

1. Problem 7.19 in Kay-I.

Comments: You may use the `randn` function in MATLAB to generate Gaussian random variables. Plot the function to be maximized for a few noisy realizations. Estimate the mean (bias) and variance of the Newton-Raphson and grid-search estimators of f_0 [along the lines of equations (7.9) and (7.10) in Kay-I].

Grid search corresponds to computing the function to be maximized ($g(f_0)$, say) over a dense uniform grid over the parameter space and choosing the parameter estimate to be the grid point that gives the maximum value of $g(f_0)$.

2. Generate conditionally independent, identically distributed (i.i.d.) random variables

$$X[0], X[1], \dots, x[N-1]$$

given θ using a lognormal distribution (see (16) in handout #3) with parameters

$$\theta = [\alpha, \mu, \sigma^2]^T = [2, 0, 1].$$

Consider the cases of $N = 10, 20$, and 100 measurements.

- (i) Plot the concentrated likelihood function in (20) of handout #3 as a function of α .
- (ii) Find the ML estimate of α . You may use `fminsearch` to maximize the concentrated likelihood or try out Newton-Raphson iteration. Comment on your results. *Hint:* See MATLAB help to learn how to use `fminsearch`.
- (iii) Try out this set of $N = 9$ measurements:

$$\mathbf{x} = [7.472, 6.872, 8.322, 4.533, 3.558, 15.294, 8.732, 2.301, 5.781]^T$$

What do you observe?