Using Nonlinear Devices in Electronic Circuits

Operating Points, Small-Signal Modeling, Linear Amplifiers
Miscellaneous Examples: Example 1

◆ Determine the small signal voltage gain $A_V = \frac{V_{OUT}}{V_{IN}}$. Assume $M_1$ and $M_2$ are operating in the saturation region and that $\lambda = 0$.
Miscellaneous Examples: Example 1

Small-signal circuit

Small-signal MOSFET model for $\lambda=0$
Miscellaneous Examples: Example 1

Small-signal circuit equivalent
### Miscellaneous Examples: Example 1

The Analysis

**By KCL**

\[ g_{m1} v_{gs1} = g_{m2} v_{gs2} \]

but

\[ v_{in} = v_{gs1} \]
\[ v_{out} = -v_{gs2} \]

**thus:**

\[ A_v = \frac{v_{out}}{v_{in}} = -\frac{g_{m1}}{g_{m2}} \]
The Analysis

\[ A_v = \frac{V_{out}}{V_{in}} = -\frac{g_{m_1}}{g_{m_2}} \]

\[ g_{m_1} = \sqrt{2\mu C_{ox} \frac{W_1}{L_1}} \times I_{DQ1}, \text{ and } g_{m_2} = \sqrt{2\mu C_{ox} \frac{W_2}{L_2}} \times I_{DQ2} \]

\[ A_v = \frac{V_{out}}{V_{in}} = -\frac{g_{m_1}}{g_{m_2}} = -\sqrt{\frac{2\mu C_{ox} \frac{W_1}{L_1}}{\frac{W_2}{L_2}}} \times I_{DQ1} \]

Note that \( I_{DQ1} = I_{DQ2} \)

Therefore \( A_v = -\frac{\sqrt{W_1}}{\sqrt{W_2}} \times \frac{\sqrt{L_2}}{\sqrt{L_1}} \)

If \( L_1 = L_2 \) \( A_v = -\frac{\sqrt{W_1}}{\sqrt{W_2}} \)
Obtain the small-signal input impedance of the diode-connected MOSFET shown below. What operation region is the MOSFET in?

\[ r_{in} = \frac{V_{in}}{i_{in}} \]
Small-signal MOSFET model

\[ r_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{in}}{V_{in} + g_m V_{in}} = \frac{1}{1 + g_m r_o} = \frac{1}{g_m} \quad (g_m \gg \frac{1}{r_o}) \]
**Q-point and Small-Signal Modeling of Transistors: MOSFET Summary**

- **NMOSFETs** → **For Q-point purposes** we use the following model of the transistor in saturation (also called the Large Signal Model of the device)

\[
\begin{align*}
I_{GQ} &= 0 \\
I_{DQ} &= \mu_n C_{OX} \frac{W}{2L} (V_{GSQ} - V_{TNQ})^2 \times (1 + \lambda V_{DSQ}) \\
I_{BQ} &= 0 \\
V_{TNQ} &= V_{TN0} + \gamma \left( \sqrt{\phi - V_{BSQ}} - \sqrt{\phi} \right)
\end{align*}
\]

We ignore the \(\lambda\) factor for Q-point purposes

\[
\begin{align*}
I_{GQ} &= 0 \\
I_{DQ} &= \mu_n C_{OX} \frac{W}{2L} (V_{GSQ} - V_{TNQ})^2 \\
I_{BQ} &= 0 \\
V_{TNQ} &= V_{TN0} + \gamma \left( \sqrt{\phi - V_{BSQ}} - \sqrt{\phi} \right)
\end{align*}
\]

If the bulk is connected to the source or IF I TELL YOU TO IGNORE the BODY EFFECT

\[
\begin{align*}
I_{GQ} &= 0 \\
I_{DQ} &= \mu_n C_{OX} \frac{W}{2L} (V_{GSQ} - V_{TNQ})^2 \\
I_{BQ} &= 0 \\
V_{TNQ} &= V_{TN0}
\end{align*}
\]
Q-point and Small-Signal Modeling of Transistors: MOSFET Summary

