EE 330
Lecture 43

- The Cascode Configuration
High-gain amplifier

Can the gain be made even larger?

The Cascode Configuration

Discuss
The Cascode Amplifier (consider npn BJT version)

Instead of just determining the voltage gain, we will obtain the two-port model for the cascode amplifier.
The Cascode Amplifier (consider npn BJT version)

From the two-port model of the cascode, the $A_{VCC}$ in the model is simply the voltage gain of the cascode amplifier and $g_{0CC}$ is the output conductance of the cascode amplifier. Instead of just determining the voltage gain, we will obtain the two-port (Thevenin) model for the cascode amplifier.
The Cascode Amplifier (consider npn BJT version)
Cascode Configuration

Two-port model of cascode amplifier

\[
\begin{align*}
(V_X + V_2) g_{02} + V_2 g_{m2} &= I_X \\
V_1 g_{m1} - V_2 (g_{01} + g_{\pi2}) &= I_X
\end{align*}
\]

Observing \( V_1 = V_{IN} \) and eliminating \( V_2 \) between these two equations, we obtain

\[
V_X = -\left[ \frac{g_{m1} (g_{02} + g_{m2})}{g_02 (g_{\pi2} + g_{01})} \right] V_{IN} + \left[ \frac{g_{01} + g_{02} + g_{\pi2} + g_{m2}}{g_{02} (g_{01} + g_{\pi2})} \right] I_X
\]

and

\[
V_{IN} = \frac{1}{g_{\pi1}} I_1
\]
Cascode Configuration

Two-port model of cascode amplifier

\[ V_X = -\left[ \frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi2}+g_{01})} \right] V_1 + \left[ \frac{g_{01}+g_{02}+g_{\pi2}+g_{m2}}{g_{02}(g_{01}+g_{\pi2})} \right] I_X \]

\[ V_{IN} = \frac{1}{g_{\pi1}} I_I \]

It thus follows for the npn bipolar structure that:

\[ A_{VCC} = -\left[ \frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi2}+g_{01})} \right] \approx -\left[ \frac{g_{m1}g_{m2}}{g_{02}g_{\pi2}} \right] \]

\[ g_{0CC} = \left[ \frac{g_{02}(g_{01}+g_{\pi2})}{g_{01}+g_{02}+g_{\pi2}+g_{m2}} \right] \approx \left[ \frac{g_{02}g_{\pi2}}{g_{m2}} \right] \]

\[ g_{\pi CC} = g_{\pi1} \]
Cascode Configuration

\[ A_{VCC} \approx - \left[ \frac{g_{m1}g_{m2}}{g_{02}g_{\pi2}} \right] \]

\[ g_{0CC} \approx \left[ \frac{g_{02}g_{\pi2}}{g_{m2}} \right] \]

\[ g_{\pi CC} = g_{\pi1} \]

\[ A_{VCC} \approx - \left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx - \left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

\[ g_{0CC} \approx \frac{g_{01}}{\beta} \]

- Voltage gain is a factor of \( \beta \) larger than that of the CE amplifier with current source load
- Output impedance is a factor of \( \beta \) larger than that of the CE amplifier
Cascode Configuration

What happens to the gain if a transistor-level current source is used for $I_B$?

This gain is very large!

$$A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{02}} \right] \beta \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta$$

$$g_{0CC} \approx \frac{g_{02}}{\beta}$$
Cascode Configuration

\[ V_{XX} \rightarrow Q_1 \rightarrow Q_2 \rightarrow V_{OUT} \]

\[ V_{IN} \rightarrow V_{SS} \]

\[ V_{CC} \rightarrow I_B \rightarrow V_{OUT} \]

\[ V_{YY} \rightarrow Q_3 \rightarrow V_{OUT} \]

\[ V_{IN} \rightarrow V_{SS} \]
Cascode Configuration

\[ V_{CC} \]
\[ V_{YY} \]
\[ Q_3 \]
\[ V_{XX} \]
\[ Q_2 \]
\[ Q_1 \]
\[ V_{SS} \]
\[ V_{IN} \]
\[ V_{OUT} \]
High-gain amplifier comparisons

It thus follows that

\[ A_V = A_{VCC} \left[ \frac{g_{0CC}}{g_{03} + g_{0CC}} \right] \]

But \[ g_{0CC} \approx \frac{g_{03}}{\beta} \]

\[ A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta} \]

