EE330
Integrated Electronics

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The Semiconductor Process

The Transistor, Reliability and Yield, Review of Basic Statistical Concepts, A Brief History
Yield Issues: Hard Faults

- Dust particles and other undesirable processes cause defects → These defects in manufacturing cause yield loss
Yield Issues: Hard Faults

- Defects in processing cause yield loss
- The probability of a defect causing a circuit failure increases with die area
- The circuit failures associated with these defects are termed **Hard Faults**
- This is the major factor limiting the size of die in integrated circuits
- Several different models have been proposed to model the hard faults
Yield Issues: Hard Fault Models

\[ Y_H = e^{-Ad} \]

- \( Y_H \) is the probability that the die does not have a hard fault
- \( A \) is the die area
- \( d \) is the defect density (typically \( 1\text{cm}^{-2} < d < 2\text{cm}^{-2} \))

- **Industry often closely guards the value of \( d \) for their process**
- **Other models, which may be better, have the same general functional form**
Yield Issues: Soft Faults

- Random parametric variations in a process can also cause circuit failure or cause circuits to not meet desired performance specifications (this is of particular concern in analog and mixed-signal circuits).

- The circuit failures associated with these parametric variations are termed **Soft Faults**.

- Decreases with area, judicious layout and routing, and clever circuit design techniques can reduce the effects of soft faults.
Yield Issues: Soft Fault Models

- Soft fault models often depend on design and application
- Often the standard deviation of a parameter depends on the reciprocal of the square root of the parameter’s sensitive area

\[ \sigma = \frac{\rho}{\sqrt{A_k}} \]

\( \rho \) is a constant dependent upon the architecture and the process

\( A_k \) is the area of the parameter’s sensitive area
Yield Issues: Soft Fault Models

\[ P_{SOFT} = \int_{X_{\text{MIN}}}^{X_{\text{MAX}}} f(x) \, dx \]

- \( P_{SOFT} \) is the soft fault yield
- \( f(x) \) is the probability density function of the parameter of interest
- \( X_{\text{MIN}} \) and \( X_{\text{MAX}} \) define the acceptable range of the parameter of interest

\[ \begin{align*}
X_{\text{MIN}} & \quad \text{X}_{\text{MAX}}
\end{align*} \]

- Some circuits may have several parameters that must meet performance requirements
Yield Issues: Soft Fault Models

- If there are $k$ parameters that must meet certain parametric performance requirements and if the random variables characterizing these parameters are uncorrelated, then the soft yield is given by

$$Y_S = \prod_{j=1}^{k} P_{\text{SOFT}_j}$$
Yield Issues: Overall Yield

If both hard and soft faults affect the yield of a circuit, the overall yield is given by the expression

\[ Y = Y_H Y_S \]
Yield Issues: Yield and Die Cost

- The overall yield can dramatically affect Die cost
- The manufacturing cost per “good” Die is given by

\[ C_{\text{Good}} = \frac{C_{\text{FabDie}}}{Y} \]

where \( C_{\text{FabDie}} \) is the manufacturing costs of a fab die and \( Y \) is the yield.

- There are other costs that must ultimately be included such as testing costs, engineering costs, etc.
- Let’s take an example!
Yield Issues: Yield and Die Cost

- Assume a die has no soft fault vulnerability, a die area of $1\text{cm}^2$, and a process that has a defect density of $1.5\text{cm}^{-2}$

  - Determine the hard yield
  - Determine the manufacturing cost per “good” Die if 8-inch wafer is used and if the cost of the wafer is $1200

\[
Y_H = e^{-Ad} = e^{-1\text{cm}^2 \times 1.5\text{cm}^{-2}} = 0.22
\]

\[
C_{\text{FabDie}} = \frac{C_{\text{Wafer}}}{A_{\text{Wafer}}} A_{\text{Die}} = \frac{$1200}{\pi (4\text{in})^2} 1\text{cm}^2 = $3.82
\]

\[
C_{\text{Good}} = \frac{C_{\text{FabDie}}}{Y} = \frac{$3.82}{0.22} = $17.37
\]
Review of Basic Concepts of Statistics

- Statistics govern what really happens throughout much of the engineering field!

- Statistics characterize what WILL happen in many processes → You MUST know the basics of it whether you like it or not!