- NMOSFETs → **For small-signal purposes** we use the following linear model of the transistor in saturation (also called the **Small-Signal Model** of the device)

- If the bulk is connected to the source or if I tell you to ignore body effect
  \[
  i_g = 0 \\
  i_d = g_m \cdot v_{gs} + g_{mb} \cdot v_{bs} + \frac{1}{r_o} \cdot v_{ds} \\
  i_b = 0
  \]

- If I tell you to ignore the output impedance of the transistor
  \[
  i_g = 0 \\
  i_d = g_m \cdot v_{gs} \\
  i_b = 0
  \]
Q-point and Small-Signal Modeling of Transistors: MOSFET Summary

- PMOSFETs → For Q-point purposes we use the following model of the transistor in saturation (also called the Large Signal Model of the device)

\[
\begin{align*}
I_{GQ} &= 0 \\
|I_{DQ}| &= \mu_p C_{OX} \frac{W}{2L} (V_{SGQ} - |V_{TPQ}|)^2 \times (1 + \lambda V_{SDQ}) \\
I_{BQ} &= 0 \\
|V_{TPQ}| &= |V_{TP0}| + \gamma (\sqrt{\varphi - V_{SBQ}} - \sqrt{\varphi})
\end{align*}
\]

We ignore the \( \lambda \) factor for Q-point purposes.

If the bulk is connected to the source or IF I TELL YOU TO IGNORE the BODY EFFECT

\[
\begin{align*}
I_{GQ} &= 0 \\
|I_{DQ}| &= \mu_p C_{OX} \frac{W}{2L} (V_{SGQ} - |V_{TPQ}|)^2 \\
I_{BQ} &= 0 \\
|V_{TPQ}| &= |V_{TP0}|
\end{align*}
\]
PMOSFETs → **For small-signal purposes** we use the following linear model of the transistor in saturation (also called the Small-Signal Model of the device)

\[ i_g = 0 \]
\[ |i_d| = g_m \cdot v_{sg} + g_{mb} \cdot v_{sb} + \frac{1}{r_o} \cdot v_{sd} \]
\[ i_b = 0 \]

If the bulk is connected to the source or if I tell you to ignore body effect

\[ i_g = 0 \]
\[ |i_d| = g_m \cdot v_{sg} + \frac{1}{r_o} \cdot v_{sd} \]
\[ i_b = 0 \]

If I tell you to ignore the output impedance of the transistor

\[ i_g = 0 \]
\[ |i_d| = g_m \cdot v_{sg} \]
\[ i_b = 0 \]
Q-point and Small-Signal Modeling of Transistors: BJTs Summary

- **NPN → For Q-point purposes** we use the following model of the transistor in forward active (also called the **Large Signal Model** of the device)

\[
I_{\text{BQ}} = \frac{J_S A_E}{\beta} e^{\frac{V_{\text{BEQ}}}{V_t}}
\]

\[
I_{\text{CQ}} = J_S A_E e^{\frac{V_{\text{BEQ}}}{V_t}} \left(1 + \frac{V_{\text{CEQ}}}{V_{\text{AF}}}\right)
\]

We ignore the \( V_{\text{AF}} \) factor for Q-point purposes.

