CE 203 Civil Engineering Synthesis I

Chapter 3

INTEREST AND EQUIVALENCE

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Economic Decision Components

- Where economic decisions are immediate we need to consider:
  - amount of expenditure
  - taxes
- Where economic decisions occur over a considerable period of time we also need to consider:
  - interest
  - inflation

Computing Cash Flows

- Cash flows have:
  - Costs (disbursements) > a negative number
  - Benefits (receipts) > a positive number

Time Value of Money

- Money has value
  - Money can be leased or rented
  - The payment is called interest
  - If you put $100 in a bank at 9% interest for one time period you will receive back your original $100 plus $9

Original amount to be returned = $100
Interest to be returned = $100 x .09 = $9

Simple Interest

- Interest that is computed only on the original sum or principal
- Total interest earned = $I = P \times i \times n$
  - Where
    - P – present sum of money
    - i – interest rate
    - n – number of periods (years)

I = $100 x .09/period x 2 periods = $18

Future Value of a Loan with Simple Interest

- Amount of money due at the end of a loan
  - $F = P + P\,i\,n$ or $F = P\,(1 + i\,n)$
  - Where
    - F = future value

• Would you accept payment with simple interest terms?
• Would a bank?

Compound Interest

- Interest that is computed on the original unpaid debt and the unpaid interest
- Total interest earned = $I_n = P\,(1+i)^n - P$
  - Where
    - P – present sum of money
    - i – interest rate
    - n – number of periods (years)

$I_2 = \$100 \times (1+.09)^2 - \$100 = \$18.81$

Future Value of a Loan with Compound Interest

- Amount of money due at the end of a loan
  - $F = P(1+i)_1(1+i)_2.....(1+i)_n$ or $F = P(1 + i)^n$
  - Where
    - F = future value

$$F = \$100\ (1 + .09)^2 = \$118.81$$

• Would you be more likely to accept payment with compound interest terms?
• Would a bank?

Comparison of Simple and Compound Interest Over Time

- If you loaned a friend money for short period of time the difference between simple and compound interest is negligible.
- If you loaned a friend money for a long period of time the difference between simple and compound interest may amount to a considerable difference.

Check the table to see the difference over time.

Short or long? When is the $ difference significant?
You pick the time period.

Simple and compound interest
Single payment

Principal =	100.00
Interest =	9.00%

Period	Simple amount factor Find Fs Given P	Compound amount factor Find F Given P
n	Fs/P	F/P
0	100.000	100.000
1	109.000	109.000
2	118.000	118.810
3	127.000	129.503
4	136.000	141.158
5	145.000	153.862
6	154.000	167.710
7	163.000	182.804
8	172.000	199.256
9	181.000	217.189
10	190.000	236.736
11	199.000	258.043
12	208.000	281.266
13	217.000	306.580
14	226.000	334.173
15	235.000	364.248
16	244.000	397.031
17	253.000	432.763
18	262.000	471.712
19	271.000	514.166
20	280.000	560.441

Four Ways to Repay a Debt

Plan	Repay Principal	Repay Interest	Interest Earned
1	Equal annual installments	Interest on unpaid balance	Declines
2	End of loan	Interest on unpaid balance	Constant
3	Equal annual installments		Declines at increasing rate
4	End of loan	Compound and pay at end of loan	Compounds at increasing rate until end of loan

Equivalence

- When an organization is indifferent as to whether it has a present sum of money now or the assurance of some other sum of money (or series of sums of money) in the future, we say that the present sum of money is *equivalent* to the future sum or series of sums.

Given the choice of these two plans which would you choose?

Year	Plan 1	Plan 2
1	$1400	$400
2	1320	400
3	1240	400
4	1160	400
5	1080	5400
Total	$6200	$7000

To make a choice the cash flows must be altered so a comparison may be made.

Technique of Equivalence

- Determine a single equivalent value at a point in time for plan 1.
- Determine a single equivalent value at a point in time for plan 2.

Both at the same interest rate.

•Judge the relative attractiveness of the two alternatives from the comparable equivalent values.

Single payment compound interest formula