This is a dramatic reduction in gain compared to what the ideal current source biasing provided.
Cascode Configuration

\[ A_V \approx A_{VCC} \left[ \frac{g_{0CC}}{g_{03}} \right] \approx \frac{A_{VCC}}{\beta} \]

But recall
\[ A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta \]

Thus
\[ A_V \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \]

- This is still a factor of 2 better than that of the CE amplifier with transistor current source \( A_{VCE} \approx -\left[ \frac{g_{m1}}{2g_{01}} \right] \)
- It only requires one additional transistor
- But its not nearly as good as the gain the cascode circuit seemed to provide
Cascode Configuration Comparisons

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V \approx \frac{-g_{m1}}{2g_{01}} \]

Can we design a better current source?

In particular, one with a higher output impedance?
Better current sources

Need a higher output impedance than $g_o$.

The output impedance of the cascode circuit itself was very large!

$$g_{0CC} \approx \frac{g_{01}}{\beta}$$

Can a current source be built with the cascode circuit?
Cascode current sources

\[ Q_1 \]
\[ Q_2 \]
\[ V_{XX} \]
\[ V_{YY} \]
\[ V_{SS} \]

\[ V_{XX} \]
\[ V_{YY} \]
\[ V_{SS} \]

\[ V_{CC} \]
\[ V_{YY} \]
\[ V_{XX} \]

\[ V_{DD} \]
\[ V_{YY} \]
\[ V_{XX} \]
Cascode current sources

All have the same small-signal model:

\[ g_{0CC} = \frac{g_{02} (g_{01} + g_{\pi2})}{g_{01} + g_{02} + g_{\pi2} + g_m} \]
For the BJT cascode current sources

\[
g_{0CC} = \frac{g_{02}(g_{01} + g_{\pi2})}{g_{01} + g_{02} + g_{\pi2} + g_{m2}} \approx \frac{g_{02}g_{\pi2}}{g_{m2}} = \frac{g_{01}}{\beta}
\]
Cascode Configuration

- \( V_{XX} \) to \( Q_1 \) to \( Q_2 \) to \( V_{OUT} \)
- \( V_{IN} \) to \( Q_1 \) to \( Q_2 \) to \( V_{OUT} \)
- \( V_{CC} \) to \( Q_3 \) to \( Q_4 \) to \( V_{OUT} \)
- \( V_{IN} \) to \( Q_3 \) to \( Q_4 \) to \( V_{OUT} \)
This gain is very large and is a factor of 2 below that obtained with an ideal current source biasing.

Although the factor of 2 is not desired, the performance of this circuit is still very good.

This factor of 2 gain reduction is the same as was observed for the CE amplifier when a transistor-level current source was used.
Cascode Configuration Comparisons

Can we use more cascoding to further increase the gain?
The double cascode

\[ A_V \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \beta^2 \]

\[ g_{0CC} \approx \frac{g_{01}}{\beta^2} \]

- Further gain enhancement
- Further output impedance increase
- Limited applications, particularly at lower voltages, because signal swings at outputs are small
The Cascode Amplifier (consider n-ch MOS version)

- Same functional form for gain and output conductance except $g_{\pi}=0$
- Simplifications functionally different!

\[ A_{\text{VCC}} = -\frac{g_{m1}(g_{02}+g_{m2})}{g_{02}(g_{\pi2}+g_{01})} \approx -\frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

\[ g_{0\text{CC}} = \frac{g_{02}(g_{01}+g_{\pi2})}{g_{01}+g_{02}+g_{\pi2}+g_{m2}} \approx \frac{g_{01}g_{02}}{g_{m2}} \]
The Cascode Amplifier (consider n-ch MOS version)

\[ A_{VCC} \approx - \begin{bmatrix} g_{m1} g_{m2} \\ g_{01} g_{02} \end{bmatrix} \]

\[ g_{0CC} \approx \begin{bmatrix} g_{01} g_{02} \\ g_{m2} \end{bmatrix} \]

Same issues for biasing with current source as for BJT case

With cascode current source, gain only drops by a factor of 2

Discuss
The Cascode Amplifier (consider n-ch MOS version)

\[ A_{\text{VCC}} \approx -\left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]

\[ A_{\text{VCC}} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_{\text{VCC}} \approx -\frac{1}{2} \left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]
High Gain Amplifier Comparisons (BJT)