**Diagram:**

\[
I_{\text{BQ}} = \frac{J_S A_E}{\beta} e^{\frac{V_{\text{BEQ}}}{V_t}}
\]

\[
I_{\text{CQ}} = J_S A_E e^{\frac{V_{\text{BEQ}}}{V_t}}
\]
**Q-point and Small-Signal Modeling of Transistors: MOSFET Summary**

- **NPN** → **For small-signal purposes** we use the following linear model of the transistor in forward active (the **Small-Signal Model** of the device)

  \[
  i_b = \frac{1}{r_{\pi}} \cdot v_{be} \\
  i_c = g_m \cdot v_{be} + \frac{1}{r_o} \cdot v_{ce}
  \]

  If I tell you to ignore the output impedance of the transistor

  \[
  i_b = \frac{1}{r_{\pi}} \cdot v_{be} \\
  i_c = \beta \cdot i_b
  \]

  \[
  i_b = \frac{1}{r_{\pi}} \cdot v_{be} \\
  i_c = g_m \cdot v_{be}
  \]

  Both are equivalent

  \[
  g_m = \frac{I_{CQ}}{V_t}, \quad r_o = \frac{1}{g_o} = \frac{V_{AF}}{I_{CQ}}, \quad r_{\pi} = \frac{1}{g_{\pi}} = \frac{V_t}{I_{BQ}} = \beta V_t
  \]

  Both are equivalent
Q-point and Small-Signal Modeling of Transistors: BJTs Summary

- **PNP** → For Q-point purposes we use the following model of the transistor in forward active (also called the **Large Signal Model** of the device)

\[
\begin{align*}
|I_{BQ}| &= \frac{J_S A_E}{\beta} e^{\frac{V_{EBQ}}{V_t}} \\
|I_{CQ}| &= J_S A_E e^{\frac{V_{EBQ}}{V_t}} \left(1 + \frac{V_{ECQ}}{V_{AF}}\right)
\end{align*}
\]

We ignore the \( V_{AF} \) factor for Q-point purposes.

\[
\begin{align*}
|I_{BQ}| &= \frac{J_S A_E}{\beta} e^{\frac{V_{EBQ}}{V_t}} \\
|I_{CQ}| &= J_S A_E e^{\frac{V_{EBQ}}{V_t}}
\end{align*}
\]
**Q-point and Small-Signal Modeling of Transistors: MOSFET Summary**

- **PNP → For small-signal purposes** we use the following linear model of the transistor in forward active (the **Small-Signal Model** of the device)

\[
|I_b| = \frac{1}{r_\pi} \cdot v_{eb}
\]

\[
|I_c| = g_m \cdot v_{eb} + \frac{1}{r_o} \cdot v_{ec}
\]

If I tell you to ignore the output impedance of the transistor

\[
|I_b| = \frac{1}{r_\pi} \cdot v_{eb}
\]

\[
|I_c| = \beta \cdot |I_b| + \frac{1}{r_o} \cdot v_{ec}
\]

Both are equivalent

\[
g_m = \frac{|l_{CQ}|}{V_t}, \quad r_o = \frac{1}{g_o} = \frac{V_{AF}}{|l_{CQ}|}, \quad r_\pi = \frac{1}{g_\pi} = \frac{V_t}{|l_{BQ}|} = \frac{\beta V_t}{|l_{CQ}|}
\]

\[
g_m = \frac{|l_{CQ}|}{V_t}, \quad r_\pi = \frac{1}{g_\pi} = \frac{V_t}{|l_{BQ}|} = \frac{\beta V_t}{|l_{CQ}|}
\]

Both are equivalent

EE330, Spring 2014, Lecture # 32
Changing the small-signal gain can be done through changing the $g_m$ of the MOSFET transistor.

$A_V = (-g_m \times R)$

How can we change $g_m$?
Insights into Small-Signal Parameters of Transistors

◆ $g_m$ of the MOSFET transistor can be represented in 3 different ways

$$g_m = \sqrt{2\mu C_{OX} \frac{W}{L} \times I_{DQ}}$$  
Function of the square root of drain current and transistor size

$$g_m = \frac{2I_{DQ}}{(V_{GSQ} - V_T)}$$  
Function of the drain current and $V_{GS}$

$$g_m = \mu C_{OX} \frac{W}{L} (V_{GSQ} - V_T)$$  
Function of $V_{GS}$ and transistor size
Insights into Small-Signal Parameters of Transistors