- For instance, you must rely on it to determine yield and subsequently cost → If you don’t use it, you can’t find out if your investment is worthwhile
Review of Basic Concepts of Statistics

Assume $x$ is a random variable of interest

$f(x) =$ Probability Density Function (PDF) for $x$

$$\int_{-\infty}^{\infty} f(x) \, dx = 1$$

$F(x) =$ Cumulative Density Function (CDF) for $x$

$$F(X_1) = \int_{-\infty}^{X_1} f(x) \, dx$$

$$0 \leq F(x) \leq 1 \quad \quad \frac{\partial F(x)}{\partial x} \geq 0$$
Review of Basic Concepts of Statistics

\[ f(x) = \text{Probability Density Function (PDF) for } x \]

\[ F(x) = \text{Cumulative Density Function (CDF) for } x \]

\[ P\{x \leq x_1\} = \int_{-\infty}^{x_1} f(x) \, dx \]

\[ P\{x \leq X_1\} = F(X_1) \]
Review of Basic Concepts of Statistics

f(x) = Probability Density Function (PDF) for x
F(x) = Cumulative Density Function (CDF) for x

\[ P\{X_1 \leq x \leq X_2\} = \int_{X_1}^{X_2} f(x) \, dx \]

\[ P\{X_1 \leq x \leq X_2\} = F(X_2) - F(X_1) \]
Review of Basic Concepts of Statistics

How to we obtain the PDF of a given random variable?

- Most parameters we deal with in microelectronic circuits have what we call a “Normal distribution”, which is also called “Gaussian distribution”

- Experimental observations confirm that and provide close agreement between theoretical and experimental results

What exactly is a Normal distribution?
What exactly is a Normal distribution?

- The PDF of a Normal distribution is fully characterized and tabulated through its Mean ($\mu$) and Standard Deviation ($\sigma$).
- A Normal distribution is referred to as $N(\mu, \sigma)$.
What exactly is a Normal distribution?

A Normal distribution is referred to as $N(\mu, \sigma)$

![Diagram showing the normal distribution with $\mu$ and $\sigma$]
Review of Basic Concepts of Statistics

◆ A Normal distribution has some interesting features

Theorem 1: If the random variable $x$ is normally distributed with mean $\mu$ and standard deviation $\sigma$, then $y = \frac{x - \mu}{\sigma}$ is also a random variable that is normally distributed with mean 0 and standard deviation of 1.

◆ Why is that an important feature?

- Because it means I don’t have to tabulate every single PDF just because it has a different $\mu$ and $\sigma$
- As long as the distribution is normal, I can convert it to a PDF with Zero mean and unity Sigma, compute my probability, then scale it back to obtain the actual probability for the specific PDF I want → How?
We know the following:

If \( x \) is a Normal (Gaussian) random variable with mean \( \mu \) and standard deviation \( \sigma \), then the probability that \( x \) is between \( x_1 \) and \( x_2 \) is given by

\[
p = \int_{x_1}^{x_2} f(x) \, dx
\]
If \( f(x) \) is the PDF of a Normal distribution with mean \( \mu \) and standard deviation \( \sigma \), while \( f_n(x) \) is the PDF of a Normal distribution with Zero mean and unity standard deviation, then we can show that

\[
p = \int_{x_1}^{x_2} f(x) \, dx = \int_{x_{1n}}^{x_{2n}} f_n(x) \, dx
\]

where \( x_{1n} = \frac{x_1 - \mu}{\sigma} \) and \( x_{2n} = \frac{x_2 - \mu}{\sigma} \).
Review of Basic Concepts of Statistics

- **We also Know that**

\[ p = \int_{x_1}^{x_2} f(x) \, dx = F(x_2) - F(x_1) \]

- **And we can show that**

\[ p = \int_{x_{1n}}^{x_{2n}} f_n(x) \, dx = F_n(x_{2n}) - F_n(x_{1n}) \]

- **Where** \( F(x) \) **and** and \( F_n(x) \) **are the CDF of both distributions**
In many electronic circuits, the random variables of interest are 0 mean Gaussian and the probabilities of interest are characterized by a region defined by the magnitude of the random variable → The variable is symmetric around zero.

\[ p = \int_{-\ln x}^{\ln x} f_n(x) \, dx = F_n(x_{1n}) - F_n(-x_{1n}) = 2F_n(x_{1n}) - 1 \]
Review of Basic Concepts of Statistics

- Tables of the CDF of the N(0,1) random variable are readily available. It is also available in Matlab, Excel, and a host of other sources

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http://www.math.unb.ca/~knight/utility/NormTble.htm
Review of Basic Concepts of Statistics

- Example: Determine the probability that the \( N(0,1) \) random variable has magnitude less than 2.6

\[
p = 2F_n(2.6) - 1
\]

*From the table of the CDF, \( F_n(2.6) = 0.9953 \) so \( p = 0.9906 \)
Example: Determine the soft yield of an operational amplifier that has an offset voltage requirement of 5mV if the standard deviation of the offset voltage is 2.5mV and the mean is 0V.

\[ p = \int_{-2}^{2} f_N(x) \, dx = F_N(2) - F_N(-2) = 2F_N(2) - 1 \]

\[ p = 2F_N(2) - 1 = 2 \times 0.9772 - 1 = 0.9544 \]