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V \approx -\frac{1}{2} \frac{g_m}{g_0} \]

\[ A_V \approx -\left[ \frac{g_m}{g_0} \right] \beta \]

\[ A_V = -\left[ \frac{g_m}{g_0} \right] \beta \]

Review
High Gain Amplifier Comparisons (n-ch MOS)

\[ A_V \approx -\frac{g_{m1}}{g_{01}} \]

\[ A_V \approx -\frac{1}{2} \left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_{VCC} = -\left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]

\[ A_{VCC} \approx -\left[ \frac{g_{m1}}{g_{01}} \right] \]

\[ A_{VCC} \approx -\frac{1}{2} \left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right] \]
The Cascode Amplifier

- Operational amplifiers often built with basic cascode configuration
- Usually configured as a differential structure when building op amps
- Have high output impedance (but can be buffered)
- Terms “telescopic cascode”, “folded-cascode”, and “regulated cascode” often refer to op amps based upon the cascode configuration

Telescopic Cascode Op Amp
(CMFB feedback biasing not shown)
Cascade Configurations

Two-stage Cascade

\[ A_{V_{CB}} = ? \]

\[ A_{V_{CM}} = ? \]
Cascade Configurations

Two-stage Cascade

\[ A_{V CB} \approx \left[ \frac{-g_{m1}}{g_{01} + g_{\pi 2}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] \approx \frac{g_{m1}g_{m2}}{g_{\pi 2}g_{02}} = \beta \frac{g_{m1}}{g_{02}} \]

\[ A_{V CM} = \left[ \frac{-g_{m1}}{g_{01}} \right] \left[ \frac{-g_{m2}}{g_{02}} \right] = \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \]

- Significant increase in gain
- Gain is noninverting
- Comparable to that obtained with the cascode
Cascade Configurations

Two-stage Cascade

\[ A_{VCB} \approx \begin{bmatrix} -g_{m1} \\ g_{01} + g_{03} + g_{\pi2} \end{bmatrix} \begin{bmatrix} -g_{m2} \\ g_{02} + g_{04} \end{bmatrix} \approx \frac{g_{m1}g_{m2}}{2g_{\pi}g_{02}} = \beta \frac{g_{m1}}{2g_{02}} \]

\[ A_{VCM} = \begin{bmatrix} -g_{m1} \\ g_{01} + g_{03} \end{bmatrix} \begin{bmatrix} -g_{m2} \\ g_{02} + g_{04} \end{bmatrix} = \frac{g_{m1}g_{m2}}{4g_{01}g_{02}} \]

Note factor or 2 and 4 reduction in gain due to actual current source bias
Cascade Configurations

Two-stage Cascade

- Large gains can be obtained by cascading
- Gains are multiplicative (when loading is included)
- Large gains used to build “Op Amps” and feedback used to control gain value
- Some attention is needed for biasing but it is manageable
- Minor variant of the two-stage cascade often used to built Op Amps

Three-stage Cascade

- Compensation of two-stage cascade needed if feedback is applied to maintain stability
- Three or more stages are seldom cascaded because no really good way to compensate to maintain stability
EE 330
Lecture 35

• Current source/mirror Layout
• Differential Amplifiers
• Other Basic Amplifier Configurations
High Gain Amplifier Comparisons (BJT)

\[ A_V = \frac{-g_m}{g_0} \]

\[ A_V \approx -\frac{1}{2} \frac{g_m}{g_0} \]

\[ A_V \approx -\left[ \frac{g_m}{g_0} \right] \beta \]

\[ A_V = -\left[ \frac{g_m}{g_0} \right] \]

\[ A_V = -\left[ \frac{g_m}{g_0} \right] \frac{\beta}{2} \]

Review
High Gain Amplifier Comparisons (n-ch MOS)

\[
A_V \approx - \frac{g_{m1}}{g_{01}}
\]

\[
A_V \approx - \frac{1}{2} \left[ \frac{g_{m1}}{g_{01}} \right]
\]

\[
A_{VCC} \approx - \frac{g_{m1}g_{m2}}{g_{01}g_{02}}
\]

\[
A_{VCC} \approx - \left[ \frac{g_{m1}}{g_{01}} \right]
\]

\[
A_{VCC} \approx - \frac{1}{2} \left[ \frac{g_{m1}g_{m2}}{g_{01}g_{02}} \right]
\]