- To make the right judgment, we must use the representation of $g_m$ as a function of independent parameters only (meaning that changing one parameters does not force the other parameters to change)

\[
g_m = \sqrt{2\mu C_{OX} \frac{W}{L} \times I_{DQ}}
\]

If the drain current and transistor size are the independent parameters while $V_{GS}$ is the dependent parameter

\[
g_m = \frac{2I_{DQ}}{(V_{GSQ} - V_T)}
\]

If the drain current and $V_{GS}$ are the independent parameters while transistor size is the dependent parameter \(\rightarrow\) unusual

\[
g_m = \mu C_{OX} \frac{W}{L} (V_{GSQ} - V_T)
\]

If $V_{GS}$ and transistor size are the independent parameters while drain current is the dependent parameter

- The first and last representations are the most common ones
Insights into Small-Signal Parameters of Transistors

◆ How do the small-signal parameters of BJT compare to a MOSFET → let’s start with $g_m$

$$A_V = (-g_m \times R)$$

$$A_V = -\left(\sqrt{2\mu C_{OX} \frac{W}{L} \times I_{DQ}}\right) \times R$$

$$A_V = -\left(\frac{I_{CQ}}{V_t}\right) \times R$$
Insights into Small-Signal Parameters of Transistors

To be able to compare, we must have basis for comparison → Bias current is a good criterion because it determines power consumption

\[ A_V = (-g_m \times R) \]

\[ A_V = -\left( \sqrt{2 \mu C_{OX} \frac{W}{L} \times I_{DQ}} \right) \times R \]
Insights into Small-Signal Parameters of Transistors

- For the same bias current, the BJT features much larger $g_m$ than a MOSFET counterpart → This is because the slope of an exponential function is much higher than the slope of a square-law function
- Note also that in order to double the $g_m$ of the MOS transistor, the drain current must be quadrupled, while doubling $g_m$ of the BJT transistor requires only doubling the collector current
- The net result is that you can get much higher gain from the BJT transistor than what you can get from the MOS transistor for the same amount of power consumption → BJT has superior $g_m$

\[
g_{m-MOS} = \sqrt{2\mu C_{OX} \frac{W}{L} \times I_{DQ}}
\]

\[
g_{m-BJT} = \frac{I_{cQ}}{V_t}
\]
Insights into Small-Signal Parameters of Transistors

◆ How do the small-signal parameters of BJT compare to a MOSFET → how about $r_o$?

$$r_{o-MOS} = \frac{1}{\lambda I_{DQ}}$$
$$r_{o-BJT} = \frac{V_{AF}}{I_{CQ}}$$

$$\frac{r_{o-BJT}}{r_{o-MOS}} = \lambda V_{AF} \approx 0.01V^{-1} \times 1200V = 2$$

◆ For the same amount of current, the BJT features slightly better output impedance → BJT and MOS have comparable output impedance
Insights into Small-Signal Parameters of Transistors

◆ How do the small-signal parameters of BJT compare to a MOSFET → how about $r_{\pi}$?

$$r_{\pi - MOS} = \infty$$

$$r_{o - BJT} = \frac{\beta V_t}{I_{CQ}}$$

◆ The MOS has superior input impedance
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

- We can understand better the role of the quiescent point by looking at the problem graphically.

\[ I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{ss} + V_T)^2 \]

\[ V_{OUTQ} = V_{DD} - I_{DQ} R = V_{DD} - \frac{\mu C_{ox} W}{2L} \left[ V_{ss} + V_T \right]^2 R \]
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

Load Line \( \rightarrow V_{\text{OUTQ}} = V_{\text{DD}} - I_{\text{DQ}} R \)

Device Current at operating point \( \rightarrow I_{\text{DQ}} = \frac{\mu C_{\text{OX}} W}{2L} (V_{\text{SS}} + V_T)^2 \)
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

\[ I_{\text{DQ}} = \mu \frac{C_{\text{OX}} W}{2L} (V_{\text{SS}} + V_T)^2, \quad V_{\text{GSQ}} = -V_{\text{SS}} \]

\[ V_{\text{OUTQ}} = V_{\text{DD}} - I_{\text{DQ}} R \]
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

As the input increases, we must maintain saturation at all times, but that does not mean we can allow the signal to swing the entire region because distortion will occur.

