensing range, the sensors deployed in the first round are connected. In practice, if under certain circumstances when the communication range is small, an additional constraint on the deployment interval should be included so the connectivity requirement could be satisfied.

Note that we study the problem of *guaranteed barrier coverage* in this paper. A related but distinct problem is how to provide barrier coverage in a probabilistic sense with multi-round sensor deployment. This problem may require different techniques to deal with. Tradeoff between the desired probability of barrier coverage and the deployment interval needs to be investigated. This is part of our future work.

VIII. Conclusion

In this paper, we conduct extensive analytical and simulation studies on reducing the number of sensors needed to provide guaranteed barrier coverage with multi-round sensor deployment strategies. We study the performance of multi-round sensor deployment and derive the optimal strategies that use fewest sensors to cover a barrier. We find that the efficient barrier coverage can be achieved with the simple two-round sensor deployment. In addition, two practical solutions are presented to deal with realistic situations when the distribution of a sensor's residence point is not fully known. The effectiveness of the proposed solutions is supported by numerical and simulation results.

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