As the input decreases, the saturation region is depicted.
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

- Linear signal swing region smaller than saturation region
- Modest nonlinear distortion provided saturation region operation maintained
- Symmetric swing about Q-point
- Signal swing can be maximized by judicious location of Q-point
Insights into Small-Signal Parameters of Transistors: The Role of the Quiescent Point

Very limited signal swing with non-optimal Q-point location
Signal swing can be maximized by judicious location of Q-point

Often selected to be at middle of load line in saturation region
Not convenient to have multiple dc power supplies and $V_{OUTQ}$ is very sensitive to $V_{EE}$

This is a better way to do it, and does produce identical small-signal model as the original $\rightarrow$ verify
Since the Thevenin equivalent circuit in red circle is $V_{IN}$, both circuits have same small-signal model.
Determine $V_{OUTQ}$, $A_V$, $r_{in}$

Determine $V_{OUT}$ and $V_{OUT}(t)$ if $V_{IN} = 0.002\sin(400t)$

assume $\beta = 100$
Miscellaneous Examples

Biasing Components: $C$, $R_B$, $V_{CC}$, in this case, all disappear in small-signal gain circuit.

Biasing Circuit

- $V_{IN}(t)$
- $R_B = 500K$
- $C = 1\mu F$
- $R_1 = 2K$
- $V_{CC} = 12V$
- $Q_1$
- $V_{OUT}$
**Miscellaneous Examples**

\[ V_{CC} = 12V \]

\[ R_B = 500K \]

\[ R_1 = 2K \]

\[ C = 1 \mu F \]

\[ \beta = 100 \]

\[ V_{IN(t)} \]

\[ V_{OUT} \]

\[ \beta \]

\[ I_B \]

\[ R_{B1} = 500K \]

\[ R_2 = 2K \]

\[ V_{OUTQ} \]

\[ I_{CQ} = \beta I_{BQ} = 100 \left( \frac{12V - 0.6V}{500K} \right) = 2.3mA \]

\[ V_{OUTQ} = 12V - I_{CQ} R_1 = 12V - 2.3mA \cdot 2K = 7.4V \]
**Miscellaneous Examples**

- $V_{CC} = 12V$
- $R_B = 500K$
- $R_1 = 2K$
- $C = 1\mu F$
- $\beta = 100$

**SS equivalent circuit**

\[
\begin{align*}
V_{OUT} &= -g_m v_{BE} R_1 \\
V_{IN} &= v_{BE} \\
A_V &= -R_1 g_m \\
A_V &\approx -\frac{I_{CQ} R_1}{V_t} \\
&\approx -\frac{2.3mA \cdot 2K}{26mV} \approx -177
\end{align*}
\]
Miscellaneous Examples

V_\text{CC}=12V

R_B=500K

R_1=2K

C=1\mu F

\beta=100

ss equivalent circuit

r_{in}=\frac{v_{IN}}{i_{IN}}

Usually R_B >> r_\pi

r_{in} = R_B / / r_\pi \simeq r_\pi

r_{in} \simeq r_\pi = \frac{I_{CQ}}{\beta V_t}
Determine $V_{OUT}$ and $V_{OUT}(t)$ if $V_{IN} = 0.002\sin(400t)$

$$V_{OUT} \approx V_{OUTQ} + A_V V_{IN}$$

$$V_{OUT} \approx 7.4V - 177 \cdot 0.002 \cdot \sin(400t)$$

$$V_{OUT} \approx 7.4V - 35 \cdot \sin(400t)$$